

# Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.6-Inverse-hyperbolic-cosecant/7.6.1-u-a+b-arccsch-c-x-^n

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 178 ]. This is test number [ 202 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$  functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 178 )	% 0.00 ( 0 )
Mathematica	% 97.19 ( 173 )	% 2.81 ( 5 )
Maple	% 56.18 ( 100 )	% 43.82 ( 78 )
Maxima	% 47.19 ( 84 )	% 52.81 ( 94 )
Fricas	% 63.48 ( 113 )	% 36.52 ( 65 )
Sympy	% 17.98 ( 32 )	% 82.02 ( 146 )
Giac	% 25.84 ( 46 )	% 74.16 ( 132 )
Mupad	% 27.53 ( 49 )	% 72.47 ( 129 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

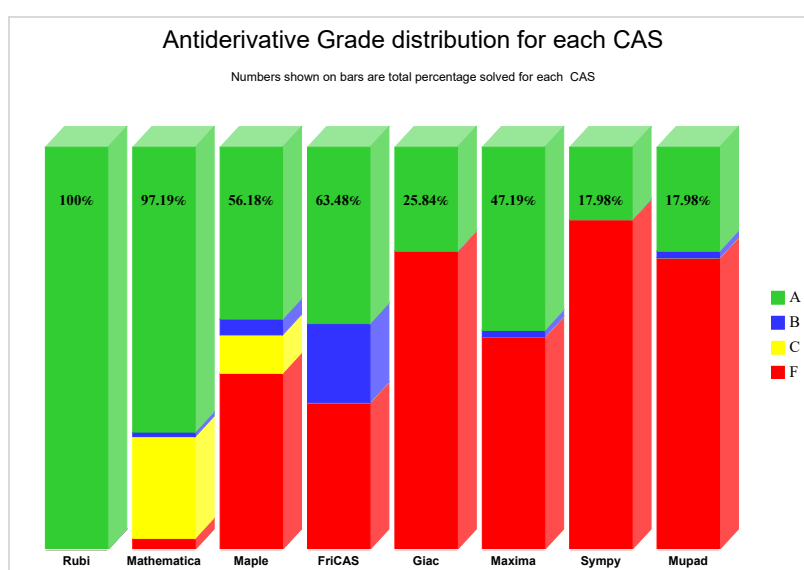
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

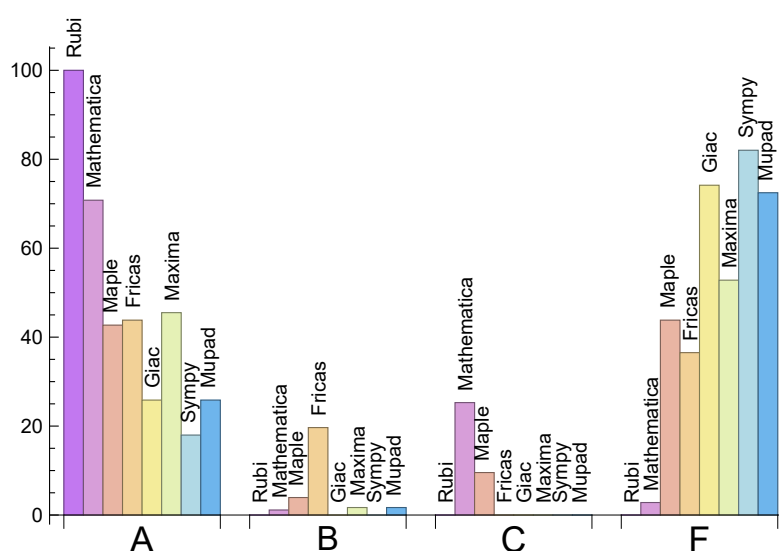
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	70.79	1.12	25.28	2.81
Maple	42.70	3.93	9.55	43.82
Maxima	45.51	1.69	0.00	52.81
Fricas	43.82	19.66	0.00	36.52
Sympy	17.98	0.00	0.00	82.02
Giac	25.84	0.00	0.00	74.16
Mupad	25.84	1.69	0.00	72.47

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	5	100.00 %	0.00 %	0.00 %
Maple	78	96.15 %	0.00 %	3.85 %
Maxima	94	94.68 %	4.26 %	1.06 %
Fricas	65	84.62 %	15.38 %	0.00 %
Sympy	146	65.75 %	34.25 %	0.00 %
Giac	132	98.48 %	0.00 %	1.52 %
Mupad	129	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

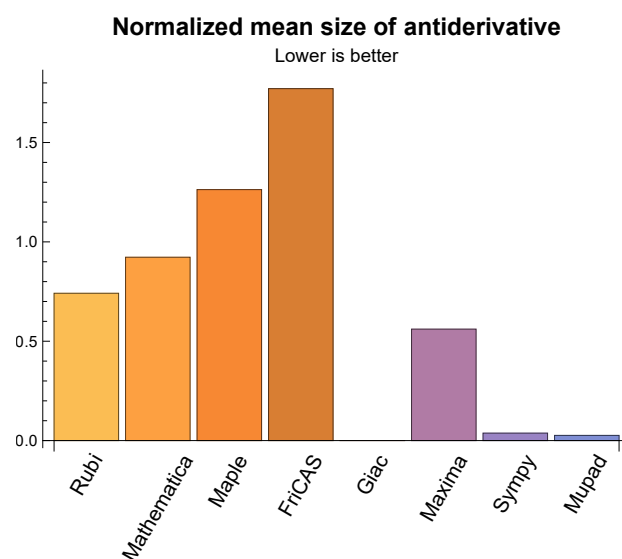
## 1.3 Performance

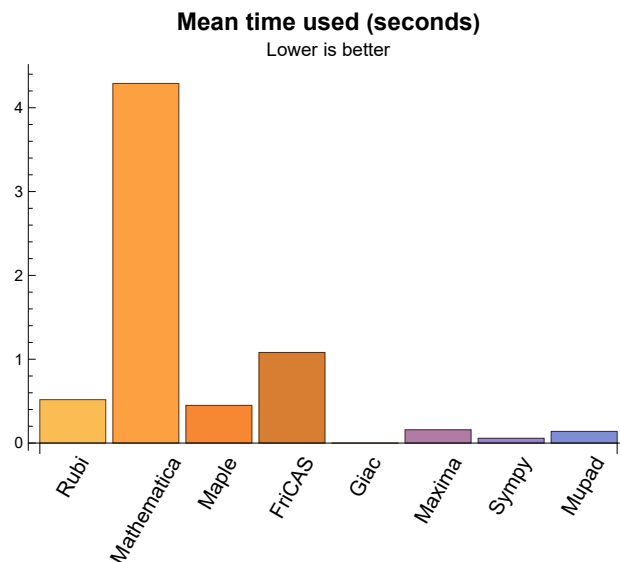
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.52	217.70	0.74	132.50	1.00
Mathematica	4.29	294.37	0.92	123.00	0.90
Maple	0.45	401.64	1.26	83.00	0.90
Maxima	0.16	65.42	0.56	0.00	0.00
Fricas	1.08	301.39	1.77	123.00	0.97
Sympy	0.06	1.13	0.04	0.00	0.00
Giac	0.00	0.00	0.00	0.00	0.00
Mupad	0.14	1.61	0.03	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





## 1.4 list of integrals that has no closed form antiderivative

{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178}

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {8, 99, 103, 104, 111}

Mathematica {8, 16, 18, 19, 24, 25, 26, 27, 28, 48, 51, 57, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 115, 116, 117, 118, 128}

Maple Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```



```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

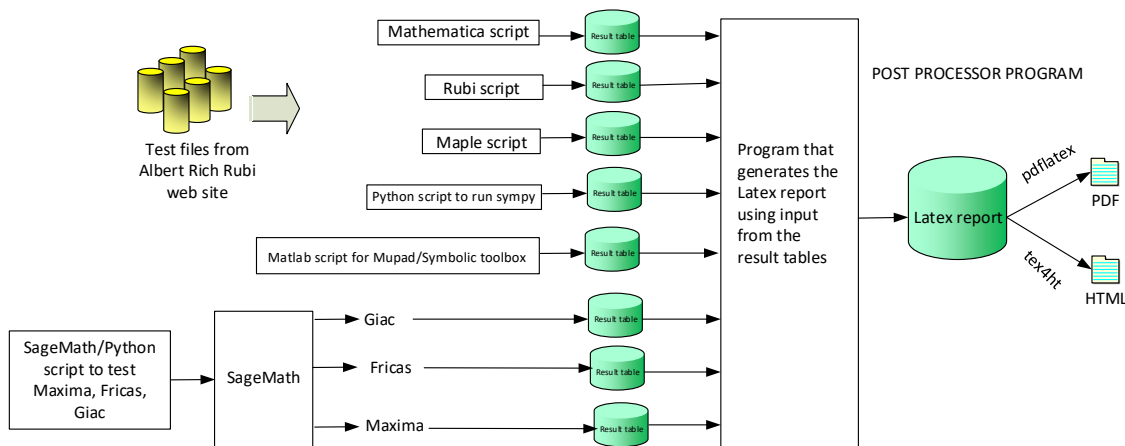
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"  
*The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.  
*The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { }

C grade: { }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { 25, 27 }

C grade: { 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 126, 127, 128, 136, 137, 146, 155, 164 }

F grade: { 106, 114, 165, 166, 167 }

#### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 54, 55, 61, 62, 67, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 9, 10, 49, 50, 105, 112, 113 }

C grade: { 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75 }

F grade: { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 68, 73, 74, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 17, 20, 29, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 10, 12, 14 }

C grade: { }

F grade: { 8, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.1.5 FriCAS

A grade: { 2, 4, 10, 11, 12, 13, 14, 23, 32, 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 80, 81, 82, 83, 84, 85, 88, 92, 93, 94, 95, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 141, 142, 143, 144, 147, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178 }

B grade: { 1, 3, 5, 6, 7, 9, 15, 17, 20, 21, 22, 29, 30, 31, 44, 45, 46, 47, 49, 50, 78, 79, 89, 90, 91, 105, 112, 113, 140, 148, 149, 156, 157, 158, 176 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 126, 127, 136, 137, 145, 146, 154, 155, 163, 164, 165, 166, 167 }

## 2.1.6 Sympy

A grade: { 9, 33, 34, 35, 39, 40, 42, 43, 61, 62, 67, 68, 121, 122, 123, 124, 125, 130, 131, 133, 134, 135, 141, 142, 143, 144, 153, 168, 171, 172, 177, 178 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 132, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 173, 174, 175, 176 }

## 2.1.7 Giac

A grade: { 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.1.8 Mupad

A grade: { 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 6, 9, 10 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	107	127	158	208	0	0	-1
normalized size	1	1.00	0.97	1.15	1.44	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.143	0.085	0.358	0.801	0.000	0.000	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	72	83	77	97	0	0	-1
normalized size	1	1.00	0.84	0.97	0.90	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.135	0.051	0.309	2.011	0.000	0.000	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	97	108	128	199	0	0	-1
normalized size	1	1.00	1.13	1.26	1.49	2.31	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.048	0.049	0.369	0.733	0.000	0.000	0.000
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	74	57	87	0	0	-1
normalized size	1	1.00	1.00	1.19	0.92	1.40	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.102	0.045	0.316	3.314	0.000	0.000	0.000
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	85	88	96	186	0	0	-1
normalized size	1	1.00	1.37	1.42	1.55	3.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.052	0.050	0.344	1.575	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	50	65	35	70	0	0	39
normalized size	1	1.00	1.32	1.71	0.92	1.84	0.00	0.00	1.03
time (sec)	N/A	0.013	0.025	0.045	0.375	0.736	0.000	0.000	2.217
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	44	36	49	143	0	0	-1
normalized size	1	1.00	1.47	1.20	1.63	4.77	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.060	0.043	0.328	0.725	0.000	0.000	0.000
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	47	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	0.045	0.068	0.000	2.073	0.000	0.000	0.000
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	40	62	32	64	36	0	35
normalized size	1	1.00	1.33	2.07	1.07	2.13	1.20	0.00	1.17
time (sec)	N/A	0.024	0.031	0.046	0.351	0.775	1.823	0.000	2.323
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	66	100	105	76	0	0	51
normalized size	1	1.00	1.32	2.00	2.10	1.52	0.00	0.00	1.02
time (sec)	N/A	0.037	0.036	0.048	0.389	0.980	0.000	0.000	2.265
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	75	56	77	0	0	-1
normalized size	1	1.00	1.02	1.29	0.97	1.33	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.047	0.049	0.370	1.004	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	120	147	89	0	0	-1
normalized size	1	1.00	1.05	1.62	1.99	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.043	0.046	0.329	1.025	0.000	0.000	0.000
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	69	83	73	87	0	0	-1
normalized size	1	1.00	0.87	1.05	0.92	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.063	0.046	0.312	0.484	0.000	0.000	0.000
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	139	185	99	0	0	-1
normalized size	1	1.00	0.90	1.42	1.89	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.074	0.056	0.307	1.122	0.000	0.000	0.000
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	122	0	0	272	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.211	0.052	0.000	1.337	0.000	0.000	0.000
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	211	0	0	0	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.427	0.052	0.000	1.370	0.000	0.000	0.000
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	87	0	82	234	0	0	-1
normalized size	1	1.00	1.61	0.00	1.52	4.33	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.143	0.055	0.360	0.783	0.000	0.000	0.000



Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	121	0	0	0	0	0	-1
normalized size	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.236	0.050	0.000	2.257	0.000	0.000	0.000
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0	-1
normalized size	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.133	0.053	0.000	0.529	0.000	0.000	0.000
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	0	78	139	0	0	-1
normalized size	1	1.00	1.43	0.00	1.59	2.84	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.168	0.066	0.334	1.075	0.000	0.000	0.000
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	163	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.143	0.056	0.000	0.668	0.000	0.000	0.000
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	106	0	0	178	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.194	0.047	0.000	0.819	0.000	0.000	0.000
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	147	0	0	202	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.181	0.051	0.000	0.829	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	271	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	1.019	0.050	0.000	0.627	0.000	0.000	0.000
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	548	0	0	0	0	0	-1
normalized size	1	1.00	2.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	7.520	0.049	0.000	1.124	0.000	0.000	0.000
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	171	0	0	0	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.471	0.049	0.000	1.391	0.000	0.000	0.000
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	246	0	0	0	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.341	0.049	0.000	2.955	0.000	0.000	0.000
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	198	0	0	0	0	0	-1
normalized size	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.227	0.055	0.000	0.410	0.000	0.000	0.000
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	132	0	144	222	0	0	-1
normalized size	1	1.00	1.69	0.00	1.85	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.249	0.052	0.353	0.826	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	182	0	0	267	0	0	-1
normalized size	1	1.00	1.48	0.00	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.326	0.052	0.000	0.404	0.000	0.000	0.000
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	200	0	0	301	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.314	0.053	0.000	0.768	0.000	0.000	0.000
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	277	0	0	346	0	0	-1
normalized size	1	1.00	1.36	0.00	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.181	0.368	0.053	0.000	0.982	0.000	0.000	0.000
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.015	2.931	0.053	0.000	0.878	0.000	0.000	0.000
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	2.821	0.054	0.000	0.975	0.000	0.000	0.000
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.026	0.307	0.062	0.000	0.901	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.076	0.049	0.000	1.590	0.000	0.000	0.000
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.141	0.076	0.048	0.000	2.149	0.000	0.000	0.000
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.165	0.052	0.000	2.138	0.000	0.000	0.000
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.024	6.870	0.049	0.000	2.686	0.000	0.000	0.000
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	4.489	0.050	0.000	0.782	0.000	0.000	0.000
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	0	0	0	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.243	0.049	0.000	0.830	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	0.931	0.048	0.000	2.983	0.000	0.000	0.000
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	1.899	0.049	0.000	1.359	0.000	0.000	0.000
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	165	269	261	419	0	0	-1
normalized size	1	1.00	0.99	1.61	1.56	2.51	0.00	0.00	-0.01
time (sec)	N/A	0.384	0.294	0.059	0.322	1.161	0.000	0.000	0.000
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	204	192	328	0	0	-1
normalized size	1	1.00	1.00	1.67	1.57	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.259	0.191	0.053	0.340	0.996	0.000	0.000	0.000
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	99	115	87	207	0	0	-1
normalized size	1	1.00	1.22	1.42	1.07	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.203	0.051	0.390	1.696	0.000	0.000	0.000
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	44	36	49	143	0	0	-1
normalized size	1	1.00	1.47	1.20	1.63	4.77	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.050	0.046	0.307	0.783	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	506	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.664	0.492	0.000	0.682	0.000	0.000	0.000
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	134	208	0	354	0	0	-1
normalized size	1	1.00	1.37	2.12	0.00	3.61	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.219	0.087	0.000	0.610	0.000	0.000	0.000
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	204	963	0	745	0	0	-1
normalized size	1	1.00	1.25	5.91	0.00	4.57	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.478	0.070	0.000	1.337	0.000	0.000	0.000
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	918	918	1094	2515	0	0	0	0	-1
normalized size	1	1.00	1.19	2.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.253	14.395	0.357	0.000	0.000	0.000	0.000	0.000
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	418	1964	0	0	0	0	-1
normalized size	1	1.00	0.62	2.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.519	1.678	0.085	0.000	0.000	0.000	0.000	0.000
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	926	840	0	0	0	0	-1
normalized size	1	1.00	2.16	1.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.708	14.077	0.079	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	19.511	6.871	0.000	0.573	0.000	0.000	0.000
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.079	7.967	5.921	0.000	0.924	0.000	0.000	0.000
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	380	1939	0	0	0	0	-1
normalized size	1	1.00	0.78	3.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.023	1.607	0.068	0.000	11.811	0.000	0.000	0.000
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	939	939	1098	2543	0	0	0	0	-1
normalized size	1	1.00	1.17	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.871	14.302	0.078	0.000	15.322	0.000	0.000	0.000
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	707	707	1012	1991	0	0	0	0	-1
normalized size	1	1.00	1.43	2.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.142	14.426	0.073	0.000	0.000	0.000	0.000	0.000
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	343	868	0	0	0	0	-1
normalized size	1	1.00	0.72	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.751	1.335	0.071	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	307	395	0	0	0	0	-1
normalized size	1	1.00	1.08	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.414	5.224	0.066	0.000	0.000	0.000	0.000	0.000
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.077	6.551	1.532	0.000	1.122	0.000	0.000	0.000
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.082	9.214	5.263	0.000	0.833	0.000	0.000	0.000
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	1042	2019	0	0	0	0	-1
normalized size	1	1.00	1.43	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.636	14.699	0.075	0.000	16.601	0.000	0.000	0.000
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	979	896	0	0	0	0	-1
normalized size	1	1.00	1.96	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.010	14.494	0.073	0.000	0.000	0.000	0.000	0.000
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	264	418	0	0	0	0	-1
normalized size	1	1.00	0.83	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.691	1.585	0.071	0.000	0.000	0.000	0.000	0.000



Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	166	328	0	0	0	0	-1
normalized size	1	1.00	1.11	2.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.284	0.682	0.072	0.000	9.003	0.000	0.000	0.000
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.090	12.169	6.987	0.000	0.809	0.000	0.000	0.000
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.099	15.678	180.000	0.000	1.347	0.000	0.000	0.000
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	777	777	1108	2726	0	0	0	0	-1
normalized size	1	1.00	1.43	3.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.186	14.487	0.089	0.000	16.418	0.000	0.000	0.000
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	1076	2497	0	0	0	0	-1
normalized size	1	1.00	1.89	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.497	14.383	0.084	0.000	0.000	0.000	0.000	0.000
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	390	2107	0	0	0	0	-1
normalized size	1	1.00	0.99	5.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.191	2.514	0.075	0.000	8.178	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	892	2079	0	0	0	0	-1
normalized size	1	1.00	2.42	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.567	14.102	0.073	0.000	0.000	0.000	0.000	0.000
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.094	28.845	180.000	0.000	3.642	0.000	0.000	0.000
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.106	28.471	180.000	0.000	2.038	0.000	0.000	0.000
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	648	785	1217	3782	0	0	0	0	-1
normalized size	1	1.21	1.88	5.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.035	14.494	0.107	0.000	9.390	0.000	0.000	0.000
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	138	211	289	295	0	0	-1
normalized size	1	1.00	0.64	0.99	1.35	1.38	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.235	0.051	0.372	1.951	0.000	0.000	0.000
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	119	171	227	273	0	0	-1
normalized size	1	1.00	0.71	1.02	1.36	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.170	0.051	0.380	3.015	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	135	126	148	245	0	0	-1
normalized size	1	1.00	1.17	1.10	1.29	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.261	0.056	0.348	1.012	0.000	0.000	0.000
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	89	107	84	222	0	0	-1
normalized size	1	1.00	0.98	1.18	0.92	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.132	0.066	0.369	0.799	0.000	0.000	0.000
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	122	91	105	0	0	-1
normalized size	1	1.00	0.62	1.12	0.83	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.084	0.069	0.350	0.946	0.000	0.000	0.000
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	93	140	132	127	0	0	-1
normalized size	1	1.00	0.59	0.89	0.84	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.124	0.065	0.315	0.950	0.000	0.000	0.000
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	109	158	165	146	0	0	-1
normalized size	1	1.00	0.53	0.77	0.80	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.122	0.146	0.058	0.326	1.045	0.000	0.000	0.000
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	114	152	176	165	0	0	-1
normalized size	1	1.00	0.56	0.75	0.86	0.81	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.249	0.055	0.336	0.822	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	97	134	137	144	0	0	-1
normalized size	1	1.00	0.61	0.84	0.86	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.211	0.050	0.315	0.888	0.000	0.000	0.000
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	77	115	95	123	0	0	-1
normalized size	1	1.00	0.53	0.79	0.65	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.101	0.049	0.343	1.126	0.000	0.000	0.000
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	93	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.291	0.141	0.106	0.000	0.952	0.000	0.000	0.000
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	138	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.567	0.107	0.000	2.671	0.000	0.000	0.000
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	182	286	396	388	0	0	-1
normalized size	1	1.00	0.70	1.10	1.52	1.49	0.00	0.00	-0.00
time (sec)	N/A	0.258	0.357	0.060	0.334	1.327	0.000	0.000	0.000
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	149	217	287	353	0	0	-1
normalized size	1	1.00	0.76	1.10	1.46	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.228	0.049	0.356	1.152	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	134	189	191	347	0	0	-1
normalized size	1	1.00	0.79	1.11	1.12	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.218	0.064	0.408	1.216	0.000	0.000	0.000
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	123	190	152	334	0	0	-1
normalized size	1	1.00	0.75	1.16	0.93	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.270	0.065	0.327	0.919	0.000	0.000	0.000
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	126	191	175	165	0	0	-1
normalized size	1	1.00	0.67	1.01	0.93	0.87	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.225	0.060	0.359	0.634	0.000	0.000	0.000
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	152	223	232	197	0	0	-1
normalized size	1	1.00	0.61	0.90	0.93	0.79	0.00	0.00	-0.00
time (sec)	N/A	0.197	0.262	0.062	0.450	1.008	0.000	0.000	0.000
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	159	214	244	224	0	0	-1
normalized size	1	1.00	0.64	0.86	0.98	0.90	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.388	0.053	0.375	1.249	0.000	0.000	0.000
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	123	182	183	189	0	0	-1
normalized size	1	1.00	0.61	0.90	0.90	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.311	0.050	0.368	1.163	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	148	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.421	0.423	0.427	0.000	0.947	0.000	0.000	0.000
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	187	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.424	0.907	0.457	0.000	0.663	0.000	0.000	0.000
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	1221	0	0	0	0	0	-1
normalized size	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.198	1.666	3.395	0.000	0.614	0.000	0.000	0.000
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	467	449	1103	0	0	0	0	0	-1
normalized size	1	0.96	2.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.106	0.351	0.462	0.000	0.945	0.000	0.000	0.000
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	1055	0	0	0	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.849	0.508	1.453	0.000	0.805	0.000	0.000	0.000
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	387	0	0	0	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	0.978	0.444	0.000	0.715	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	1211	0	0	0	0	0	-1
normalized size	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.060	1.687	2.759	0.000	1.042	0.000	0.000	0.000
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	591	571	1447	0	0	0	0	0	-1
normalized size	1	0.97	2.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.282	6.365	0.444	0.000	0.565	0.000	0.000	0.000
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	553	535	1410	0	0	0	0	0	-1
normalized size	1	0.97	2.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.249	2.414	0.440	0.000	1.947	0.000	0.000	0.000
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	271	360	0	615	0	0	-1
normalized size	1	1.00	1.95	2.59	0.00	4.42	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.792	0.074	0.000	0.943	0.000	0.000	0.000
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.153	44.155	0.482	0.000	1.670	0.000	0.000	0.000
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	756	756	1583	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.468	6.172	15.131	0.000	0.854	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	719	719	1442	0	0	0	0	0	-1
normalized size	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.242	2.736	3.510	0.000	0.572	0.000	0.000	0.000
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	713	713	1437	0	0	0	0	0	-1
normalized size	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.187	3.648	4.649	0.000	0.814	0.000	0.000	0.000
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	758	758	1487	0	0	0	0	0	-1
normalized size	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.246	3.119	14.660	0.000	2.023	0.000	0.000	0.000
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	694	676	2023	0	0	0	0	0	-1
normalized size	1	0.97	2.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.408	7.854	0.490	0.000	2.920	0.000	0.000	0.000
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	375	1922	0	1381	0	0	-1
normalized size	1	1.00	2.25	11.51	0.00	8.27	0.00	0.00	-0.01
time (sec)	N/A	0.215	1.476	0.094	0.000	3.046	0.000	0.000	0.000
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	368	1892	0	1256	0	0	-1
normalized size	1	1.00	1.80	9.23	0.00	6.13	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.957	0.079	0.000	1.430	0.000	0.000	0.000



Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	657	657	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.322	70.345	0.444	0.000	0.841	0.000	0.000	0.000
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1106	1106	2045	0	0	0	0	0	-1
normalized size	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.741	6.214	3.803	0.000	0.956	0.000	0.000	0.000
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1106	1106	2053	0	0	0	0	0	-1
normalized size	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.046	6.148	5.009	0.000	0.657	0.000	0.000	0.000
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1096	1096	2038	0	0	0	0	0	-1
normalized size	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.743	6.066	3.774	0.000	0.876	0.000	0.000	0.000
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	345	0	0	1951	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	4.72	0.00	0.00	-0.00
time (sec)	N/A	1.412	0.705	0.434	0.000	5.103	0.000	0.000	0.000
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	337	0	0	1625	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	5.38	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.752	0.440	0.000	1.798	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	278	0	0	1342	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	6.61	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.511	0.479	0.000	1.105	0.000	0.000	0.000
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.100	5.625	0.449	0.000	0.524	0.000	0.000	0.000
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.106	5.399	0.450	0.000	0.644	0.000	0.000	0.000
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.101	9.309	0.426	0.000	0.656	0.000	0.000	0.000
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	2.656	0.422	0.000	0.426	0.000	0.000	0.000
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.090	1.779	0.455	0.000	0.531	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	237	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.427	0.619	0.438	0.000	0.656	0.000	0.000	0.000
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	314	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	0.696	0.434	0.000	0.714	0.000	0.000	0.000
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	318	0	0	1943	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	5.06	0.00	0.00	-0.00
time (sec)	N/A	0.514	0.769	0.479	0.000	3.407	0.000	0.000	0.000
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	314	0	0	1625	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	6.02	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.798	0.431	0.000	2.007	0.000	0.000	0.000
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.117	6.559	0.433	0.000	0.653	0.000	0.000	0.000
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.123	5.860	0.420	0.000	0.727	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.120	9.669	0.613	0.000	0.704	0.000	0.000	0.000
Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	3.523	0.440	0.000	0.426	0.000	0.000	0.000
Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.106	5.794	0.447	0.000	0.651	0.000	0.000	0.000
Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.108	15.969	0.446	0.000	0.594	0.000	0.000	0.000
Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	291	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.575	0.703	0.427	0.000	0.580	0.000	0.000	0.000
Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	372	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.866	0.828	0.463	0.000	0.630	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	339	0	0	1633	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	4.96	0.00	0.00	-0.00
time (sec)	N/A	1.179	0.789	0.449	0.000	1.824	0.000	0.000	0.000
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	280	0	0	1341	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	5.86	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.565	0.441	0.000	1.023	0.000	0.000	0.000
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	223	0	0	1064	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	7.88	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.240	0.433	0.000	0.689	0.000	0.000	0.000
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.096	1.745	0.422	0.000	0.665	0.000	0.000	0.000
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.108	23.255	0.463	0.000	0.440	0.000	0.000	0.000
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.092	6.333	0.437	0.000	0.497	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	1.046	0.433	0.000	0.657	0.000	0.000	0.000
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	139	0	0	0	0	0	-1
normalized size	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.207	0.506	0.000	0.613	0.000	0.000	0.000
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	239	0	0	0	0	0	-1
normalized size	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	0.636	0.493	0.000	0.900	0.000	0.000	0.000
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	311	0	0	1719	0	0	-1
normalized size	1	1.00	1.21	0.00	0.00	6.71	0.00	0.00	-0.00
time (sec)	N/A	1.135	0.644	0.470	0.000	1.034	0.000	0.000	0.000
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	233	0	0	1274	0	0	-1
normalized size	1	1.00	1.46	0.00	0.00	7.96	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.375	0.437	0.000	0.840	0.000	0.000	0.000
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	122	0	0	368	0	0	-1
normalized size	1	1.00	1.49	0.00	0.00	4.49	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.165	0.443	0.000	0.690	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.121	32.960	0.443	0.000	0.676	0.000	0.000	0.000
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.133	38.594	0.457	0.000	0.563	0.000	0.000	0.000
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.111	8.805	0.439	0.000	0.614	0.000	0.000	0.000
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.107	5.019	0.429	0.000	0.500	0.000	0.000	0.000
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.197	0.459	0.000	0.993	0.000	0.000	0.000
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	201	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	0.457	0.448	0.000	0.612	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	327	0	0	2421	0	0	-1
normalized size	1	1.00	1.30	0.00	0.00	9.65	0.00	0.00	-0.00
time (sec)	N/A	1.229	0.562	0.440	0.000	1.106	0.000	0.000	0.000
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	201	0	0	786	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	4.65	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.298	0.452	0.000	0.938	0.000	0.000	0.000
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	185	0	0	698	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	4.85	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.234	0.430	0.000	0.658	0.000	0.000	0.000
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.124	45.844	0.502	0.000	0.694	0.000	0.000	0.000
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.152	59.836	0.509	0.000	0.761	0.000	0.000	0.000
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.132	13.403	0.477	0.000	0.789	0.000	0.000	0.000



Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.110	12.602	0.442	0.000	0.615	0.000	0.000	0.000
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	189	0	0	0	0	0	-1
normalized size	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.296	0.428	0.000	0.507	0.000	0.000	0.000
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	248	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.177	0.560	0.436	0.000	0.522	0.000	0.000	0.000
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	596	577	0	0	0	0	0	0	-1
normalized size	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.575	0.228	0.425	0.000	0.776	0.000	0.000	0.000
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	379	360	0	0	0	0	0	0	-1
normalized size	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.491	0.156	0.409	0.000	0.493	0.000	0.000	0.000
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	208	0	0	0	0	0	0	-1
normalized size	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	0.114	0.059	0.000	0.565	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.079	1.867	0.445	0.000	0.634	0.000	0.000	0.000
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	6.956	0.430	0.000	0.884	0.000	0.000	0.000
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.116	1.145	0.450	0.000	0.559	0.000	0.000	0.000
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.104	0.117	0.464	0.000	0.607	0.000	0.000	0.000
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.103	1.459	0.443	0.000	0.711	0.000	0.000	0.000
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.115	1.872	0.516	0.000	0.549	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	214	0	0	382	0	0	-1
normalized size	1	1.00	0.54	0.00	0.00	0.97	0.00	0.00	-0.00
time (sec)	N/A	2.344	0.314	0.547	0.000	0.609	0.000	0.000	0.000
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	180	0	0	324	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	1.916	0.418	0.477	0.000	0.590	0.000	0.000	0.000
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	133	141	0	0	265	0	0	-1
normalized size	1	1.02	1.08	0.00	0.00	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.337	0.464	0.000	0.680	0.000	0.000	0.000
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	0.411	0.446	0.000	0.455	0.000	0.000	0.000
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.104	3.487	0.456	0.000	0.747	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [69] had the largest ratio of [.8571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	12	0.417
2	A	4	3	1.00	12	0.250
3	A	6	5	1.00	12	0.417
4	A	3	3	1.00	12	0.250
5	A	5	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	5	4	1.00	8	0.500
8	A	6	6	1.00	12	0.500
9	A	2	2	1.00	12	0.167
10	A	4	4	1.00	12	0.333
11	A	4	3	1.00	12	0.250
12	A	5	4	1.00	12	0.333
13	A	4	3	1.00	12	0.250
14	A	6	4	1.00	12	0.333
15	A	5	5	1.00	14	0.357
16	A	8	6	1.00	14	0.429
17	A	4	4	1.00	12	0.333
18	A	7	5	1.00	10	0.500
19	A	6	6	1.00	14	0.429
20	A	4	3	1.00	14	0.214
21	A	4	3	1.00	14	0.214
22	A	5	5	1.00	14	0.357
23	A	5	3	1.00	14	0.214
24	A	10	10	1.00	14	0.714
25	A	11	8	1.00	14	0.571
26	A	7	7	1.00	12	0.583
27	A	9	6	1.00	10	0.600
28	A	7	7	1.00	14	0.500
29	A	5	3	1.00	14	0.214
30	A	6	6	1.00	14	0.429
31	A	8	6	1.00	14	0.429
32	A	10	6	1.00	14	0.429
33	A	0	0	0.00	0	0.000
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	4	4	1.00	14	0.286
37	A	6	6	1.00	14	0.429
38	A	9	5	1.00	14	0.357
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	3	3	1.00	14	0.214
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	11	9	1.00	16	0.562
45	A	10	9	1.00	16	0.562
46	A	9	9	1.00	14	0.643
47	A	5	4	1.00	8	0.500
48	A	4	2	1.00	16	0.125
49	A	7	7	1.00	16	0.438
50	A	8	8	1.00	16	0.500
51	A	31	16	1.00	21	0.762
52	A	24	15	1.00	19	0.790
53	A	15	11	1.00	18	0.611
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	22	13	1.00	18	0.722
57	A	27	17	1.00	21	0.810
58	A	20	15	1.00	21	0.714
59	A	14	13	1.00	19	0.684
60	A	9	9	1.00	18	0.500
61	A	0	0	0.00	0	0.000
62	A	0	0	0.00	0	0.000
63	A	23	15	1.00	21	0.714
64	A	16	13	1.00	21	0.619
65	A	11	11	1.00	19	0.579
66	A	6	6	1.00	18	0.333
67	A	0	0	0.00	0	0.000
68	A	0	0	0.00	0	0.000
69	A	31	18	1.00	21	0.857
70	A	25	17	1.00	21	0.810
71	A	19	14	1.00	19	0.737

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	12	11	1.00	18	0.611
73	A	0	0	0.00	0	0.000
74	A	0	0	0.00	0	0.000
75	A	19	14	1.21	18	0.778
76	A	7	7	1.00	19	0.368
77	A	6	7	1.00	19	0.368
78	A	5	5	1.00	16	0.312
79	A	4	5	1.00	19	0.263
80	A	4	5	1.00	19	0.263
81	A	5	6	1.00	19	0.316
82	A	6	6	1.00	19	0.316
83	A	5	5	1.00	19	0.263
84	A	5	5	1.00	19	0.263
85	A	6	5	1.00	17	0.294
86	A	11	11	1.00	19	0.579
87	A	13	13	1.00	19	0.684
88	A	7	8	1.00	21	0.381
89	A	6	7	1.00	18	0.389
90	A	6	7	1.00	21	0.333
91	A	6	7	1.00	21	0.333
92	A	5	6	1.00	21	0.286
93	A	6	7	1.00	21	0.333
94	A	5	6	1.00	21	0.286
95	A	6	5	1.00	19	0.263
96	A	12	13	1.00	21	0.619
97	A	14	15	1.00	21	0.714
98	A	25	12	1.00	21	0.571
99	A	26	9	0.96	19	0.474
100	A	19	7	1.00	18	0.389
101	A	19	7	1.00	21	0.333
102	A	24	10	1.00	21	0.476
103	A	31	14	0.97	21	0.667
104	A	29	12	0.97	21	0.571
105	A	7	6	1.00	19	0.316
106	A	24	10	1.00	21	0.476
107	A	51	15	1.00	21	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	27	10	1.00	21	0.476
109	A	47	11	1.00	18	0.611
110	A	50	13	1.00	21	0.619
111	A	33	13	0.97	21	0.619
112	A	6	7	1.00	21	0.333
113	A	8	7	1.00	19	0.368
114	A	28	11	1.00	21	0.524
115	A	35	11	1.00	21	0.524
116	A	63	12	1.00	21	0.571
117	A	81	12	1.00	18	0.667
118	A	12	12	1.00	23	0.522
119	A	11	12	1.00	23	0.522
120	A	9	9	1.00	21	0.429
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	0	0	0.00	0	0.000
124	A	0	0	0.00	0	0.000
125	A	0	0	0.00	0	0.000
126	A	8	9	1.00	23	0.391
127	A	9	10	1.00	23	0.435
128	A	12	12	1.00	23	0.522
129	A	10	10	1.00	21	0.476
130	A	0	0	0.00	0	0.000
131	A	0	0	0.00	0	0.000
132	A	0	0	0.00	0	0.000
133	A	0	0	0.00	0	0.000
134	A	0	0	0.00	0	0.000
135	A	0	0	0.00	0	0.000
136	A	9	10	1.00	23	0.435
137	A	10	10	1.00	23	0.435
138	A	11	12	1.00	23	0.522
139	A	10	12	1.00	23	0.522
140	A	8	8	1.00	21	0.381
141	A	0	0	0.00	0	0.000
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	0	0	0.00	0	0.000
145	A	8	9	1.00	23	0.391
146	A	8	10	1.00	23	0.435
147	A	10	11	1.00	23	0.478
148	A	9	11	1.00	23	0.478
149	A	4	4	1.00	21	0.190
150	A	0	0	0.00	0	0.000
151	A	0	0	0.00	0	0.000
152	A	0	0	0.00	0	0.000
153	A	0	0	0.00	0	0.000
154	A	3	4	1.00	20	0.200
155	A	7	9	1.00	23	0.391
156	A	10	11	1.00	23	0.478
157	A	7	8	1.00	23	0.348
158	A	5	5	1.00	21	0.238
159	A	0	0	0.00	0	0.000
160	A	0	0	0.00	0	0.000
161	A	0	0	0.00	0	0.000
162	A	0	0	0.00	0	0.000
163	A	7	8	1.00	23	0.348
164	A	5	7	1.00	20	0.350
165	A	6	7	0.97	23	0.304
166	A	6	7	0.95	23	0.304
167	A	5	6	0.95	21	0.286
168	A	0	0	0.00	0	0.000
169	A	0	0	0.00	0	0.000
170	A	0	0	0.00	0	0.000
171	A	0	0	0.00	0	0.000
172	A	0	0	0.00	0	0.000
173	A	0	0	0.00	0	0.000
174	A	16	11	1.00	26	0.423
175	A	13	11	1.00	26	0.423
176	A	8	9	1.02	26	0.346
177	A	0	0	0.00	0	0.000
178	A	0	0	0.00	0	0.000



# Chapter 3

## Listing of integrals

### 3.1 $\int x^6 (a + b \operatorname{csch}^{-1}(cx)) dx$

Optimal. Leaf size=110

$$\frac{1}{7}x^7 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^6 \sqrt{\frac{1}{c^2 x^2} + 1}}{42c} - \frac{5b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{112c^7} + \frac{5bx^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{112c^5} - \frac{5bx^4 \sqrt{\frac{1}{c^2 x^2} + 1}}{168c^3}$$

[Out]  $\frac{1}{7}x^7(a+b\operatorname{arccsch}(cx)) - \frac{5}{112}b\operatorname{arctanh}\left(\frac{(1+1/c^2/x^2)^{1/2}}{c}\right) + \frac{5}{112}bx^2\sqrt{1+1/c^2/x^2} - \frac{5}{168}bx^4\sqrt{1+1/c^2/x^2} + \frac{1}{42}bx^6\sqrt{1+1/c^2/x^2}$

**Rubi [A]** time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6284, 266, 51, 63, 208}

$$\frac{1}{7}x^7 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^6 \sqrt{\frac{1}{c^2 x^2} + 1}}{42c} - \frac{5bx^4 \sqrt{\frac{1}{c^2 x^2} + 1}}{168c^3} + \frac{5bx^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{112c^5} - \frac{5b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{112c^7}$$

Antiderivative was successfully verified.

[In] Int[x^6\*(a + b\*ArcCsch[c\*x]), x]

[Out]  $(5*b*\sqrt{1 + 1/(c^2*x^2)}*x^2)/(112*c^5) - (5*b*\sqrt{1 + 1/(c^2*x^2)}*x^4)/(168*c^3) + (b*\sqrt{1 + 1/(c^2*x^2)}*x^6)/(42*c) + (x^7*(a + b*ArcCsch[c*x]))/7 - (5*b*ArcTanh[\sqrt{1 + 1/(c^2*x^2)}])/(112*c^7)$

#### Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 6284

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCsch[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 + 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int x^6 (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{7} x^7 (a + b \operatorname{csch}^{-1}(cx)) + \frac{b \int \frac{x^5}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{7c} \\
 &= \frac{1}{7} x^7 (a + b \operatorname{csch}^{-1}(cx)) - \frac{b \operatorname{Subst}\left(\int \frac{1}{x^4 \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{14c} \\
 &= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \operatorname{csch}^{-1}(cx)) + \frac{(5b) \operatorname{Subst}\left(\int \frac{1}{x^3 \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{84c^3} \\
 &= -\frac{5b \sqrt{1 + \frac{1}{c^2 x^2}} x^4}{168c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(5b) \operatorname{Subst}\left(\int \frac{1}{x^2 \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{112c^5} \\
 &= \frac{5b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{112c^5} - \frac{5b \sqrt{1 + \frac{1}{c^2 x^2}} x^4}{168c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \operatorname{csch}^{-1}(cx)) + \frac{5b \operatorname{csch}^{-1}(cx)}{112c^5} \quad (5b) \\
 &= \frac{5b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{112c^5} - \frac{5b \sqrt{1 + \frac{1}{c^2 x^2}} x^4}{168c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \operatorname{csch}^{-1}(cx)) + \frac{5b \operatorname{csch}^{-1}(cx)}{112c^5} \quad (5b) \\
 &= \frac{5b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{112c^5} - \frac{5b \sqrt{1 + \frac{1}{c^2 x^2}} x^4}{168c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^6}{42c} + \frac{1}{7} x^7 (a + b \operatorname{csch}^{-1}(cx)) - \frac{5b \operatorname{csch}^{-1}(cx)}{112c^5} \quad (5b)
 \end{aligned}$$

**Mathematica** [A] time = 0.14, size = 107, normalized size = 0.97

$$\frac{ax^7}{7} - \frac{5b \log\left(x \left(\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1\right)\right)}{112c^7} + b \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} \left(\frac{5x^2}{112c^5} - \frac{5x^4}{168c^3} + \frac{x^6}{42c}\right) + \frac{1}{7} b x^7 \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^6\*(a + b\*ArcCsch[c\*x]),x]

[Out] (a\*x^7)/7 + b\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)]\*((5\*x^2)/(112\*c^5) - (5\*x^4)/(168\*c^3) + x^6/(42\*c)) + (b\*x^7\*ArcCsch[c\*x])/7 - (5\*b\*Log[x\*(1 + Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)])])/(112\*c^7)

**fricas** [B] time = 0.80, size = 208, normalized size = 1.89

$$48 ac^7 x^7 + 48 bc^7 \log\left(cx\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx + 1\right) - 48 bc^7 \log\left(cx\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx - 1\right) + 15 b \log\left(cx\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx\right) + \frac{336 c^7}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 1/336\*(48\*a\*c^7\*x^7 + 48\*b\*c^7\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - 48\*b\*c^7\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + 15\*b\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) + 48\*(b\*c^7\*x^7 - b\*c^7)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (8\*b\*c^6\*x^6 - 10\*b\*c^4\*x^4 + 15\*b\*c^2\*x^2)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^7

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^6, x)

**maple** [A] time = 0.08, size = 127, normalized size = 1.15

$$\frac{c^7 x^7 a}{7} + b \left( \frac{c^7 x^7 \operatorname{arccsch}(cx)}{7} + \frac{\sqrt{c^2 x^2 + 1} \left( 8c^5 x^5 \sqrt{c^2 x^2 + 1} - 10c^3 x^3 \sqrt{c^2 x^2 + 1} + 15cx \sqrt{c^2 x^2 + 1} - 15 \operatorname{arcsinh}(cx) \right)}{336 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} cx} \right) \frac{1}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c^7\*(1/7\*c^7\*x^7\*a+b\*(1/7\*c^7\*x^7\*arccsch(c\*x)+1/336\*(c^2\*x^2+1)^(1/2)\*(8\*c^5\*x^5\*(c^2\*x^2+1)^(1/2)-10\*c^3\*x^3\*(c^2\*x^2+1)^(1/2)+15\*c\*x\*(c^2\*x^2+1)^(1/2)-15\*arcsinh(c\*x))/(c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x)

**maxima** [A] time = 0.36, size = 158, normalized size = 1.44

$$\frac{1}{7} ax^7 + \frac{1}{672} \left( 96 x^7 \operatorname{arcsch}(cx) + \frac{2 \left( 15 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} - 40 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 33 \sqrt{\frac{1}{c^2 x^2} + 1} \right)}{c^6 \left( \frac{1}{c^2 x^2} + 1 \right)^3 - 3c^6 \left( \frac{1}{c^2 x^2} + 1 \right)^2 + 3c^6 \left( \frac{1}{c^2 x^2} + 1 \right) - c^6} - \frac{15 \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right)}{c^6} + \frac{15 \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} - 1\right)}{c^6} \right) \frac{1}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/7\*a\*x^7 + 1/672\*(96\*x^7\*arccsch(c\*x) + (2\*(15\*(1/(c^2\*x^2) + 1)^(5/2) - 40\*(1/(c^2\*x^2) + 1)^(3/2) + 33\*sqrt(1/(c^2\*x^2) + 1))/(c^6\*(1/(c^2\*x^2) + 1)^3 - 3\*c^6\*(1/(c^2\*x^2) + 1)^2 + 3\*c^6\*(1/(c^2\*x^2) + 1) - c^6) - 15\*log(sqrt(1/(c^2\*x^2) + 1) + 1)/c^6 + 15\*log(sqrt(1/(c^2\*x^2) + 1) - 1)/c^6)/c)\*b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^6 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(a + b*asinh(1/(c*x))),x)`

[Out] `int(x^6*(a + b*asinh(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6 (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(a+b*acsch(c*x)),x)`

[Out] `Integral(x**6*(a + b*acsch(c*x)), x)`

### 3.2 $\int x^5 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=86

$$\frac{1}{6}x^6(a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^5\sqrt{\frac{1}{c^2x^2} + 1}}{30c} + \frac{4bx\sqrt{\frac{1}{c^2x^2} + 1}}{45c^5} - \frac{2bx^3\sqrt{\frac{1}{c^2x^2} + 1}}{45c^3}$$

[Out]  $\frac{1}{6}x^6(a + b \operatorname{arccsch}(cx)) + \frac{4}{45}bx^5(1 + 1/c^2/x^2)^{(1/2)}/c^5 - \frac{2}{45}bx^3(1 + 1/c^2/x^2)^{(1/2)}/c^3 + \frac{1}{30}bx^5(1 + 1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.04, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {6284, 271, 191}

$$\frac{1}{6}x^6(a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^5\sqrt{\frac{1}{c^2x^2} + 1}}{30c} - \frac{2bx^3\sqrt{\frac{1}{c^2x^2} + 1}}{45c^3} + \frac{4bx\sqrt{\frac{1}{c^2x^2} + 1}}{45c^5}$$

Antiderivative was successfully verified.

[In] `Int[x^5*(a + b*ArcCsch[c*x]), x]`

[Out]  $\frac{(4*b*\sqrt{1 + 1/(c^2*x^2)}*x)/(45*c^5) - (2*b*\sqrt{1 + 1/(c^2*x^2)}*x^3)/(45*c^3) + (b*\sqrt{1 + 1/(c^2*x^2)}*x^5)/(30*c) + (x^6*(a + b*ArcCsch[c*x]))}{6}$

#### Rule 191

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]`

#### Rule 271

`Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]`

#### Rule 6284

`Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m + 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]`

#### Rubi steps

$$\begin{aligned}
\int x^5 (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{6} x^6 (a + b \operatorname{csch}^{-1}(cx)) + \frac{b \int \frac{x^4}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^5}{30c} + \frac{1}{6} x^6 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(2b) \int \frac{x^2}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{15c^3} \\
&= -\frac{2b \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{45c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^5}{30c} + \frac{1}{6} x^6 (a + b \operatorname{csch}^{-1}(cx)) + \frac{(4b) \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{45c^5} \\
&= \frac{4b \sqrt{1 + \frac{1}{c^2 x^2}} x}{45c^5} - \frac{2b \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{45c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^5}{30c} + \frac{1}{6} x^6 (a + b \operatorname{csch}^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 72, normalized size = 0.84

$$\frac{ax^6}{6} + b \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} \left( \frac{4x}{45c^5} - \frac{2x^3}{45c^3} + \frac{x^5}{30c} \right) + \frac{1}{6} b x^6 \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(a + b\*ArcCsch[c\*x]),x]

[Out] (a\*x^6)/6 + b\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)]\*((4\*x)/(45\*c^5) - (2\*x^3)/(45\*c^3) + x^5/(30\*c)) + (b\*x^6\*ArcCsch[c\*x])/6

**fricas [A]** time = 2.01, size = 97, normalized size = 1.13

$$\frac{15bc^5x^6 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + 15ac^5x^6 + (3bc^4x^5 - 4bc^2x^3 + 8bx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{90c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 1/90\*(15\*b\*c^5\*x^6\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 15\*a\*c^5\*x^6 + (3\*b\*c^4\*x^5 - 4\*b\*c^2\*x^3 + 8\*b\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^5

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^5, x)

**maple [A]** time = 0.05, size = 83, normalized size = 0.97

$$\frac{c^6 x^6 a}{6} + b \left( \frac{c^6 x^6 \operatorname{arccsch}(cx)}{6} + \frac{(c^2 x^2 + 1)(3c^4 x^4 - 4c^2 x^2 + 8)}{90 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} cx} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a+b*arccsch(c*x)),x)`

[Out]  $1/c^6*(1/6*c^6*x^6*a+b*(1/6*c^6*x^6*arccsch(c*x)+1/90*(c^2*x^2+1)*(3*c^4*x^4-4*c^2*x^2+8)/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/c/x))$

**maxima** [A] time = 0.31, size = 77, normalized size = 0.90

$$\frac{1}{6}ax^6 + \frac{1}{90} \left( 15x^6 \operatorname{arcsch}(cx) + \frac{3c^4x^5\left(\frac{1}{c^2x^2} + 1\right)^{\frac{5}{2}} - 10c^2x^3\left(\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} + 15x\sqrt{\frac{1}{c^2x^2} + 1}}{c^5} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="maxima")`

[Out]  $1/6*a*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^{(5/2)} - 10*c^2*x^3*(1/(c^2*x^2) + 1)^{(3/2)} + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a + b*asinh(1/(c*x))),x)`

[Out] `int(x^5*(a + b*asinh(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(a+b*acsch(c*x)),x)`

[Out] `Integral(x**5*(a + b*acsch(c*x)), x)`

### 3.3 $\int x^4 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=86

$$\frac{1}{5}x^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^4 \sqrt{\frac{1}{c^2 x^2} + 1}}{20c} + \frac{3b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{40c^5} - \frac{3bx^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{40c^3}$$

[Out]  $1/5*x^5*(a+b*\operatorname{arccsch}(c*x))+3/40*b*\operatorname{arctanh}((1+1/c^2/x^2)^{(1/2)})/c^5-3/40*b*x^2*(1+1/c^2/x^2)^{(1/2)}/c^3+1/20*b*x^4*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6284, 266, 51, 63, 208}

$$\frac{1}{5}x^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^4 \sqrt{\frac{1}{c^2 x^2} + 1}}{20c} - \frac{3bx^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{40c^3} + \frac{3b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{40c^5}$$

Antiderivative was successfully verified.

[In] `Int[x^4*(a + b*ArcCsch[c*x]),x]`

[Out]  $(-3*b*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x^2)/(40*c^3) + (b*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x^4)/(20*c) + (x^5*(a + b*\operatorname{ArcCsch}[c*x])/5 + (3*b*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + 1/(c^2*x^2)]])/(40*c^5)$

#### Rule 51

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[ ((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]`

#### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

#### Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

#### Rule 266

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

#### Rule 6284

`Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m + 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]`



Rubi steps

$$\begin{aligned}
\int x^4 (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{5} x^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{b \int \frac{x^3}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{5c} \\
&= \frac{1}{5} x^5 (a + b \operatorname{csch}^{-1}(cx)) - \frac{b \operatorname{Subst}\left(\int \frac{1}{x^3 \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{10c} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{x^2 \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{40c^3} \\
&= -\frac{3b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{40c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{80c^5} \\
&= -\frac{3b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{40c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{-c^2 + \frac{x}{c^2}} dx, x, \frac{1}{x^2}\right)}{40c^5} \\
&= -\frac{3b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{40c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^4}{20c} + \frac{1}{5} x^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{3b \tanh^{-1}\left(\sqrt{1 + \frac{x}{c^2}}\right)}{40c^5}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 97, normalized size = 1.13

$$\frac{ax^5}{5} + \frac{3b \log\left(x \left(\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1\right)\right)}{40c^5} + b \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} \left(\frac{x^4}{20c} - \frac{3x^2}{40c^3}\right) + \frac{1}{5} b x^5 \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(a + b\*ArcCsch[c\*x]), x]

[Out] (a\*x^5)/5 + b\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)]\*((-3\*x^2)/(40\*c^3) + x^4/(20\*c)) + (b\*x^5\*ArcCsch[c\*x])/5 + (3\*b\*Log[x\*(1 + Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)])]/(40\*c^5))

**fricas [B]** time = 0.73, size = 199, normalized size = 2.31

$$\frac{8ac^5x^5 + 8bc^5 \log\left(cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 8bc^5 \log\left(cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - 3b \log\left(cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + 8(bc^5x^5 - bc^5)}{40c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] 1/40\*(8\*a\*c^5\*x^5 + 8\*b\*c^5\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - 8\*b\*c^5\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) - 3\*b\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) + 8\*(b\*c^5\*x^5 - b\*c^5)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (2\*b\*c^4\*x^4 - 3\*b\*c^2\*x^2)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^5

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^4, x)

**maple** [A] time = 0.05, size = 108, normalized size = 1.26

$$\frac{c^5 x^5 a}{5} + b \left( \frac{c^5 x^5 \operatorname{arccsch}(cx)}{5} + \frac{\sqrt{c^2 x^2 + 1} \left( 2c^3 x^3 \sqrt{c^2 x^2 + 1} - 3cx \sqrt{c^2 x^2 + 1} + 3 \operatorname{arcsinh}(cx) \right)}{40 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} cx} \right) \frac{1}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c^5\*(1/5\*c^5\*x^5\*a+b\*(1/5\*c^5\*x^5\*arccsch(c\*x)+1/40\*(c^2\*x^2+1)^(1/2)\*(2\*c^3\*x^3\*(c^2\*x^2+1)^(1/2)-3\*c\*x\*(c^2\*x^2+1)^(1/2)+3\*arcsinh(c\*x))/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x))

**maxima** [A] time = 0.37, size = 128, normalized size = 1.49

$$\frac{1}{5} ax^5 + \frac{1}{80} \left( 16x^5 \operatorname{arcsch}(cx) - \frac{2 \left( 3 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 5 \sqrt{\frac{1}{c^2 x^2} + 1} \right)}{c^4 \left( \frac{1}{c^2 x^2} + 1 \right)^2 - 2c^4 \left( \frac{1}{c^2 x^2} + 1 \right) + c^4} - \frac{3 \log \left( \sqrt{\frac{1}{c^2 x^2} + 1} + 1 \right)}{c^4} + \frac{3 \log \left( \sqrt{\frac{1}{c^2 x^2} + 1} - 1 \right)}{c^4} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/5\*a\*x^5 + 1/80\*(16\*x^5\*arccsch(c\*x) - (2\*(3\*(1/(c^2\*x^2) + 1)^(3/2) - 5\*sqrt(1/(c^2\*x^2) + 1))/(c^4\*(1/(c^2\*x^2) + 1)^2 - 2\*c^4\*(1/(c^2\*x^2) + 1) + c^4) - 3\*log(sqrt(1/(c^2\*x^2) + 1) + 1)/c^4 + 3\*log(sqrt(1/(c^2\*x^2) + 1) - 1)/c^4)/c)\*b

**mpad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x^4\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*acsch(c\*x)),x)

[Out] Integral(x\*\*4\*(a + b\*acsch(c\*x)), x)

### 3.4 $\int x^3 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=62

$$\frac{1}{4}x^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^3 \sqrt{\frac{1}{c^2 x^2} + 1}}{12c} - \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1}}{6c^3}$$

[Out]  $1/4*x^4*(a+b*\operatorname{arccsch}(c*x))-1/6*b*x*(1+1/c^2/x^2)^{(1/2)}/c^3+1/12*b*x^3*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {6284, 271, 191}

$$\frac{1}{4}x^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^3 \sqrt{\frac{1}{c^2 x^2} + 1}}{12c} - \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1}}{6c^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $-(b*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x^3)/(12*c) + (x^4*(a + b*\operatorname{ArcCsch}[c*x]))/4$

#### Rule 191

$\operatorname{Int}[(a_.) + (b_.)*(x_)^{(n_)}]^{(p_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(x*(a + b*x^n)^{(p + 1)})/a, x] /;$   $\operatorname{FreeQ}\{a, b, n, p, x\} \ \&\& \ \operatorname{EqQ}[1/n + p + 1, 0]$

#### Rule 271

$\operatorname{Int}[(x_)^{(m_)}*((a_.) + (b_.)*(x_)^{(n_)}]^{(p_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(x^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(a*(m + 1)), x] - \operatorname{Dist}[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), \operatorname{Int}[x^{(m + n)}*(a + b*x^n)^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, m, n, p, x\} \ \&\& \ \operatorname{ILtQ}[\operatorname{Simplify}[(m + 1)/n + p + 1], 0] \ \&\& \ \operatorname{NeQ}[m, -1]$

#### Rule 6284

$\operatorname{Int}[(a_.) + \operatorname{ArcCsch}[(c_.)*(x_)]*(b_.)]^{(d_)}*(x_)^{(m_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m + 1)}*(a + b*\operatorname{ArcCsch}[c*x])]/(d*(m + 1)), x] + \operatorname{Dist}[(b*d)/(c*(m + 1)), \operatorname{Int}[(d*x)^{(m - 1)}/\operatorname{Sqrt}[1 + 1/(c^2*x^2)], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \operatorname{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int x^3 (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{4}x^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{b \int \frac{x^2}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{4c} \\ &= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{12c} + \frac{1}{4}x^4 (a + b \operatorname{csch}^{-1}(cx)) - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c^3} \\ &= -\frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{6c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{12c} + \frac{1}{4}x^4 (a + b \operatorname{csch}^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 62, normalized size = 1.00

$$\frac{ax^4}{4} + b\sqrt{\frac{c^2x^2+1}{c^2x^2}} \left( \frac{x^3}{12c} - \frac{x}{6c^3} \right) + \frac{1}{4}bx^4\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcCsch[c\*x]),x]

[Out] (a\*x^4)/4 + b\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)]\*(-1/6\*x/c^3 + x^3/(12\*c)) + (b\*x^4\*ArcCsch[c\*x])/4

**fricas [A]** time = 3.31, size = 87, normalized size = 1.40

$$\frac{3bc^3x^4 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + 3ac^3x^4 + (bc^2x^3 - 2bx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 1/12\*(3\*b\*c^3\*x^4\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 3\*a\*c^3\*x^4 + (b\*c^2\*x^3 - 2\*b\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^3, x)

**maple [A]** time = 0.04, size = 74, normalized size = 1.19

$$\frac{\frac{c^4x^4a}{4} + b \left( \frac{c^4x^4\operatorname{arccsch}(cx)}{4} + \frac{(c^2x^2+1)(c^2x^2-2)}{12\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx} \right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c^4\*(1/4\*c^4\*x^4\*a+b\*(1/4\*c^4\*x^4\*arccsch(c\*x)+1/12\*(c^2\*x^2+1)\*(c^2\*x^2-2)/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x))

**maxima [A]** time = 0.32, size = 57, normalized size = 0.92

$$\frac{1}{4}ax^4 + \frac{1}{12} \left( 3x^4 \operatorname{arcsch}(cx) + \frac{c^2x^3 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2} + 1}}{c^3} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{4}ax^4 + \frac{1}{12}(3x^4 \operatorname{arccsch}(cx) + (c^2x^3(1/(c^2x^2) + 1))^{3/2} - 3x\sqrt{1/(c^2x^2) + 1})/c^3)b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b*asinh(1/(c*x))), x)`

[Out] `int(x^3*(a + b*asinh(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*acsch(c*x)), x)`

[Out] `Integral(x**3*(a + b*acsch(c*x)), x)`

### 3.5 $\int x^2 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=62

$$\frac{1}{3}x^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{6c} - \frac{b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{6c^3}$$

[Out]  $\frac{1}{3}x^3(a+b\operatorname{arccsch}(c*x))-\frac{1}{6}b*\operatorname{arctanh}((1+1/c^2/x^2)^{(1/2)})/c^3+\frac{1}{6}b*x^2*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {6284, 266, 51, 63, 208}

$$\frac{1}{3}x^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{6c} - \frac{b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{6c^3}$$

Antiderivative was successfully verified.

[In] `Int[x^2*(a + b*ArcCsch[c*x]),x]`

[Out]  $(b*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x^2)/(6*c) + (x^3*(a + b*\operatorname{ArcCsch}[c*x]))/3 - (b*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + 1/(c^2*x^2)]])/(6*c^3)$

#### Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 6284

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*((d_.)*(x_)^(m_.)), x_Symbol] := Si
mp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m
+ 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{b \int \frac{x}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{3c} \\
&= \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx)) - \frac{b \operatorname{Subst} \left( \int \frac{1}{x^2 \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{6c} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{6c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{b \operatorname{Subst} \left( \int \frac{1}{x \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{12c^3} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{6c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{b \operatorname{Subst} \left( \int \frac{1}{-c^2 + c^2 x^2} dx, x, \sqrt{1 + \frac{1}{c^2 x^2}} \right)}{6c} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{6c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx)) - \frac{b \tanh^{-1} \left( \sqrt{1 + \frac{1}{c^2 x^2}} \right)}{6c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 85, normalized size = 1.37

$$\frac{ax^3}{3} + \frac{bx^2 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}}{6c} - \frac{b \log \left( x \left( \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1 \right) \right)}{6c^3} + \frac{1}{3} bx^3 \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*ArcCsch[c\*x]),x]

[Out] (a\*x^3)/3 + (b\*x^2\*sqrt[(1 + c^2\*x^2)/(c^2\*x^2)])/(6\*c) + (b\*x^3\*ArcCsch[c\*x])/3 - (b\*Log[x\*(1 + sqrt[(1 + c^2\*x^2)/(c^2\*x^2)])])/(6\*c^3)

**fricas [B]** time = 1.58, size = 186, normalized size = 3.00

$$\frac{2ac^3x^3 + bc^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 2bc^3 \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 2bc^3 \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) + b \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 1/6\*(2\*a\*c^3\*x^3 + b\*c^2\*x^2\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 2\*b\*c^3\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - 2\*b\*c^3\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + b\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) + 2\*(b\*c^3\*x^3 - b\*c^3)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/c^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2, x)

**maple** [A] time = 0.05, size = 88, normalized size = 1.42

$$\frac{\frac{ac^3x^3}{3} + b \left( \frac{c^3x^3 \operatorname{arccsch}(cx)}{3} + \frac{\sqrt{c^2x^2+1} (cx\sqrt{c^2x^2+1} - \operatorname{arcsinh}(cx))}{6\sqrt{\frac{c^2x^2+1}{c^2}} cx} \right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arccsch(c*x)),x)`

[Out]  $\frac{1}{c^3} * \left( \frac{1}{3} * a * c^3 * x^3 + b * \left( \frac{1}{3} * c^3 * x^3 * \operatorname{arccsch}(c * x) + \frac{1}{6} * (c^2 * x^2 + 1)^{(1/2)} * (c * x * (c^2 * x^2 + 1)^{(1/2)} - \operatorname{arcsinh}(c * x)) \right) \right) / \left( (c^2 * x^2 + 1) / c^2 / x^2 \right)^{(1/2)} / c / x$

**maxima** [A] time = 0.34, size = 96, normalized size = 1.55

$$\frac{1}{3} ax^3 + \frac{1}{12} \left( 4x^3 \operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="maxima")`

[Out]  $\frac{1}{3} * a * x^3 + \frac{1}{12} * (4 * x^3 * \operatorname{arccsch}(c * x) + (2 * \sqrt{1 / (c^2 * x^2) + 1} / (c^2 * (1 / (c^2 * x^2) + 1) - c^2) - \log(\sqrt{1 / (c^2 * x^2) + 1} + 1) / c^2 + \log(\sqrt{1 / (c^2 * x^2) + 1} - 1) / c^2) / c) * b$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*asinh(1/(c*x))),x)`

[Out] `int(x^2*(a + b*asinh(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*acsch(c*x)),x)`

[Out] `Integral(x**2*(a + b*acsch(c*x)), x)`



### 3.6 $\int x (a + b \operatorname{csch}^{-1}(cx)) dx$

Optimal. Leaf size=38

$$\frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1}}{2c}$$

[Out]  $1/2*x^2*(a+b*\operatorname{arccsch}(c*x))+1/2*b*x*(1+1/c^2/x^2)^{(1/2)}/c$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {6284, 191}

$$\frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1}}{2c}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcCsch[c\*x]), x]

[Out] (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x)/(2\*c) + (x^2\*(a + b\*ArcCsch[c\*x]))/2

Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 6284

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.)\*(x\_)^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcCsch[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 + 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int x (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx)) + \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{2c} \\ &= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2c} + \frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx)) \end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.32

$$\frac{ax^2}{2} + \frac{bx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}}{2c} + \frac{1}{2}bx^2 \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcCsch[c\*x]), x]

[Out] (a\*x^2)/2 + (b\*x\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)])/(2\*c) + (b\*x^2\*ArcCsch[c\*x])/2

**fricas** [B] time = 0.74, size = 70, normalized size = 1.84

$$\frac{bcx^2 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + acx^2 + bx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 1/2\*(b\*c\*x^2\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + a\*c\*x^2 + b\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x, x)

**maple** [A] time = 0.04, size = 65, normalized size = 1.71

$$\frac{\frac{c^2x^2a}{2} + b\left(\frac{c^2x^2\operatorname{arccsch}(cx)}{2} + \frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c^2\*(1/2\*c^2\*x^2\*a+b\*(1/2\*c^2\*x^2\*arccsch(c\*x)+1/2/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x\*(c^2\*x^2+1)))

**maxima** [A] time = 0.38, size = 35, normalized size = 0.92

$$\frac{1}{2}ax^2 + \frac{1}{2}\left(x^2 \operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/2\*a\*x^2 + 1/2\*(x^2\*arccsch(c\*x) + x\*sqrt(1/(c^2\*x^2) + 1)/c)\*b

**mupad** [B] time = 2.22, size = 39, normalized size = 1.03

$$\frac{ax^2}{2} + \frac{bx^2 \operatorname{asinh}\left(\frac{1}{cx}\right)}{2} + \frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*asinh(1/(c\*x))),x)

[Out] (a\*x^2)/2 + (b\*x^2\*asinh(1/(c\*x)))/2 + (b\*x\*(1/(c^2\*x^2) + 1)^(1/2))/(2\*c)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x)), x)

[Out] Integral(x\*(a + b\*acsch(c\*x)), x)

### 3.7 $\int (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=30

$$ax + \frac{b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{c} + bx \operatorname{csch}^{-1}(cx)$$

[Out] a\*x+b\*x\*arccsch(c\*x)+b\*arctanh((1+1/c^2/x^2)^(1/2))/c

**Rubi [A]** time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6278, 266, 63, 208}

$$ax + \frac{b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{c} + bx \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcCsch[c\*x], x]

[Out] a\*x + b\*x\*ArcCsch[c\*x] + (b\*ArcTanh[Sqrt[1 + 1/(c^2\*x^2)]])/c

#### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 6278

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

#### Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{csch}^{-1}(cx)) dx &= ax + b \int \operatorname{csch}^{-1}(cx) dx \\
&= ax + bx \operatorname{csch}^{-1}(cx) + \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} dx}}{c} \\
&= ax + bx \operatorname{csch}^{-1}(cx) - \frac{b \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{2c} \\
&= ax + bx \operatorname{csch}^{-1}(cx) - (bc) \operatorname{Subst}\left(\int \frac{1}{-c^2 + c^2 x^2} dx, x, \sqrt{1 + \frac{1}{c^2 x^2}}\right) \\
&= ax + bx \operatorname{csch}^{-1}(cx) + \frac{b \tanh^{-1}\left(\sqrt{1 + \frac{1}{c^2 x^2}}\right)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 44, normalized size = 1.47

$$ax + \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1} \sinh^{-1}(cx)}{\sqrt{c^2 x^2 + 1}} + bx \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcCsch[c\*x], x]

[Out] a\*x + b\*x\*ArcCsch[c\*x] + (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*ArcSinh[c\*x])/Sqrt[1 + c^2\*x^2]

**fricas [B]** time = 0.73, size = 143, normalized size = 4.77

$$\frac{acx + bc \log\left(cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx + 1\right) - bc \log\left(cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx - 1\right) - b \log\left(cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx\right) + (bcx - bc) \log\left(\frac{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx + 1}{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx - 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsch(c\*x), x, algorithm="fricas")

[Out] (a\*c\*x + b\*c\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - b\*c\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) - b\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) + (b\*c\*x - b\*c)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/c

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int b \operatorname{arcsch}(cx) + a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsch(c\*x), x, algorithm="giac")

[Out] integrate(b\*arccsch(c\*x) + a, x)

**maple [A]** time = 0.04, size = 36, normalized size = 1.20

$$ax + bx \operatorname{arcsch}(cx) + \frac{b \ln\left(cx + cx \sqrt{1 + \frac{1}{c^2 x^2}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arccsch(c*x),x)`

[Out] `a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))`

**maxima** [A] time = 0.33, size = 49, normalized size = 1.63

$$ax + \frac{\left(2cx \operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2x^2} + 1} - 1\right)\right)b}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arccsch(c*x),x, algorithm="maxima")`

[Out] `a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b/c`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int a + b \operatorname{asinh}\left(\frac{1}{cx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + b*asinh(1/(c*x)),x)`

[Out] `int(a + b*asinh(1/(c*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*acsch(c*x),x)`

[Out] `Integral(a + b*acsch(c*x), x)`

### 3.8 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x} dx$

**Optimal.** Leaf size=56

$$-\frac{(a+b\operatorname{csch}^{-1}(cx))^2}{2b} - \log\left(1 - e^{-2\operatorname{csch}^{-1}(cx)}\right)(a+b\operatorname{csch}^{-1}(cx)) + \frac{1}{2}b\operatorname{Li}_2\left(e^{-2\operatorname{csch}^{-1}(cx)}\right)$$

[Out]  $-1/2*(a+b*\operatorname{arccsch}(c*x))^2/b - (a+b*\operatorname{arccsch}(c*x))*\ln(1-1/(1/c/x+(1+1/c^2/x^2)^(1/2)))^2 + 1/2*b*\operatorname{polylog}(2, 1/(1/c/x+(1+1/c^2/x^2)^(1/2)))^2$

**Rubi [A]** time = 0.09, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6282, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}b\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right) + \frac{(a+b\operatorname{csch}^{-1}(cx))^2}{2b} - \log\left(1 - e^{2\operatorname{csch}^{-1}(cx)}\right)(a+b\operatorname{csch}^{-1}(cx))$$

Warning: Unable to verify antiderivative.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])/x, x]$

[Out]  $(a + b*\operatorname{ArcCsch}[c*x])^2/(2*b) - (a + b*\operatorname{ArcCsch}[c*x])*Log[1 - E^{(2*\operatorname{ArcCsch}[c*x])}] - (b*\operatorname{PolyLog}[2, E^{(2*\operatorname{ArcCsch}[c*x])}])/2$

**Rule 2190**

$\operatorname{Int}[\frac{((F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))})^{(n_*)}*((c_*) + (d_*)*(x_*))^{(m_*)})}{((a_*) + (b_*)*((F_*)^{((g_*)*((e_*) + (f_*)*(x_*)))})^{(n_*)})}, x\_Symbol] \rightarrow \operatorname{Simp}[\frac{(c + d*x)^m * Log[1 + (b*(F^{(g*(e + f*x)))^n)/a]}{(b*f*g*n*Log[F])}, x] - \operatorname{Dist}[\frac{(d*m)}{(b*f*g*n*Log[F])}, \operatorname{Int}[(c + d*x)^{(m-1)} * Log[1 + (b*(F^{(g*(e + f*x)))^n)/a}], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \operatorname{IGtQ}[m, 0]$

**Rule 2279**

$\operatorname{Int}[Log[(a_*) + (b_*)*((F_*)^{((e_*)*((c_*) + (d_*)*(x_*)))})^{(n_*)}], x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*e*n*Log[F]), \operatorname{Subst}[\operatorname{Int}[Log[a + b*x]/x, x], x, (F^{(e*(c + d*x)))^n}], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \operatorname{GtQ}[a, 0]$

**Rule 2391**

$\operatorname{Int}[Log[(c_*)*((d_*) + (e_*)*(x_*)^{(n_*)})]/(x_*)], x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x\} \&\& \operatorname{EqQ}[c*d, 1]$

**Rule 3716**

$\operatorname{Int}[\frac{((c_*) + (d_*)*(x_*))^{(m_*)} * \tan[(e_*) + \operatorname{Pi}*(k_*) + (\operatorname{Complex}[0, fz_*])*(f_*)*(x_*)]}{(a_*) + (b_*)*((F_*)^{((e_*)*((c_*) + (d_*)*(x_*)))})^{(n_*)})}, x\_Symbol] \rightarrow -\operatorname{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] + \operatorname{Dist}[2*I, \operatorname{Int}[\frac{(c + d*x)^m * E^{(2*(-(I*e) + f*fz*x))}}{(E^{(2*I*k*Pi)}*(1 + E^{(2*(-(I*e) + f*fz*x))})/E^{(2*I*k*Pi)})}], x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \operatorname{Integ erQ}[4*k] \&\& \operatorname{IGtQ}[m, 0]$

**Rule 5659**

$\operatorname{Int}[\frac{((a_*) + \operatorname{ArcSinh}[(c_*)*(x_*)])*(b_*)^{(n_*)}}{(x_*)}, x\_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n / \operatorname{Tanh}[x], x], x, \operatorname{ArcSinh}[c*x]] /; \operatorname{FreeQ}\{a, b, c\}, x\} \&\& \operatorname{IGtQ}[n, 0]$

**Rule 6282**

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))/(x_), x_Symbol] := -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int (a + bx) \operatorname{coth}(x) dx, x, \operatorname{csch}^{-1}(cx) \right) \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2b} + 2 \operatorname{Subst} \left( \int \frac{e^{2x}(a + bx)}{1 - e^{2x}} dx, x, \operatorname{csch}^{-1}(cx) \right) \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2b} - (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - e^{2 \operatorname{csch}^{-1}(cx)} \right) + b \operatorname{Subst} \left( \int \log(1 - e^{2x}) \right) \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2b} - (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - e^{2 \operatorname{csch}^{-1}(cx)} \right) + \frac{1}{2} b \operatorname{Subst} \left( \int \frac{\log(1 - x)}{x} \right) \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2b} - (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - e^{2 \operatorname{csch}^{-1}(cx)} \right) - \frac{1}{2} b \operatorname{Li}_2 \left( e^{2 \operatorname{csch}^{-1}(cx)} \right)
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 47, normalized size = 0.84

$$a \log(x) + \frac{1}{2} b \left( \operatorname{Li}_2 \left( e^{-2 \operatorname{csch}^{-1}(cx)} \right) - \operatorname{csch}^{-1}(cx) \left( \operatorname{csch}^{-1}(cx) + 2 \log \left( 1 - e^{-2 \operatorname{csch}^{-1}(cx)} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x,x]
```

```
[Out] a*Log[x] + (b*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[c*x])])
))) + PolyLog[2, E^(-2*ArcCsch[c*x])])/2
```

**fricas** [F] time = 2.07, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \operatorname{arcsch}(cx) + a}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/x, x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x, x)
```

**maple** [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsch}(cx)}{x} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x,x)

[Out] int((a+b\*arccsch(c\*x))/x,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( 4c^2 \int \frac{x^2 \log(x)}{c^2 x^3 + x} dx - 2c^2 \int \frac{x \log(x)}{c^2 x^2 + (c^2 x^2 + 1)^{\frac{3}{2}} + 1} dx - (\log(c^2 x^2 + 1) - 2 \log(x)) \log(c) + \log(c^2 x^2 + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x,x, algorithm="maxima")

[Out] -1/2\*(4\*c^2\*integrate(x^2\*log(x)/(c^2\*x^3 + x), x) - 2\*c^2\*integrate(x\*log(x)/(c^2\*x^2 + (c^2\*x^2 + 1)^(3/2) + 1), x) - (log(c^2\*x^2 + 1) - 2\*log(x))\*log(c) + log(c^2\*x^2 + 1)\*log(c) - 2\*log(x)\*log(sqrt(c^2\*x^2 + 1) + 1) + 2\*integrate(log(x)/(c^2\*x^3 + x), x))\*b + a\*log(x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/x,x)

[Out] int((a + b\*asinh(1/(c\*x)))/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x,x)

[Out] Integral((a + b\*acsch(c\*x))/x, x)

### 3.9 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2} dx$

**Optimal.** Leaf size=30

$$bc\sqrt{\frac{1}{c^2x^2} + 1} - \frac{a + b\operatorname{csch}^{-1}(cx)}{x}$$

[Out]  $(-a-b*\operatorname{arccsch}(c*x))/x+b*c*(1+1/c^2/x^2)^(1/2)$

**Rubi [A]** time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {6284, 261}

$$bc\sqrt{\frac{1}{c^2x^2} + 1} - \frac{a + b\operatorname{csch}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])/x^2, x]$

[Out]  $b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)] - (a + b*\operatorname{ArcCsch}[c*x])/x$

**Rule 261**

$\operatorname{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x^n)^{(p+1)}/(b*n*(p+1)), x] /; \operatorname{FreeQ}\{a, b, m, n, p\}, x\} \&\& \operatorname{EqQ}[m, n-1] \&\& \operatorname{NeQ}[p, -1]$

**Rule 6284**

$\operatorname{Int}[(a_. + \operatorname{ArcCsch}[c_.*(x_.)]*(b_.))*((d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcCsch}[c*x])/(d*(m+1)), x] + \operatorname{Dist}[(b*d)/(c*(m+1)), \operatorname{Int}[(d*x)^{(m-1)}/\operatorname{Sqrt}[1 + 1/(c^2*x^2)], x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x\} \&\& \operatorname{NeQ}[m, -1]$

**Rubi steps**

$$\begin{aligned} \int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2} dx &= -\frac{a + b\operatorname{csch}^{-1}(cx)}{x} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2x^2}} x^3} dx}{c} \\ &= bc\sqrt{1 + \frac{1}{c^2x^2}} - \frac{a + b\operatorname{csch}^{-1}(cx)}{x} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 40, normalized size = 1.33

$$-\frac{a}{x} + bc\sqrt{\frac{c^2x^2 + 1}{c^2x^2}} - \frac{b\operatorname{csch}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[(a + b*\operatorname{ArcCsch}[c*x])/x^2, x]$

[Out]  $-(a/x) + b*c*\operatorname{Sqrt}[(1 + c^2*x^2)/(c^2*x^2)] - (b*\operatorname{ArcCsch}[c*x])/x$

**fricas [B]** time = 0.78, size = 64, normalized size = 2.13

$$\frac{bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - b \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2,x, algorithm="fricas")

[Out] (b\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - b\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - a)/x

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/x^2, x)

**maple** [B] time = 0.05, size = 62, normalized size = 2.07

$$c \left( -\frac{a}{cx} + b \left( -\frac{\operatorname{arcsch}(cx)}{cx} + \frac{c^2 x^2 + 1}{\sqrt{c^2 x^2 + 1} c^2 x^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^2,x)

[Out] c\*(-a/c/x+b\*(-1/c/x\*arccsch(c\*x)+1/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c^2/x^2\*(c^2\*x^2+1)))

**maxima** [A] time = 0.35, size = 32, normalized size = 1.07

$$\left( c \sqrt{\frac{1}{c^2 x^2} + 1} - \frac{\operatorname{arcsch}(cx)}{x} \right) b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2,x, algorithm="maxima")

[Out] (c\*sqrt(1/(c^2\*x^2) + 1) - arccsch(c\*x)/x)\*b - a/x

**mupad** [B] time = 2.32, size = 35, normalized size = 1.17

$$bc \sqrt{\frac{1}{c^2 x^2} + 1} - \frac{a}{x} - \frac{b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/x^2,x)

[Out] b\*c\*(1/(c^2\*x^2) + 1)^(1/2) - a/x - (b\*asinh(1/(c\*x)))/x

**sympy** [A] time = 1.82, size = 36, normalized size = 1.20

$$\begin{cases} -\frac{a}{x} + bc \sqrt{1 + \frac{1}{c^2 x^2}} - \frac{b \operatorname{arcsch}(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a + \infty b}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*2,x)

[Out] Piecewise((-a/x + b\*c\*sqrt(1 + 1/(c\*\*2\*x\*\*2)) - b\*acsch(c\*x)/x, Ne(c, 0)), (-a + zoo\*b)/x, True)

### 3.10 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3} dx$

Optimal. Leaf size=50

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2} + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{4x} - \frac{1}{4}bc^2\operatorname{csch}^{-1}(cx)$$

[Out]  $-1/4*b*c^2*\operatorname{arccsch}(c*x)+1/2*(-a-b*\operatorname{arccsch}(c*x))/x^2+1/4*b*c*(1+1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6284, 335, 321, 215}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{2x^2} + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{4x} - \frac{1}{4}bc^2\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])/x^3, x]$

[Out]  $(b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*\operatorname{ArcCsch}[c*x])/4 - (a + b*\operatorname{ArcCsch}[c*x])/(2*x^2)$

#### Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x\_Symbol] := \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ \operatorname{PosQ}[b]$

#### Rule 321

$\operatorname{Int}[(c_)*(x_)^m*((a_) + (b_)*(x_)^n)^p, x\_Symbol] := \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \operatorname{Dist}[(a*c^{(n-1)}*(c*x)^{(m-n+1)})/(b*(m+n*p+1)), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{GtQ}[m, n-1] \ \&\& \ \operatorname{NeQ}[m+n*p+1, 0] \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 335

$\operatorname{Int}[(x_)^m*((a_) + (b_)*(x_)^n)^p, x\_Symbol] := -\operatorname{Subst}[\operatorname{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x] /;$   $\operatorname{FreeQ}\{a, b, p\}, x \ \&\& \ \operatorname{ILtQ}[n, 0] \ \&\& \ \operatorname{IntegerQ}[m]$

#### Rule 6284

$\operatorname{Int}[(a_) + \operatorname{ArcCsch}[c_)*(x_)]*(b_)*((d_)*(x_)^m, x\_Symbol] := \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcCsch}[c*x])/(d*(m+1)), x] + \operatorname{Dist}[(b*d)/(c*(m+1)), \operatorname{Int}[(d*x)^{(m-1)}/\operatorname{Sqrt}[1 + 1/(c^2*x^2)], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \operatorname{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2x^2} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^4} dx}{2c} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2x^2} + \frac{b \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2c} \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{4x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2x^2} - \frac{1}{4}(bc) \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{4x} - \frac{1}{4} bc^2 \operatorname{csch}^{-1}(cx) - \frac{a + b \operatorname{csch}^{-1}(cx)}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 66, normalized size = 1.32

$$-\frac{a}{2x^2} + \frac{bc \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}}{4x} - \frac{1}{4} bc^2 \sinh^{-1} \left( \frac{1}{cx} \right) - \frac{b \operatorname{csch}^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/x^3, x]

[Out] -1/2\*a/x^2 + (b\*c\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)]/(4\*x) - (b\*ArcCsch[c\*x])/(2\*x^2) - (b\*c^2\*ArcSinh[1/(c\*x)]/4

**fricas [A]** time = 0.98, size = 76, normalized size = 1.52

$$\frac{bcx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - (bc^2 x^2 + 2b) \log \left( \frac{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1}{cx} \right) - 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3,x, algorithm="fricas")

[Out] 1/4\*(b\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - (b\*c^2\*x^2 + 2\*b)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - 2\*a)/x^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/x^3, x)

**maple [B]** time = 0.05, size = 100, normalized size = 2.00

$$c^2 \left( -\frac{a}{2c^2 x^2} + b \left( -\frac{\operatorname{arccsch}(cx)}{2c^2 x^2} - \frac{\sqrt{c^2 x^2 + 1} \left( \operatorname{arctanh} \left( \frac{1}{\sqrt{c^2 x^2 + 1}} \right) c^2 x^2 - \sqrt{c^2 x^2 + 1} \right)}{4 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} c^3 x^3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsch(c*x))/x^3,x)`

[Out]  $c^2*(-1/2*a/c^2/x^2+b*(-1/2/c^2/x^2*arccsch(c*x)-1/4*(c^2*x^2+1)^{(1/2)}*(arc$   
 $\tanh(1/(c^2*x^2+1)^{(1/2)})*c^2*x^2-(c^2*x^2+1)^{(1/2)})/((c^2*x^2+1)/c^2/x^2)^{($   
 $1/2)/c^3/x^3))$

**maxima** [B] time = 0.39, size = 105, normalized size = 2.10

$$\frac{1}{8} b \left( \frac{\frac{2c^4x\sqrt{\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}+1\right)^{-1}} - c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}+1} + 1\right) + c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}+1} - 1\right)}{c} - \frac{4 \operatorname{arcsch}(cx)}{x^2} \right) - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsch(c*x))/x^3,x, algorithm="maxima")`

[Out]  $1/8*b*((2*c^4*x*\sqrt{1/(c^2*x^2)+1})/(c^2*x^2*(1/(c^2*x^2)+1)-1) - c^3$   
 $*\log(c*x*\sqrt{1/(c^2*x^2)+1}+1) + c^3*\log(c*x*\sqrt{1/(c^2*x^2)+1}-1$   
 $))/c - 4*arccsch(c*x)/x^2) - 1/2*a/x^2$

**mupad** [B] time = 2.27, size = 51, normalized size = 1.02

$$\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{4x} - \frac{b \operatorname{asinh}\left(\frac{1}{cx}\right)\left(\frac{c^2x}{4} + \frac{1}{2x}\right)}{x} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))/x^3,x)`

[Out]  $(b*c*(1/(c^2*x^2)+1)^{(1/2)})/(4*x) - (b*asinh(1/(c*x))*((c^2*x)/4 + 1/(2*x$   
 $)))/x - a/(2*x^2)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))/x**3,x)`

[Out] `Integral((a + b*acsch(c*x))/x**3, x)`

$$3.11 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^4} dx$$

Optimal. Leaf size=58

$$-\frac{a + b \operatorname{csch}^{-1}(cx)}{3x^3} + \frac{1}{9}bc^3 \left( \frac{1}{c^2x^2} + 1 \right)^{3/2} - \frac{1}{3}bc^3 \sqrt{\frac{1}{c^2x^2} + 1}$$

[Out]  $1/9*b*c^3*(1+1/c^2/x^2)^(3/2)+1/3*(-a-b*\operatorname{arccsch}(c*x))/x^3-1/3*b*c^3*(1+1/c^2/x^2)^(1/2)$

**Rubi [A]** time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {6284, 266, 43}

$$-\frac{a + b \operatorname{csch}^{-1}(cx)}{3x^3} + \frac{1}{9}bc^3 \left( \frac{1}{c^2x^2} + 1 \right)^{3/2} - \frac{1}{3}bc^3 \sqrt{\frac{1}{c^2x^2} + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/x^4, x]

[Out]  $-(b*c^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/3 + (b*c^3*(1 + 1/(c^2*x^2))^(3/2))/9 - (a + b*\operatorname{ArcCsch}[c*x])/(3*x^3)$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 6284

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.))\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d\*x)^(m + 1)\*(a + b\*ArcCsch[c\*x]))/(d\*(m + 1)), x] + Dist[(b\*d)/(c\*(m + 1)), Int[(d\*x)^(m - 1)/Sqrt[1 + 1/(c^2\*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^4} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{3x^3} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^5} dx}{3c} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{3x^3} + \frac{b \operatorname{Subst}\left(\int \frac{x}{\sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{6c} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{3x^3} + \frac{b \operatorname{Subst}\left(\int \left(-\frac{c^2}{\sqrt{1 + \frac{x}{c^2}}} + c^2 \sqrt{1 + \frac{x}{c^2}}\right) dx, x, \frac{1}{x^2}\right)}{6c} \\
&= -\frac{1}{3} b c^3 \sqrt{1 + \frac{1}{c^2 x^2}} + \frac{1}{9} b c^3 \left(1 + \frac{1}{c^2 x^2}\right)^{3/2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{3x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 59, normalized size = 1.02

$$-\frac{a}{3x^3} + b \left( \frac{c}{9x^2} - \frac{2c^3}{9} \right) \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - \frac{b \operatorname{csch}^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/x^4, x]

[Out] -1/3\*a/x^3 + b\*((-2\*c^3)/9 + c/(9\*x^2))\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcCsch[c\*x])/(3\*x^3)

**fricas [A]** time = 1.00, size = 77, normalized size = 1.33

$$\frac{3 b \log\left(\frac{c x \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1}{c x}\right) + (2 b c^3 x^3 - b c x) \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 3 a}{9 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^4,x, algorithm="fricas")

[Out] -1/9\*(3\*b\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (2\*b\*c^3\*x^3 - b\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 3\*a)/x^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^4,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/x^4, x)

**maple [A]** time = 0.05, size = 75, normalized size = 1.29

$$c^3 \left( -\frac{a}{3c^3 x^3} + b \left( -\frac{\operatorname{arccsch}(cx)}{3c^3 x^3} - \frac{(c^2 x^2 + 1)(2c^2 x^2 - 1)}{9 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} c^4 x^4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int((a+b\*arccsch(c\*x))/x^4,x)

[Out]  $c^3*(-1/3/c^3/x^3*a+b*(-1/3/c^3/x^3*arccsch(c*x)-1/9*(c^2*x^2+1)*(2*c^2*x^2-1)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^4/x^4))$

**maxima** [A] time = 0.37, size = 56, normalized size = 0.97

$$\frac{1}{9}b \left( \frac{c^4 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{\frac{1}{c^2 x^2} + 1}}{c} - \frac{3 \operatorname{arcsch}(cx)}{x^3} \right) - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^4,x, algorithm="maxima")

[Out]  $1/9*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3) - 1/3*a/x^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/x^4,x)

[Out] int((a + b\*asinh(1/(c\*x)))/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*4,x)

[Out] Integral((a + b\*acsch(c\*x))/x\*\*4, x)

### 3.12 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^5} dx$

Optimal. Leaf size=74

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4} + \frac{3}{32}bc^4\operatorname{csch}^{-1}(cx) + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{16x^3} - \frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}}{32x}$$

[Out]  $3/32*b*c^4*\operatorname{arccsch}(c*x)+1/4*(-a-b*\operatorname{arccsch}(c*x))/x^4+1/16*b*c*(1+1/c^2/x^2)^{(1/2)}/x^3-3/32*b*c^3*(1+1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6284, 335, 321, 215}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{4x^4} - \frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}}{32x} + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{16x^3} + \frac{3}{32}bc^4\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])/x^5, x]$

[Out]  $(b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(16*x^3) - (3*b*c^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(32*x) + (3*b*c^4*\operatorname{ArcCsch}[c*x])/32 - (a + b*\operatorname{ArcCsch}[c*x])/(4*x^4)$

#### Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x\_Symbol] := \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ \operatorname{PosQ}[b]$

#### Rule 321

$\operatorname{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] := \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \operatorname{Dist}[(a*c^{(n*(m-n+1))})/(b*(m+n*p+1)), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{GtQ}[m, n-1] \ \&\& \ \operatorname{NeQ}[m+n*p+1, 0] \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 335

$\operatorname{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] := -\operatorname{Subst}[\operatorname{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x] /;$   $\operatorname{FreeQ}\{a, b, p\}, x \ \&\& \ \operatorname{ILtQ}[n, 0] \ \&\& \ \operatorname{IntegerQ}[m]$

#### Rule 6284

$\operatorname{Int}[((a_) + \operatorname{ArcCsch}[(c_)*(x_)]*(b_))*((d_)*(x_))^{(m_)}, x\_Symbol] := \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcCsch}[c*x])/(d*(m+1)), x] + \operatorname{Dist}[(b*d)/(c*(m+1)), \operatorname{Int}[(d*x)^{(m-1)}/\operatorname{Sqrt}[1 + 1/(c^2*x^2)], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \operatorname{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^5} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{4x^4} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{4c} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{4x^4} + \frac{b \operatorname{Subst} \left( \int \frac{x^4}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4c} \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{16x^3} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4x^4} - \frac{1}{16} (3bc) \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{16x^3} - \frac{3bc^3 \sqrt{1 + \frac{1}{c^2 x^2}}}{32x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4x^4} + \frac{1}{32} (3bc^3) \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{16x^3} - \frac{3bc^3 \sqrt{1 + \frac{1}{c^2 x^2}}}{32x} + \frac{3}{32} bc^4 \operatorname{csch}^{-1}(cx) - \frac{a + b \operatorname{csch}^{-1}(cx)}{4x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 78, normalized size = 1.05

$$-\frac{a}{4x^4} + \frac{3}{32} bc^4 \sinh^{-1} \left( \frac{1}{cx} \right) + b \left( \frac{c}{16x^3} - \frac{3c^3}{32x} \right) \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - \frac{b \operatorname{csch}^{-1}(cx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/x^5, x]

[Out] -1/4\*a/x^4 + b\*(c/(16\*x^3) - (3\*c^3)/(32\*x))\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcCsch[c\*x])/(4\*x^4) + (3\*b\*c^4\*ArcSinh[1/(c\*x)])/32

**fricas [A]** time = 1.02, size = 89, normalized size = 1.20

$$\frac{(3bc^4x^4 - 8b) \log \left( \frac{cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx} \right) - (3bc^3x^3 - 2bcx) \sqrt{\frac{c^2x^2+1}{c^2x^2}} - 8a}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^5, x, algorithm="fricas")

[Out] 1/32\*((3\*b\*c^4\*x^4 - 8\*b)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - (3\*b\*c^3\*x^3 - 2\*b\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - 8\*a)/x^4

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^5, x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/x^5, x)

**maple [A]** time = 0.05, size = 120, normalized size = 1.62

$$c^4 \left( -\frac{a}{4c^4x^4} + b \left( -\frac{\operatorname{arccsch}(cx)}{4c^4x^4} + \frac{\sqrt{c^2x^2+1} \left( 3 \operatorname{arctanh} \left( \frac{1}{\sqrt{c^2x^2+1}} \right) c^4x^4 - 3c^2x^2\sqrt{c^2x^2+1} + 2\sqrt{c^2x^2+1} \right)}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}} c^5x^5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsch(c*x))/x^5,x)`

[Out]  $c^4*(-1/4*a/c^4/x^4+b*(-1/4/c^4/x^4*arccsch(c*x)+1/32*(c^2*x^2+1)^{(1/2)}*(3*arctanh(1/(c^2*x^2+1)^{(1/2)})*c^4*x^4-3*c^2*x^2*(c^2*x^2+1)^{(1/2)}+2*(c^2*x^2+1)^{(1/2)}))/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/c^5/x^5)$

**maxima** [B] time = 0.33, size = 147, normalized size = 1.99

$$\frac{1}{64} b \left( \frac{3c^5 \log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right) - 3c^5 \log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right) - \frac{2\left(3c^8x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 5c^6x\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2 - 2c^2x^2\left(\frac{1}{c^2x^2}+1\right)+1}}{c} - \frac{16 \operatorname{arcsch}(cx)}{x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsch(c*x))/x^5,x, algorithm="maxima")`

[Out]  $1/64*b*((3*c^5*\log(c*x*\sqrt{1/(c^2*x^2)+1})+1)-3*c^5*\log(c*x*\sqrt{1/(c^2*x^2)+1}-1)-2*(3*c^8*x^3*(1/(c^2*x^2)+1)^{(3/2)}-5*c^6*x*\sqrt{1/(c^2*x^2)+1}))/((c^4*x^4*(1/(c^2*x^2)+1)^2-2*c^2*x^2*(1/(c^2*x^2)+1)+1))/c-16*arccsch(c*x)/x^4-1/4*a/x^4$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))/x^5,x)`

[Out] `int((a + b*asinh(1/(c*x)))/x^5, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))/x**5,x)`

[Out] `Integral((a + b*acsch(c*x))/x**5, x)`

### 3.13 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^6} dx$

**Optimal.** Leaf size=79

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{5x^5} + \frac{1}{25}bc^5\left(\frac{1}{c^2x^2}+1\right)^{5/2} - \frac{2}{15}bc^5\left(\frac{1}{c^2x^2}+1\right)^{3/2} + \frac{1}{5}bc^5\sqrt{\frac{1}{c^2x^2}+1}$$

[Out]  $-2/15*b*c^5*(1+1/c^2/x^2)^(3/2)+1/25*b*c^5*(1+1/c^2/x^2)^(5/2)+1/5*(-a-b*\operatorname{arcsch}(c*x))/x^5+1/5*b*c^5*(1+1/c^2/x^2)^(1/2)$

**Rubi [A]** time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$ , Rules used = {6284, 266, 43}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{5x^5} + \frac{1}{25}bc^5\left(\frac{1}{c^2x^2}+1\right)^{5/2} - \frac{2}{15}bc^5\left(\frac{1}{c^2x^2}+1\right)^{3/2} + \frac{1}{5}bc^5\sqrt{\frac{1}{c^2x^2}+1}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcCsch[c*x])/x^6, x]`

[Out]  $(b*c^5*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]/5 - (2*b*c^5*(1 + 1/(c^2*x^2))^(3/2))/15 + (b*c^5*(1 + 1/(c^2*x^2))^(5/2))/25 - (a + b*\operatorname{ArcCsch}[c*x])/(5*x^5)$

#### Rule 43

`Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

#### Rule 266

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

#### Rule 6284

`Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(d*(m + 1)), x] + Dist[(b*d)/(c*(m + 1)), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]`

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^6} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{5x^5} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^7} dx}{5c} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{5x^5} + \frac{b \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2} \right)}{10c} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{5x^5} + \frac{b \operatorname{Subst} \left( \int \left( \frac{c^4}{\sqrt{1 + \frac{x}{c^2}}} - 2c^4 \sqrt{1 + \frac{x}{c^2}} + c^4 \left(1 + \frac{x}{c^2}\right)^{3/2} \right) dx, x, \frac{1}{x^2} \right)}{10c} \\
&= \frac{1}{5} b c^5 \sqrt{1 + \frac{1}{c^2 x^2}} - \frac{2}{15} b c^5 \left(1 + \frac{1}{c^2 x^2}\right)^{3/2} + \frac{1}{25} b c^5 \left(1 + \frac{1}{c^2 x^2}\right)^{5/2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{5x^5}
\end{aligned}$$

**Mathematica** [A] time = 0.06, size = 69, normalized size = 0.87

$$-\frac{a}{5x^5} + b \left( \frac{8c^5}{75} - \frac{4c^3}{75x^2} + \frac{c}{25x^4} \right) \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - \frac{b \operatorname{csch}^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/x^6,x]

[Out] -1/5\*a/x^5 + b\*((8\*c^5)/75 + c/(25\*x^4) - (4\*c^3)/(75\*x^2))\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcCsch[c\*x])/(5\*x^5)

**fricas** [A] time = 0.48, size = 87, normalized size = 1.10

$$\frac{15 b \log \left( \frac{c x \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1}{c x} \right) - (8 b c^5 x^5 - 4 b c^3 x^3 + 3 b c x) \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 15 a}{75 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^6,x, algorithm="fricas")

[Out] -1/75\*(15\*b\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - (8\*b\*c^5\*x^5 - 4\*b\*c^3\*x^3 + 3\*b\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 15\*a)/x^5

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^6,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/x^6, x)

**maple** [A] time = 0.05, size = 83, normalized size = 1.05

$$c^5 \left( -\frac{a}{5c^5 x^5} + b \left( -\frac{\operatorname{arcsch}(cx)}{5c^5 x^5} + \frac{(c^2 x^2 + 1)(8c^4 x^4 - 4c^2 x^2 + 3)}{75 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} c^6 x^6} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^6,x)

[Out]  $c^5*(-1/5*a/c^5/x^5+b*(-1/5/c^5/x^5*arccsch(c*x)+1/75*(c^2*x^2+1)*(8*c^4*x^4-4*c^2*x^2+3)/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/c^6/x^6))$

**maxima** [A] time = 0.31, size = 73, normalized size = 0.92

$$\frac{1}{75} b \left( \frac{3c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} - 10c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 15c^6 \sqrt{\frac{1}{c^2x^2} + 1}}{c} - \frac{15 \operatorname{arcsch}(cx)}{x^5} \right) - \frac{a}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^6,x, algorithm="maxima")

[Out]  $1/75*b*((3*c^6*(1/(c^2*x^2) + 1)^{(5/2)} - 10*c^6*(1/(c^2*x^2) + 1)^{(3/2)} + 15*c^6*\sqrt{1/(c^2*x^2) + 1})/c - 15*arccsch(c*x)/x^5) - 1/5*a/x^5$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/x^6,x)

[Out] int((a + b\*asinh(1/(c\*x)))/x^6, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*6,x)

[Out] Integral((a + b\*acsch(c\*x))/x\*\*6, x)

### 3.14 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7} dx$

**Optimal.** Leaf size=98

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6} - \frac{5}{96}bc^6\operatorname{csch}^{-1}(cx) + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{36x^5} + \frac{5bc^5\sqrt{\frac{1}{c^2x^2}+1}}{96x} - \frac{5bc^3\sqrt{\frac{1}{c^2x^2}+1}}{144x^3}$$

[Out]  $-5/96*b*c^6*\operatorname{arccsch}(c*x)+1/6*(-a-b*\operatorname{arccsch}(c*x))/x^6+1/36*b*c*(1+1/c^2/x^2)^{(1/2)}/x^5-5/144*b*c^3*(1+1/c^2/x^2)^{(1/2)}/x^3+5/96*b*c^5*(1+1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6284, 335, 321, 215}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6} + \frac{5bc^5\sqrt{\frac{1}{c^2x^2}+1}}{96x} - \frac{5bc^3\sqrt{\frac{1}{c^2x^2}+1}}{144x^3} + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{36x^5} - \frac{5}{96}bc^6\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])/x^7, x]$

[Out]  $(b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(36*x^5) - (5*b*c^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(144*x^3) + (5*b*c^5*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(96*x) - (5*b*c^6*\operatorname{ArcCsch}[c*x])/96 - (a + b*\operatorname{ArcCsch}[c*x])/(6*x^6)$

#### Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*(x_)^2], x\_Symbol] := \operatorname{Simp}[\operatorname{ArcSinh}[\operatorname{Rt}[b, 2]*x]/\operatorname{Sqrt}[a]/\operatorname{Rt}[b, 2], x] /;$   $\operatorname{FreeQ}\{a, b\}, x \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ \operatorname{PosQ}[b]$

#### Rule 321

$\operatorname{Int}[((c_)*(x_))^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] := \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \operatorname{Dist}[(a*c^{(m-n+1)})/(b*(m+n*p+1)), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, p\}, x \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{GtQ}[m, n-1] \ \&\& \ \operatorname{NeQ}[m+n*p+1, 0] \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

#### Rule 335

$\operatorname{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] := -\operatorname{Subst}[\operatorname{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x] /;$   $\operatorname{FreeQ}\{a, b, p\}, x \ \&\& \ \operatorname{ILtQ}[n, 0] \ \&\& \ \operatorname{IntegerQ}[m]$

#### Rule 6284

$\operatorname{Int}[((a_) + \operatorname{ArcCsch}[(c_)*(x_)])*(b_)*((d_)*(x_)^{(m_)}), x\_Symbol] := \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcCsch}[c*x])]/(d*(m+1)), x] + \operatorname{Dist}[(b*d)/(c*(m+1)), \operatorname{Int}[(d*x)^{(m-1)}/\operatorname{Sqrt}[1 + 1/(c^2*x^2)], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m\}, x \ \&\& \ \operatorname{NeQ}[m, -1]$

#### Rubi steps



$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^7} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{6x^6} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^8} dx}{6c} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{6x^6} + \frac{b \operatorname{Subst} \left( \int \frac{x^6}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{6c} \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{36x^5} - \frac{a + b \operatorname{csch}^{-1}(cx)}{6x^6} - \frac{1}{36} (5bc) \operatorname{Subst} \left( \int \frac{x^4}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{36x^5} - \frac{5bc^3 \sqrt{1 + \frac{1}{c^2 x^2}}}{144x^3} - \frac{a + b \operatorname{csch}^{-1}(cx)}{6x^6} + \frac{1}{48} (5bc^3) \operatorname{Subst} \left( \int \frac{x^2}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{36x^5} - \frac{5bc^3 \sqrt{1 + \frac{1}{c^2 x^2}}}{144x^3} + \frac{5bc^5 \sqrt{1 + \frac{1}{c^2 x^2}}}{96x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{6x^6} - \frac{1}{96} (5bc^5) \operatorname{Subst} \left( \int \frac{x^0}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right) \\
&= \frac{bc \sqrt{1 + \frac{1}{c^2 x^2}}}{36x^5} - \frac{5bc^3 \sqrt{1 + \frac{1}{c^2 x^2}}}{144x^3} + \frac{5bc^5 \sqrt{1 + \frac{1}{c^2 x^2}}}{96x} - \frac{5}{96} bc^6 \operatorname{csch}^{-1}(cx) - \frac{a + b \operatorname{csch}^{-1}(cx)}{6x^6}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 88, normalized size = 0.90

$$-\frac{a}{6x^6} - \frac{5}{96} bc^6 \sinh^{-1} \left( \frac{1}{cx} \right) + b \left( \frac{5c^5}{96x} - \frac{5c^3}{144x^3} + \frac{c}{36x^5} \right) \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - \frac{b \operatorname{csch}^{-1}(cx)}{6x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/x^7, x]

[Out] -1/6\*a/x^6 + b\*(c/(36\*x^5) - (5\*c^3)/(144\*x^3) + (5\*c^5)/(96\*x))\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)] - (b\*ArcCsch[c\*x])/(6\*x^6) - (5\*b\*c^6\*ArcSinh[1/(c\*x)])/96

**fricas [A]** time = 1.12, size = 99, normalized size = 1.01

$$\frac{3(5bc^6x^6 + 16b) \log \left( \frac{cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx} \right) - (15bc^5x^5 - 10bc^3x^3 + 8bcx) \sqrt{\frac{c^2x^2+1}{c^2x^2}} + 48a}{288x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^7, x, algorithm="fricas")

[Out] -1/288\*(3\*(5\*b\*c^6\*x^6 + 16\*b)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - (15\*b\*c^5\*x^5 - 10\*b\*c^3\*x^3 + 8\*b\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 48\*a)/x^6

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^7,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/x^7, x)

**maple** [A] time = 0.06, size = 139, normalized size = 1.42

$$c^6 \left( -\frac{a}{6c^6x^6} + b \left( -\frac{\operatorname{arccsch}(cx)}{6c^6x^6} - \frac{\sqrt{c^2x^2+1} \left( 15 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right) c^6x^6 - 15c^4x^4\sqrt{c^2x^2+1} + 10c^2x^2\sqrt{c^2x^2+1} - 1 \right)}{288\sqrt{\frac{c^2x^2+1}{c^2x^2}} c^7x^7} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^7,x)

[Out] c^6\*(-1/6\*a/c^6/x^6+b\*(-1/6/c^6/x^6\*arccsch(c\*x)-1/288\*(c^2\*x^2+1)^(1/2)\*(15\*arctanh(1/(c^2\*x^2+1)^(1/2))\*c^6\*x^6-15\*c^4\*x^4\*(c^2\*x^2+1)^(1/2)+10\*c^2\*x^2\*(c^2\*x^2+1)^(1/2)-8\*(c^2\*x^2+1)^(1/2))/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c^7/x^7))

**maxima** [B] time = 0.31, size = 185, normalized size = 1.89

$$-\frac{1}{576} b \left( \frac{15c^7 \log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right) - 15c^7 \log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right) - \frac{2\left(15c^{12}x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 40c^{10}x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 33c^8x\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6x^6\left(\frac{1}{c^2x^2}+1\right)^3 - 3c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2 + 3c^2x^2\left(\frac{1}{c^2x^2}+1\right) - 1}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^7,x, algorithm="maxima")

[Out] -1/576\*b\*((15\*c^7\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) + 1) - 15\*c^7\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) - 1) - 2\*(15\*c^12\*x^5\*(1/(c^2\*x^2) + 1)^(5/2) - 40\*c^10\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) + 33\*c^8\*x\*sqrt(1/(c^2\*x^2) + 1)))/(c^6\*x^6\*(1/(c^2\*x^2) + 1)^3 - 3\*c^4\*x^4\*(1/(c^2\*x^2) + 1)^2 + 3\*c^2\*x^2\*(1/(c^2\*x^2) + 1) - 1))/c + 96\*arccsch(c\*x)/x^6 - 1/6\*a/x^6

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/x^7,x)

[Out] int((a + b\*asinh(1/(c\*x)))/x^7, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*7,x)

[Out] Integral((a + b\*acsch(c\*x))/x\*\*7, x)

### 3.15 $\int x^3 (a + b \operatorname{csch}^{-1}(cx))^2 dx$

**Optimal.** Leaf size=105

$$\frac{bx^3 \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{6c} - \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{3c^3} + \frac{1}{4} x^4 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{b^2 \log(x)}{3c^4} + \frac{b^2 x^2}{12c^2}$$

[Out]  $1/12*b^2*x^2/c^2+1/4*x^4*(a+b*\operatorname{arccsch}(c*x))^2-1/3*b^2*\ln(x)/c^4-1/3*b*x*(a+b*\operatorname{arccsch}(c*x))*(1+1/c^2/x^2)^{(1/2)}/c^3+1/6*b*x^3*(a+b*\operatorname{arccsch}(c*x))*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6286, 5452, 4185, 4184, 3475}

$$\frac{bx^3 \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{6c} - \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{3c^3} + \frac{1}{4} x^4 (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{b^2 x^2}{12c^2} - \frac{b^2 \log(x)}{3c^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*(a + b*\operatorname{ArcCsch}[c*x])^2, x]$

[Out]  $(b^2*x^2)/(12*c^2) - (b*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*(a + b*\operatorname{ArcCsch}[c*x]))/(3*c^3) + (b*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x^3*(a + b*\operatorname{ArcCsch}[c*x]))/(6*c) + (x^4*(a + b*\operatorname{ArcCsch}[c*x])^2)/4 - (b^2*\operatorname{Log}[x])/(3*c^4)$

#### Rule 3475

$\operatorname{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[\operatorname{Cos}[c + d*x], x]], d, x] /;$   $\operatorname{FreeQ}\{c, d, x\}$

#### Rule 4184

$\operatorname{Int}[\operatorname{csc}[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\operatorname{Cot}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Cot}[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, x\}$  &&  $\operatorname{GtQ}[m, 0]$

#### Rule 4185

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(n_.)}*((c_.) + (d_.)*(x_.)), x\_Symbol] \rightarrow -\operatorname{Simp}[(b^2*(c + d*x)*\operatorname{Cot}[e + f*x]*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x] + (\operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(c + d*x)*(b*\operatorname{Csc}[e + f*x])^{(n-2)}, x], x] - \operatorname{Simp}[(b^2*d*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f^2*(n-1)*(n-2)), x]) /;$   $\operatorname{FreeQ}\{b, c, d, e, f, x\}$  &&  $\operatorname{GtQ}[n, 1]$  &&  $\operatorname{NeQ}[n, 2]$

#### Rule 5452

$\operatorname{Int}[\operatorname{Coth}[(a_.) + (b_.)*(x_.)]^{(p_.)}*\operatorname{Csch}[(a_.) + (b_.)*(x_.)]^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\operatorname{Csch}[a + b*x]^n/(b*n), x] + \operatorname{Dist}[(d*m)/(b*n), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Csch}[a + b*x]^n, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n, x\}$  &&  $\operatorname{EqQ}[p, 1]$  &&  $\operatorname{GtQ}[m, 0]$

#### Rule 6286

$\operatorname{Int}[(c + d*x)^m*\operatorname{ArcCsch}[(c + d*x)/x]^n*(x)^{(m_.)}, x\_Symbol] \rightarrow -\operatorname{Dist}[(c + d*x)^{(m+1)}*(-1), \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Csch}[x]^{(m+1)}*\operatorname{Coth}[x], x], x, \operatorname{ArcCsch}[c*x]], x] /;$   $\operatorname{FreeQ}\{a, b, c, x\}$  &&  $\operatorname{IntegerQ}[n]$  &&  $\operatorname{IntegerQ}[m]$  &&  $(\operatorname{GtQ}[n, 0] \mid \mid \operatorname{LtQ}[m, -1])$

Rubi steps

$$\begin{aligned}
\int x^3 (a + b \operatorname{csch}^{-1}(cx))^2 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^2 \coth(x) \operatorname{csch}^4(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^4} \\
&= \frac{1}{4} x^4 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{csch}^4(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{2c^4} \\
&= \frac{b^2 x^2}{12c^2} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^3 (a + b \operatorname{csch}^{-1}(cx))}{6c} + \frac{1}{4} x^4 (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{csch}^4(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{2c^4} \\
&= \frac{b^2 x^2}{12c^2} - \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x (a + b \operatorname{csch}^{-1}(cx))}{3c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^3 (a + b \operatorname{csch}^{-1}(cx))}{6c} + \frac{1}{4} x^4 (a + b \operatorname{csch}^{-1}(cx))^2 \\
&= \frac{b^2 x^2}{12c^2} - \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x (a + b \operatorname{csch}^{-1}(cx))}{3c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^3 (a + b \operatorname{csch}^{-1}(cx))}{6c} + \frac{1}{4} x^4 (a + b \operatorname{csch}^{-1}(cx))^2
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 122, normalized size = 1.16

$$\frac{cx \left( 3a^2 c^3 x^3 + 2ab \sqrt{\frac{1}{c^2 x^2} + 1} (c^2 x^2 - 2) + b^2 cx \right) + 2bcx \operatorname{csch}^{-1}(cx) \left( 3ac^3 x^3 + b \sqrt{\frac{1}{c^2 x^2} + 1} (c^2 x^2 - 2) \right) + 3b^2 c^4 x^4 \operatorname{csch}^{-2}(cx)}{12c^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + b\*ArcCsch[c\*x])^2,x]

[Out] (c\*x\*(b^2\*c\*x + 3\*a^2\*c^3\*x^3 + 2\*a\*b\*Sqrt[1 + 1/(c^2\*x^2)]\*(-2 + c^2\*x^2)) + 2\*b\*c\*x\*(3\*a\*c^3\*x^3 + b\*Sqrt[1 + 1/(c^2\*x^2)]\*(-2 + c^2\*x^2))\*ArcCsch[c\*x] + 3\*b^2\*c^4\*x^4\*ArcCsch[c\*x]^2 - 4\*b^2\*Log[x])/(12\*c^4)

**fricas [B]** time = 1.34, size = 272, normalized size = 2.59

$$3b^2c^4x^4 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 + 3a^2c^4x^4 + 6abc^4 \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 6abc^4 \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) + b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))^2,x, algorithm="fricas")

[Out] 1/12\*(3\*b^2\*c^4\*x^4\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 + 3\*a^2\*c^4\*x^4 + 6\*a\*b\*c^4\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - 6\*a\*b\*c^4\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + b^2\*c^2\*x^2 - 4\*b^2\*log(x) + 2\*(3\*a\*b\*c^4\*x^4 - 3\*a\*b\*c^4 + (b^2\*c^3\*x^3 - 2\*b^2\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*(a\*b\*c^3\*x^3 - 2\*a\*b\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^4

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^2 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2\*x^3, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{arccsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsch(c\*x))^2,x)

[Out] int(x^3\*(a+b\*arccsch(c\*x))^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))^2,x, algorithm="maxima")

[Out]  $\frac{1}{4}a^2x^4 + \frac{1}{6}(3x^4\operatorname{arccsch}(cx) + (c^2x^3(1/(c^2x^2) + 1))^{3/2} - 3x\sqrt{1/(c^2x^2) + 1})/c^3)ab + \frac{1}{288}(72x^4\log(\sqrt{c^2x^2 + 1}) + 1)^2 + 1152c^2\int \frac{1}{2}x^5\log(x)/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x)\log(c) - 1152c^2\int \frac{1}{2}x^5\log(\sqrt{c^2x^2 + 1} + 1)/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x)\log(c) + 576c^2\int \frac{1}{2}\sqrt{c^2x^2 + 1}x^5\log(x)^2/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 1152c^2\int \frac{1}{2}\sqrt{c^2x^2 + 1}x^5\log(x)\log(\sqrt{c^2x^2 + 1} + 1)/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 576c^2\int \frac{1}{2}x^5\log(x)^2/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 1152c^2\int \frac{1}{2}x^5\log(x)\log(\sqrt{c^2x^2 + 1} + 1)/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 1152\int \frac{1}{2}x^3\log(x)/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x)\log(c) - 1152\int \frac{1}{2}x^3\log(\sqrt{c^2x^2 + 1} + 1)/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x)\log(c) - 24(6c^2x^2 - 3(c^2x^2 + 1)^2 + 4(c^2x^2 + 1)^{3/2} - 12\sqrt{c^2x^2 + 1} + 6)\log(c)^2/c^4 - 48(3c^2x^2 - 2(c^2x^2 + 1)^{3/2} + 6\sqrt{c^2x^2 + 1} - 3\log(c^2x^2 + 1) + 3)\log(c)^2/c^4 + 144(c^2x^2 - 2\sqrt{c^2x^2 + 1} + 1)\log(c)^2/c^4 + 144(2\sqrt{c^2x^2 + 1} - \log(c^2x^2 + 1))\log(c)^2/c^4 - 48(6c^2x^2 - 3(c^2x^2 + 1)^2 + 4(c^2x^2 + 1)^{3/2}) - 12\sqrt{c^2x^2 + 1} + 6)\log(c)\log(x)/c^4 + 288(c^2x^2 - 2\sqrt{c^2x^2 + 1} + 1)\log(c)\log(x)/c^4 + 48(6c^2x^2 - 3(c^2x^2 + 1)^2 + 4(c^2x^2 + 1)^{3/2} - 12\sqrt{c^2x^2 + 1} + 6)\log(c)\log(\sqrt{c^2x^2 + 1} + 1)/c^4 - 288(c^2x^2 - 2\sqrt{c^2x^2 + 1} + 1)\log(c)\log(\sqrt{c^2x^2 + 1} + 1)/c^4 + 4(18c^2x^2 - 9(c^2x^2 + 1)^2 + 16(c^2x^2 + 1)^{3/2} - 96\sqrt{c^2x^2 + 1} + 66\log(\sqrt{c^2x^2 + 1} + 1) - 30\log(\sqrt{c^2x^2 + 1} - 1) + 18)\log(c)/c^4 + 4(6c^2x^2 + 9(c^2x^2 + 1)^2 - 28(c^2x^2 + 1)^{3/2} + 132\sqrt{c^2x^2 + 1} - 132\log(\sqrt{c^2x^2 + 1} + 1) + 6)\log(c)/c^4 - 144(c^2x^2 - 4\sqrt{c^2x^2 + 1} + 3\log(\sqrt{c^2x^2 + 1} + 1) - \log(\sqrt{c^2x^2 + 1} - 1) + 1)\log(c)/c^4 + 144(c^2x^2 - 6\sqrt{c^2x^2 + 1} + 6\log(\sqrt{c^2x^2 + 1} + 1) + 1)\log(c)/c^4 + 12(6c^2x^2 - 3(c^2x^2 + 1)^2 + 4(c^2x^2 + 1)^{3/2} - 12\sqrt{c^2x^2 + 1} + 6)\log(\sqrt{c^2x^2 + 1} + 1)/c^4 + (6c^2x^2 + 9(c^2x^2 + 1)^2 - 28(c^2x^2 + 1)^{3/2} + 132\sqrt{c^2x^2 + 1} - 132\log(\sqrt{c^2x^2 + 1} + 1) + 6)/c^4 + 576\int \frac{1}{2}\sqrt{c^2x^2 + 1}x^3\log(x)^2/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 1152\int \frac{1}{2}\sqrt{c^2x^2 + 1}x^3\log(x)\log(\sqrt{c^2x^2 + 1} + 1)/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 576\int \frac{1}{2}x^3\log(x)^2/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 1152\int \frac{1}{2}x^3\log(x)\log(\sqrt{c^2x^2 + 1} + 1)/(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x))b^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a + b*asinh(1/(c*x)))^2,x)`

[Out] `int(x^3*(a + b*asinh(1/(c*x)))^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{acsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*acsch(c*x))**2,x)`

[Out] `Integral(x**3*(a + b*acsch(c*x))**2, x)`

### 3.16 $\int x^2 (a + b \operatorname{csch}^{-1}(cx))^2 dx$

**Optimal.** Leaf size=122

$$\frac{2b \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{3c^3} + \frac{bx^2 \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{3c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{b^2 \operatorname{Li}_2\left(\frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right)}{3c^3} - \frac{b^2 \operatorname{Li}_2\left(\frac{1}{c/x - (1 + 1/c^2/x^2)^{1/2}}\right)}{3c^3}$$

[Out]  $\frac{1}{3} b^2 x / c^2 + \frac{1}{3} x^3 (a + b \operatorname{arccsch}(c x))^2 - \frac{2}{3} b (a + b \operatorname{arccsch}(c x)) \operatorname{arctanh}\left(\frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right) / c^3 - \frac{1}{3} b^2 \operatorname{polylog}\left(2, -\frac{1}{c/x - (1 + 1/c^2/x^2)^{1/2}}\right) / c^3 + \frac{1}{3} b^2 \operatorname{polylog}\left(2, \frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right) / c^3 + \frac{1}{3} b x^2 (a + b \operatorname{arccsch}(c x)) (1 + 1/c^2/x^2)^{1/2} / c$

**Rubi [A]** time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6286, 5452, 4185, 4182, 2279, 2391}

$$\frac{b^2 \operatorname{PolyLog}\left(2, -e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3} + \frac{b^2 \operatorname{PolyLog}\left(2, e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3} + \frac{bx^2 \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{3c} - \frac{2b \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{3c^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 (a + b \operatorname{ArcCsch}[c x])^2, x]$

[Out]  $\frac{(b^2 x)/(3c^2) + (b \sqrt{1 + 1/(c^2 x^2)}) x^2 (a + b \operatorname{ArcCsch}[c x])}{(3c)} + \frac{(x^3 (a + b \operatorname{ArcCsch}[c x])^2)/3 - (2b (a + b \operatorname{ArcCsch}[c x]) \operatorname{ArcTanh}[E^{\operatorname{ArcCsch}[c x]}])}{(3c^3)} - \frac{(b^2 \operatorname{PolyLog}[2, -E^{\operatorname{ArcCsch}[c x]}])}{(3c^3)} + \frac{(b^2 \operatorname{PolyLog}[2, E^{\operatorname{ArcCsch}[c x]}])}{(3c^3)}$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[a + (b \cdot ((F)^{(e \cdot ((c \cdot) + (d \cdot)(x))))^{(n \cdot)}], x\_Symbol] :> \operatorname{Dist}[1/(d \cdot e \cdot n \cdot \operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b \cdot x]/x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c \cdot) \cdot ((d \cdot) + (e \cdot)(x)^{(n \cdot)})]/(x \cdot), x\_Symbol] :> -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c \cdot e \cdot x^n)]/n, x] /;$   $\operatorname{FreeQ}\{c, d, e, n\}, x \ \&\& \operatorname{EqQ}[c \cdot d, 1]$

#### Rule 4182

$\operatorname{Int}[\operatorname{csc}[(e \cdot) + (\operatorname{Complex}[0, fz]) \cdot (f \cdot)(x)] \cdot ((c \cdot) + (d \cdot)(x))^{(m \cdot)}, x\_Symbol] :> \operatorname{Simp}[(-2 \cdot (c + d \cdot x)^m \operatorname{ArcTanh}[E^{-(I \cdot e) + f \cdot fz \cdot x}]) / (f \cdot fz \cdot I), x] + (-\operatorname{Dist}[(d \cdot m) / (f \cdot fz \cdot I), \operatorname{Int}[(c + d \cdot x)^{(m - 1)} \operatorname{Log}[1 - E^{-(I \cdot e) + f \cdot fz \cdot x}], x], x] + \operatorname{Dist}[(d \cdot m) / (f \cdot fz \cdot I), \operatorname{Int}[(c + d \cdot x)^{(m - 1)} \operatorname{Log}[1 + E^{-(I \cdot e) + f \cdot fz \cdot x}], x], x]) /;$   $\operatorname{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 4185

$\operatorname{Int}[(\operatorname{csc}[(e \cdot) + (f \cdot)(x)] \cdot (b \cdot))^{(n \cdot)} \cdot ((c \cdot) + (d \cdot)(x)), x\_Symbol] :> -\operatorname{Simp}[(b^2 \cdot (c + d \cdot x) \operatorname{Cot}[e + f \cdot x] \cdot (b \cdot \operatorname{Csc}[e + f \cdot x])^{(n - 2)}) / (f \cdot (n - 1)), x] + (\operatorname{Dist}[(b^2 \cdot (n - 2)) / (n - 1), \operatorname{Int}[(c + d \cdot x) \cdot (b \cdot \operatorname{Csc}[e + f \cdot x])^{(n - 2)}, x], x] - \operatorname{Simp}[(b^2 \cdot d \cdot (b \cdot \operatorname{Csc}[e + f \cdot x])^{(n - 2)}) / (f^2 \cdot (n - 1) \cdot (n - 2)), x]) /;$   $\operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{NeQ}[n, 2]$

#### Rule 5452

$\operatorname{Int}[\operatorname{Coth}[(a \cdot) + (b \cdot)(x)]^{(p \cdot)} \operatorname{Csch}[(a \cdot) + (b \cdot)(x)]^{(n \cdot)} \cdot ((c \cdot) + (d \cdot)(x))^{(m \cdot)}, x\_Symbol] :> -\operatorname{Simp}[(c + d \cdot x)^m \operatorname{Csch}[a + b \cdot x]^n / (b \cdot n),$

$x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^(m - 1)*\text{Csch}[a + b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \&\& \text{EqQ}[p, 1] \&\& \text{GtQ}[m, 0]$

### Rule 6286

$\text{Int}[(a + \text{ArcCsch}[c*x])*(b*x)^(n_1)*(x)^(m_1), x\_Symbol] :> -\text{Dist}[(c^(m + 1))^(n_1), \text{Subst}[\text{Int}[(a + b*x)^n*\text{Csch}[x]^(m + 1)*\text{Coth}[x], x], x, \text{ArcCsch}[c*x]], x] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[m] \&\& (\text{GtQ}[n, 0] \mid \mid \text{LtQ}[m, -1])$

### Rubi steps

$$\begin{aligned} \int x^2 (a + b \operatorname{csch}^{-1}(cx))^2 dx &= -\frac{\text{Subst}\left(\int (a + bx)^2 \coth(x) \operatorname{csch}^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3} \\ &= \frac{1}{3}x^3 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{(2b) \text{Subst}\left(\int (a + bx) \operatorname{csch}^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{3c^3} \\ &= \frac{b^2x}{3c^2} + \frac{b\sqrt{1 + \frac{1}{c^2x^2}}x^2 (a + b \operatorname{csch}^{-1}(cx))}{3c} + \frac{1}{3}x^3 (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{b \text{Subst}\left(\int (a + bx) \operatorname{csch}^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{3c^3} \\ &= \frac{b^2x}{3c^2} + \frac{b\sqrt{1 + \frac{1}{c^2x^2}}x^2 (a + b \operatorname{csch}^{-1}(cx))}{3c} + \frac{1}{3}x^3 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{2b(a + b \operatorname{csch}^{-1}(cx))}{3c^2} \\ &= \frac{b^2x}{3c^2} + \frac{b\sqrt{1 + \frac{1}{c^2x^2}}x^2 (a + b \operatorname{csch}^{-1}(cx))}{3c} + \frac{1}{3}x^3 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{2b(a + b \operatorname{csch}^{-1}(cx))}{3c^2} \\ &= \frac{b^2x}{3c^2} + \frac{b\sqrt{1 + \frac{1}{c^2x^2}}x^2 (a + b \operatorname{csch}^{-1}(cx))}{3c} + \frac{1}{3}x^3 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{2b(a + b \operatorname{csch}^{-1}(cx))}{3c^2} \end{aligned}$$

**Mathematica [A]** time = 1.43, size = 211, normalized size = 1.73

$$a^2c^3x^3 + 2abc^3x^3\operatorname{csch}^{-1}(cx) + abc^2x^2\sqrt{\frac{1}{c^2x^2} + 1} - \frac{abcx\sqrt{\frac{1}{2x^2} + 1}\sinh^{-1}(cx)}{\sqrt{c^2x^2 + 1}} + b^2c^3x^3\operatorname{csch}^{-1}(cx)^2 + b^2c^2x^2\sqrt{\frac{1}{c^2x^2} + 1} \operatorname{csch}^{-1}(cx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcCsch[c\*x])^2,x]

[Out]  $(b^2*c*x + a*b*c^2*\text{Sqrt}[1 + 1/(c^2*x^2)])*x^2 + a^2*c^3*x^3 + b^2*c^2*\text{Sqrt}[1 + 1/(c^2*x^2)]*x^2*\text{ArcCsch}[c*x] + 2*a*b*c^3*x^3*\text{ArcCsch}[c*x] + b^2*c^3*x^3*\text{ArcCsch}[c*x]^2 - (a*b*c*\text{Sqrt}[1 + 1/(c^2*x^2)]*x*\text{ArcSinh}[c*x])/\text{Sqrt}[1 + c^2*x^2] + b^2*\text{ArcCsch}[c*x]*\text{Log}[1 - \text{E}^{\text{ArcCsch}[c*x]}] - b^2*\text{ArcCsch}[c*x]*\text{Log}[1 + \text{E}^{\text{ArcCsch}[c*x]}] + b^2*\text{PolyLog}[2, -\text{E}^{\text{ArcCsch}[c*x]}] - b^2*\text{PolyLog}[2, \text{E}^{\text{ArcCsch}[c*x]}])/(3*c^3)$

**fricas [F]** time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}(b^2x^2 \operatorname{arcsch}(cx)^2 + 2abx^2 \operatorname{arcsch}(cx) + a^2x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))^2,x, algorithm="fricas")

[Out] integral(b^2\*x^2\*arccsch(c\*x)^2 + 2\*a\*b\*x^2\*arccsch(c\*x) + a^2\*x^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^2 x^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2\*x^2, x)

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{arccsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))^2,x)

[Out] int(x^2\*(a+b\*arccsch(c\*x))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a^2 x^3 + \frac{1}{6} \left( 4 x^3 \operatorname{arcsch}(cx) + \frac{2 \sqrt{\frac{1}{c^2 x^2} + 1} \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right) + \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} - 1\right)}{c^2 \left(\frac{1}{c^2 x^2} + 1\right) - c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} - 1\right)}{c^2} \right) ab + \frac{1}{3} x^3 \log\left(\sqrt{c^2 x^2 + 1} + 1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))^2,x, algorithm="maxima")

[Out] 1/3\*a^2\*x^3 + 1/6\*(4\*x^3\*arccsch(c\*x) + (2\*sqrt(1/(c^2\*x^2) + 1))/(c^2\*(1/(c^2\*x^2) + 1) - c^2) - log(sqrt(1/(c^2\*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*a\*b + 1/3\*(x^3\*log(sqrt(c^2\*x^2 + 1) + 1)^2 - 3\*integrate(-1/3\*(3\*c^2\*x^4\*log(c)^2 + 3\*x^2\*log(c)^2 + 3\*(c^2\*x^4 + x^2)\*log(x)^2 + 6\*(c^2\*x^4\*log(c) + x^2\*log(c))\*log(x) - 2\*(3\*c^2\*x^4\*log(c) + 3\*x^2\*log(c) + 3\*(c^2\*x^4 + x^2)\*log(x) + (c^2\*x^4\*(3\*log(c) + 1) + 3\*x^2\*log(c) + 3\*(c^2\*x^4 + x^2)\*log(x))\*sqrt(c^2\*x^2 + 1))\*log(sqrt(c^2\*x^2 + 1) + 1) + 3\*(c^2\*x^4\*log(c)^2 + x^2\*log(c)^2 + (c^2\*x^4 + x^2)\*log(x)^2 + 2\*(c^2\*x^4\*log(c) + x^2\*log(c))\*log(x))\*sqrt(c^2\*x^2 + 1))/(c^2\*x^2 + (c^2\*x^2 + 1)^(3/2) + 1), x))\*b^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*asinh(1/(c\*x)))^2,x)

[Out] int(x^2\*(a + b\*asinh(1/(c\*x)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{acsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))\*\*2,x)

[Out] Integral(x\*\*2\*(a + b\*acsch(c\*x))\*\*2, x)

### 3.17 $\int x \left( a + b \operatorname{csch}^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=54

$$\frac{bx\sqrt{\frac{1}{c^2x^2} + 1} \left( a + b \operatorname{csch}^{-1}(cx) \right)}{c} + \frac{1}{2}x^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)^2 + \frac{b^2 \log(x)}{c^2}$$

[Out]  $\frac{1}{2}x^2(a+b\operatorname{arccsch}(cx))^2 + b^2 \ln(x)/c^2 + bxx(a+b\operatorname{arccsch}(cx))(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6286, 5452, 4184, 3475}

$$\frac{bx\sqrt{\frac{1}{c^2x^2} + 1} \left( a + b \operatorname{csch}^{-1}(cx) \right)}{c} + \frac{1}{2}x^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)^2 + \frac{b^2 \log(x)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*ArcCsch[c\*x])^2,x]

[Out]  $(b\sqrt{1 + 1/(c^2x^2)}xx(a + b\operatorname{ArcCsch}[c*x]))/c + (x^2(a + b\operatorname{ArcCsch}[c*x])^2)/2 + (b^2\operatorname{Log}[x])/c^2$

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^2\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 5452

Int[Coth[(a\_.) + (b\_.)\*(x\_.)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_.)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Csch[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m-1)\*Csch[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := -Dist[(c^(m+1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m+1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rubi steps

$$\begin{aligned}
\int x (a + b \operatorname{csch}^{-1}(cx))^2 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^2 \coth(x) \operatorname{csch}^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{1}{2} x^2 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{csch}^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x (a + b \operatorname{csch}^{-1}(cx))}{c} + \frac{1}{2} x^2 (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{b^2 \operatorname{Subst}\left(\int \coth(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^2} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x (a + b \operatorname{csch}^{-1}(cx))}{c} + \frac{1}{2} x^2 (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{b^2 \log(x)}{c^2}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 87, normalized size = 1.61

$$\frac{acx \left( acx + 2b \sqrt{\frac{1}{c^2 x^2} + 1} \right) + 2bcx \operatorname{csch}^{-1}(cx) \left( acx + b \sqrt{\frac{1}{c^2 x^2} + 1} \right) + b^2 c^2 x^2 \operatorname{csch}^{-1}(cx)^2 + 2b^2 \log(cx)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*ArcCsch[c\*x])^2,x]

[Out] (a\*c\*x\*(2\*b\*Sqrt[1 + 1/(c^2\*x^2)] + a\*c\*x) + 2\*b\*c\*x\*(b\*Sqrt[1 + 1/(c^2\*x^2)] + a\*c\*x)\*ArcCsch[c\*x] + b^2\*c^2\*x^2\*ArcCsch[c\*x]^2 + 2\*b^2\*Log[c\*x])/(2\*c^2)

**fricas [B]** time = 0.78, size = 234, normalized size = 4.33

$$\frac{b^2 c^2 x^2 \log\left(\frac{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1}{cx}\right)^2 + a^2 c^2 x^2 + 2 abc^2 \log\left(cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx + 1\right) - 2 abc^2 \log\left(cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx - 1\right) + 2 a^2 \log(cx)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))^2,x, algorithm="fricas")

[Out] 1/2\*(b^2\*c^2\*x^2\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 + a^2\*c^2\*x^2 + 2\*a\*b\*c^2\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - 2\*a\*b\*c^2\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + 2\*a\*b\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 2\*b^2\*log(x) + 2\*(a\*b\*c^2\*x^2 + b^2\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - a\*b\*c^2)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/c^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2\*x, x)

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{arcsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))^2,x)

[Out] int(x\*(a+b\*arccsch(c\*x))^2,x)

**maxima** [A] time = 0.36, size = 82, normalized size = 1.52

$$\frac{1}{2} b^2 x^2 \operatorname{arcsch}(cx)^2 + \frac{1}{2} a^2 x^2 + \left( x^2 \operatorname{arcsch}(cx) + \frac{x \sqrt{\frac{1}{c^2 x^2} + 1}}{c} \right) ab + \left( \frac{x \sqrt{\frac{1}{c^2 x^2} + 1} \operatorname{arcsch}(cx)}{c} + \frac{\log(x)}{c^2} \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))^2,x, algorithm="maxima")

[Out] 1/2\*b^2\*x^2\*arccsch(c\*x)^2 + 1/2\*a^2\*x^2 + (x^2\*arccsch(c\*x) + x\*sqrt(1/(c^2\*x^2) + 1)/c)\*a\*b + (x\*sqrt(1/(c^2\*x^2) + 1)\*arccsch(c\*x)/c + log(x)/c^2)\*b^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*asinh(1/(c\*x)))^2,x)

[Out] int(x\*(a + b\*asinh(1/(c\*x)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{acsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))\*\*2,x)

[Out] Integral(x\*(a + b\*acsch(c\*x))\*\*2, x)

### 3.18 $\int (a + b \operatorname{csch}^{-1}(cx))^2 dx$

**Optimal.** Leaf size=68

$$x(a + b \operatorname{csch}^{-1}(cx))^2 + \frac{4b \tanh^{-1}(e^{\operatorname{csch}^{-1}(cx)})(a + b \operatorname{csch}^{-1}(cx))}{c} + \frac{2b^2 \operatorname{Li}_2(-e^{\operatorname{csch}^{-1}(cx)})}{c} - \frac{2b^2 \operatorname{Li}_2(e^{\operatorname{csch}^{-1}(cx)})}{c}$$

[Out]  $x*(a+b*\operatorname{arccsch}(c*x))^2+4*b*(a+b*\operatorname{arccsch}(c*x))*\operatorname{arctanh}(1/c/x+(1+1/c^2/x^2)^(1/2))/c+2*b^2*\operatorname{polylog}(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c-2*b^2*\operatorname{polylog}(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c$

**Rubi [A]** time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6280, 5452, 4182, 2279, 2391}

$$\frac{2b^2 \operatorname{PolyLog}(2, -e^{\operatorname{csch}^{-1}(cx)})}{c} - \frac{2b^2 \operatorname{PolyLog}(2, e^{\operatorname{csch}^{-1}(cx)})}{c} + x(a + b \operatorname{csch}^{-1}(cx))^2 + \frac{4b \tanh^{-1}(e^{\operatorname{csch}^{-1}(cx)})(a + b \operatorname{csch}^{-1}(cx))}{c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])^2, x]$

[Out]  $x*(a + b*\operatorname{ArcCsch}[c*x])^2 + (4*b*(a + b*\operatorname{ArcCsch}[c*x])* \operatorname{ArcTanh}[E^{\operatorname{ArcCsch}[c*x]}])/c + (2*b^2*\operatorname{PolyLog}[2, -E^{\operatorname{ArcCsch}[c*x]}])/c - (2*b^2*\operatorname{PolyLog}[2, E^{\operatorname{ArcCsch}[c*x]}])/c$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x\_Symbol]$   
 $\rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /;$   $\operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /;$   $\operatorname{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \operatorname{EqQ}[c*d, 1]$

#### Rule 4182

$\operatorname{Int}[\operatorname{csc}[(e_) + (\operatorname{Complex}[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x\_Symbol] \rightarrow \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}])/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^(m-1)*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^(m-1)*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}], x], x)] /;$   $\operatorname{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 5452

$\operatorname{Int}[\operatorname{Coth}[(a_) + (b_)*(x_)]^(p_)*\operatorname{Csch}[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_))^(m_), x\_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\operatorname{Csch}[a + b*x]^n/(b*n), x] + \operatorname{Dist}[(d*m)/(b*n), \operatorname{Int}[(c + d*x)^(m-1)*\operatorname{Csch}[a + b*x]^n, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n\}, x\} \ \&\& \ \operatorname{EqQ}[p, 1] \ \&\& \ \operatorname{GtQ}[m, 0]$

#### Rule 6280

$\operatorname{Int}[(a_) + \operatorname{ArcCsch}[(c_)*(x_)]*(b_)]^(n_), x\_Symbol] \rightarrow -\operatorname{Dist}[c^{(-1)}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Csch}[x]*\operatorname{Coth}[x], x], x, \operatorname{ArcCsch}[c*x]], x] /;$   $\operatorname{FreeQ}\{a, b, c, n\}, x\} \ \&\& \ \operatorname{IGtQ}[n, 0]$

#### Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{csch}^{-1}(cx))^2 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^2 \coth(x) \operatorname{csch}(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c} \\
&= x (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{(2b) \operatorname{Subst}\left(\int (a + bx) \operatorname{csch}(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c} \\
&= x (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{4b (a + b \operatorname{csch}^{-1}(cx)) \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} + \frac{(2b^2) \operatorname{Subst}\left(\int \log\right)}{c} \\
&= x (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{4b (a + b \operatorname{csch}^{-1}(cx)) \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} + \frac{(2b^2) \operatorname{Subst}\left(\int \log\right)}{c} \\
&= x (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{4b (a + b \operatorname{csch}^{-1}(cx)) \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} + \frac{2b^2 \operatorname{Li}_2\left(-e^{\operatorname{csch}^{-1}(cx)}\right)}{c}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 121, normalized size = 1.78

$$\frac{a^2 cx + 2abcx \operatorname{csch}^{-1}(cx) - 2ab \log\left(\tanh\left(\frac{1}{2} \operatorname{csch}^{-1}(cx)\right)\right) - 2b^2 \operatorname{Li}_2\left(-e^{-\operatorname{csch}^{-1}(cx)}\right) + 2b^2 \operatorname{Li}_2\left(e^{-\operatorname{csch}^{-1}(cx)}\right) + b^2 cx \operatorname{csch}^{-1}(cx)}{c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])^2, x]

[Out] (a^2\*c\*x + 2\*a\*b\*c\*x\*ArcCsch[c\*x] + b^2\*c\*x\*ArcCsch[c\*x]^2 - 2\*b^2\*ArcCsch[c\*x]\*Log[1 - E^(-ArcCsch[c\*x])] + 2\*b^2\*ArcCsch[c\*x]\*Log[1 + E^(-ArcCsch[c\*x])] - 2\*a\*b\*Log[Tanh[ArcCsch[c\*x]/2]] - 2\*b^2\*PolyLog[2, -E^(-ArcCsch[c\*x])] + 2\*b^2\*PolyLog[2, E^(-ArcCsch[c\*x])])/c

**fricas [F]** time = 2.26, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^2 \operatorname{arcsch}(cx)^2 + 2ab \operatorname{arcsch}(cx) + a^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2,x, algorithm="fricas")

[Out] integral(b^2\*arccsch(c\*x)^2 + 2\*a\*b\*arccsch(c\*x) + a^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2, x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^2,x)

[Out] int((a+b\*arccsch(c\*x))^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left( x \log \left( \sqrt{c^2 x^2 + 1} + 1 \right)^2 - \int \frac{c^2 x^2 \log(c)^2 + (c^2 x^2 + 1) \log(x)^2 + \log(c)^2 + 2 (c^2 x^2 \log(c) + \log(c)) \log(x) - 1}{c^2 x^2 + 1} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2,x, algorithm="maxima")

[Out] (x\*log(sqrt(c^2\*x^2 + 1) + 1)^2 - integrate(-(c^2\*x^2\*log(c)^2 + (c^2\*x^2 + 1)\*log(x)^2 + log(c)^2 + 2\*(c^2\*x^2\*log(c) + log(c))\*log(x) - 2\*(c^2\*x^2\*log(c) + (c^2\*x^2 + 1)\*log(x) + (c^2\*x^2\*(log(c) + 1) + (c^2\*x^2 + 1)\*log(x) + log(c))\*sqrt(c^2\*x^2 + 1) + log(c))\*log(sqrt(c^2\*x^2 + 1) + 1) + (c^2\*x^2\*log(c)^2 + (c^2\*x^2 + 1)\*log(x)^2 + log(c)^2 + 2\*(c^2\*x^2\*log(c) + log(c))\*log(x))\*sqrt(c^2\*x^2 + 1))/(c^2\*x^2 + (c^2\*x^2 + 1)^(3/2) + 1), x))\*b^2 + a^2\*x + (2\*c\*x\*arccsch(c\*x) + log(sqrt(1/(c^2\*x^2) + 1) + 1) - log(sqrt(1/(c^2\*x^2) + 1) - 1))\*a\*b/c

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + b \operatorname{asinh} \left( \frac{1}{c x} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))^2,x)

[Out] int((a + b\*asinh(1/(c\*x)))^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*\*2,x)

[Out] Integral((a + b\*acsch(c\*x))\*\*2, x)

$$3.19 \quad \int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x} dx$$

**Optimal.** Leaf size=81

$$-b \operatorname{Li}_2\left(e^{2 \operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx)) + \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3b} - \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{1}{2} b^2 \operatorname{Li}_3\left(e^{2 \operatorname{csch}^{-1}(cx)}\right)$$

[Out]  $1/3*(a+b*\operatorname{arccsch}(c*x))^3/b - (a+b*\operatorname{arccsch}(c*x))^2*\ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2) - b*(a+b*\operatorname{arccsch}(c*x))*\operatorname{polylog}(2, (1/c/x+(1+1/c^2/x^2)^(1/2))^2) + 1/2*b^2*\operatorname{polylog}(3, (1/c/x+(1+1/c^2/x^2)^(1/2))^2)$

**Rubi [A]** time = 0.14, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6286, 3716, 2190, 2531, 2282, 6589}

$$-b \operatorname{PolyLog}\left(2, e^{2 \operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{2} b^2 \operatorname{PolyLog}\left(3, e^{2 \operatorname{csch}^{-1}(cx)}\right) + \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3b} - \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])^2/x, x]$

[Out]  $(a + b*\operatorname{ArcCsch}[c*x])^3/(3*b) - (a + b*\operatorname{ArcCsch}[c*x])^2*\operatorname{Log}[1 - E^(2*\operatorname{ArcCsch}[c*x])] - b*(a + b*\operatorname{ArcCsch}[c*x])* \operatorname{PolyLog}[2, E^(2*\operatorname{ArcCsch}[c*x])] + (b^2*\operatorname{PolyLog}[3, E^(2*\operatorname{ArcCsch}[c*x])])/2$

#### Rule 2190

$\operatorname{Int}[(((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^(g_)*((e_) + (f_)*(x_)))^(n_)), x\_Symbol] :> \operatorname{Simp}[(c + d*x)^m*\operatorname{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*\operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^(m-1)*\operatorname{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a]], x], x] /; \operatorname{FreeQ}[\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] :> \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& !\operatorname{MatchQ}[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; \operatorname{FreeQ}[\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n] \&\& !\operatorname{MatchQ}[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_] /; \operatorname{FreeQ}[\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_)*((F_)^(c_)*((a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)*(x_))^(m_), x\_Symbol] :> -\operatorname{Simp}[(f + g*x)^m*\operatorname{PolyLog}[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*\operatorname{Log}[F]), x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f + g*x)^(m-1)*\operatorname{PolyLog}[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; \operatorname{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0]$

#### Rule 3716

$\operatorname{Int}[((c_) + (d_)*(x_))^(m_)*\tan[(e_) + \operatorname{Pi}*(k_) + (\operatorname{Complex}[0, fz_])*(f_)*(x_)], x\_Symbol] :> -\operatorname{Simp}[(I*(c + d*x)^(m+1))/(d*(m+1)), x] + \operatorname{Dist}[2*I, \operatorname{Int}[(c + d*x)^m*E^(2*(-(I*e) + f*fz*x))]/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; \operatorname{FreeQ}[\{c, d, e, f, fz\}, x] \&\& \operatorname{IntegerQ}[4*k] \&\& \operatorname{IGtQ}[m, 0]$



Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m + 1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x} dx &= -\operatorname{Subst}\left(\int (a + bx)^2 \operatorname{coth}(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3b} + 2 \operatorname{Subst}\left(\int \frac{e^{2x}(a + bx)^2}{1 - e^{2x}} dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3b} - (a + b \operatorname{csch}^{-1}(cx))^2 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) + (2b) \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{coth}(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3b} - (a + b \operatorname{csch}^{-1}(cx))^2 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) - b(a + b \operatorname{csch}^{-1}(cx))^2 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) \\ &= \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3b} - (a + b \operatorname{csch}^{-1}(cx))^2 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) - b(a + b \operatorname{csch}^{-1}(cx))^2 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) \\ &= \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3b} - (a + b \operatorname{csch}^{-1}(cx))^2 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) - b(a + b \operatorname{csch}^{-1}(cx))^2 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) \end{aligned}$$

**Mathematica** [A] time = 0.13, size = 115, normalized size = 1.42

$$a^2 \log(cx) + ab \left( \operatorname{Li}_2\left(e^{-2 \operatorname{csch}^{-1}(cx)}\right) - \operatorname{csch}^{-1}(cx) \left( \operatorname{csch}^{-1}(cx) + 2 \log\left(1 - e^{-2 \operatorname{csch}^{-1}(cx)}\right) \right) \right) + b^2 \left( -\operatorname{csch}^{-1}(cx) \operatorname{Li}_2\left(e^{2 \operatorname{csch}^{-1}(cx)}\right) - \operatorname{csch}^{-1}(cx) \left( \operatorname{csch}^{-1}(cx) + 2 \log\left(1 - e^{-2 \operatorname{csch}^{-1}(cx)}\right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])^2/x, x]

[Out] a^2\*Log[c\*x] + a\*b\*(-(ArcCsch[c\*x]\*(ArcCsch[c\*x] + 2\*Log[1 - E^(-2\*ArcCsch[c\*x])])) + PolyLog[2, E^(-2\*ArcCsch[c\*x])]) + b^2\*(ArcCsch[c\*x]^3/3 - ArcCsch[c\*x]^2\*Log[1 - E^(2\*ArcCsch[c\*x])] - ArcCsch[c\*x]\*PolyLog[2, E^(2\*ArcCsch[c\*x])]) + PolyLog[3, E^(2\*ArcCsch[c\*x])]/2)

**fricas** [F] time = 0.53, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b^2 \operatorname{arcsch}(cx)^2 + 2ab \operatorname{arcsch}(cx) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x, x, algorithm="fricas")

[Out] integral((b^2\*arccsch(c\*x)^2 + 2\*a\*b\*arccsch(c\*x) + a^2)/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsch(c\*x))^2/x,x, algorithm="giac")

[Out] integrate((b\*arcsch(c\*x) + a)^2/x, x)

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsch}(cx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arcsch(c\*x))^2/x,x)

[Out] int((a+b\*arcsch(c\*x))^2/x,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 \log(x) \log\left(\sqrt{c^2 x^2 + 1} + 1\right)^2 + a^2 \log(x) - \int \frac{b^2 \log(c)^2 + (b^2 c^2 \log(c)^2 - 2abc^2 \log(c))x^2 - 2ab \log(c) + (b^2 c^2}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arcsch(c\*x))^2/x,x, algorithm="maxima")

[Out] b^2\*log(x)\*log(sqrt(c^2\*x^2 + 1) + 1)^2 + a^2\*log(x) - integrate(-(b^2\*log(c)^2 + (b^2\*c^2\*log(c)^2 - 2\*a\*b\*c^2\*log(c))\*x^2 - 2\*a\*b\*log(c) + (b^2\*c^2\*x^2 + b^2)\*log(x)^2 + 2\*((b^2\*c^2\*log(c) - a\*b\*c^2)\*x^2 + b^2\*log(c) - a\*b)\*log(x) - 2\*((b^2\*c^2\*log(c) - a\*b\*c^2)\*x^2 + b^2\*log(c) - a\*b + (b^2\*c^2\*x^2 + b^2)\*log(x) + sqrt(c^2\*x^2 + 1))\*((b^2\*c^2\*log(c) - a\*b\*c^2)\*x^2 + b^2\*log(c) - a\*b + (2\*b^2\*c^2\*x^2 + b^2)\*log(x)))\*log(sqrt(c^2\*x^2 + 1) + 1) + sqrt(c^2\*x^2 + 1)\*(b^2\*log(c)^2 + (b^2\*c^2\*log(c)^2 - 2\*a\*b\*c^2\*log(c))\*x^2 - 2\*a\*b\*log(c) + (b^2\*c^2\*x^2 + b^2)\*log(x)^2 + 2\*((b^2\*c^2\*log(c) - a\*b\*c^2)\*x^2 + b^2\*log(c) - a\*b)\*log(x)))/(c^2\*x^3 + (c^2\*x^3 + x)\*sqrt(c^2\*x^2 + 1) + x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))^2/x,x)

[Out] int((a + b\*asinh(1/(c\*x)))^2/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*2/x,x)

[Out] Integral((a + b\*acsch(c\*x))\*2/x, x)

$$3.20 \quad \int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x^2} dx$$

Optimal. Leaf size=49

$$2bc\sqrt{\frac{1}{c^2x^2} + 1} (a + b \operatorname{csch}^{-1}(cx)) - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x} - \frac{2b^2}{x}$$

[Out]  $-2*b^2/x - (a + b*\operatorname{arccsch}(c*x))^2/x + 2*b*c*(a + b*\operatorname{arccsch}(c*x))*(1 + 1/c^2/x^2)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6286, 3296, 2637}

$$2bc\sqrt{\frac{1}{c^2x^2} + 1} (a + b \operatorname{csch}^{-1}(cx)) - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x} - \frac{2b^2}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])^2/x^2, x]

[Out]  $(-2*b^2)/x + 2*b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*(a + b*\operatorname{ArcCsch}[c*x]) - (a + b*\operatorname{ArcCsch}[c*x])^2/x$

Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[(c + d\*x)^m\*cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := -Dist[(c^(m+1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m+1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x^2} dx &= -\left(c \operatorname{Subst}\left(\int (a + bx)^2 \cosh(x) dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x} + (2bc) \operatorname{Subst}\left(\int (a + bx) \sinh(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= 2bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx)) - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x} - (2b^2c) \operatorname{Subst}\left(\int \cosh(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= -\frac{2b^2}{x} + 2bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx)) - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x} \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 70, normalized size = 1.43

$$\frac{a^2 - 2abcx\sqrt{\frac{1}{c^2x^2} + 1} + 2bcsch^{-1}(cx)\left(a - bcx\sqrt{\frac{1}{c^2x^2} + 1}\right) + b^2csch^{-1}(cx)^2 + 2b^2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])^2/x^2,x]

[Out] -((a^2 + 2\*b^2 - 2\*a\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x + 2\*b\*(a - b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x)\*ArcCsch[c\*x] + b^2\*ArcCsch[c\*x]^2)/x)

**fricas [B]** time = 1.07, size = 139, normalized size = 2.84

$$\frac{2abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - b^2 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - a^2 - 2b^2 + 2\left(b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - ab\right) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^2,x, algorithm="fricas")

[Out] (2\*a\*b\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - b^2\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 - a^2 - 2\*b^2 + 2\*(b^2\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - a\*b)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/x

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2/x^2, x)

**maple [F]** time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^2/x^2,x)

[Out] int((a+b\*arccsch(c\*x))^2/x^2,x)

**maxima [A]** time = 0.33, size = 78, normalized size = 1.59

$$2\left(c\sqrt{\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsch}(cx)}{x}\right)ab + 2\left(c\sqrt{\frac{1}{c^2x^2} + 1} \operatorname{arcsch}(cx) - \frac{1}{x}\right)b^2 - \frac{b^2 \operatorname{arcsch}(cx)^2}{x} - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^2,x, algorithm="maxima")

[Out] 2\*(c\*sqrt(1/(c^2\*x^2) + 1) - arccsch(c\*x)/x)\*a\*b + 2\*(c\*sqrt(1/(c^2\*x^2) + 1)\*arccsch(c\*x) - 1/x)\*b^2 - b^2\*arccsch(c\*x)^2/x - a^2/x

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))^2/x^2, x)

[Out] int((a + b\*asinh(1/(c\*x)))^2/x^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*\*2/x\*\*2, x)

[Out] Integral((a + b\*acsch(c\*x))\*\*2/x\*\*2, x)

$$3.21 \quad \int \frac{(a+b\operatorname{csch}^{-1}(cx))^2}{x^3} dx$$

Optimal. Leaf size=87

$$\frac{bc\sqrt{\frac{1}{c^2x^2}+1}(a+b\operatorname{csch}^{-1}(cx))}{2x} - \frac{1}{2}abc^2\operatorname{csch}^{-1}(cx) - \frac{(a+b\operatorname{csch}^{-1}(cx))^2}{2x^2} - \frac{1}{4}b^2c^2\operatorname{csch}^{-1}(cx)^2 - \frac{b^2}{4x^2}$$

[Out]  $-1/4*b^2/x^2-1/2*a*b*c^2*\operatorname{arccsch}(c*x)-1/4*b^2*c^2*\operatorname{arccsch}(c*x)^2-1/2*(a+b*\operatorname{arccsch}(c*x))^2/x^2+1/2*b*c*(a+b*\operatorname{arccsch}(c*x))*(1+1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6286, 5446, 3310}

$$\frac{bc\sqrt{\frac{1}{c^2x^2}+1}(a+b\operatorname{csch}^{-1}(cx))}{2x} - \frac{1}{2}abc^2\operatorname{csch}^{-1}(cx) - \frac{(a+b\operatorname{csch}^{-1}(cx))^2}{2x^2} - \frac{1}{4}b^2c^2\operatorname{csch}^{-1}(cx)^2 - \frac{b^2}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])^2/x^3,x]

[Out]  $-b^2/(4*x^2) - (a*b*c^2*ArcCsch[c*x])/2 - (b^2*c^2*ArcCsch[c*x]^2)/4 + (b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(2*x) - (a + b*ArcCsch[c*x])^2/(2*x^2)$

#### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

#### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

#### Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*CsCh[x]^(m + 1)*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x^3} dx &= -\left(c^2 \operatorname{Subst}\left(\int (a + bx)^2 \cosh(x) \sinh(x) dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\
&= -\frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2x^2} + (bc^2) \operatorname{Subst}\left(\int (a + bx) \sinh^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= -\frac{b^2}{4x^2} + \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx))}{2x} - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2x^2} - \frac{1}{2}(bc^2) \operatorname{Subst}\left(\int \right) \\
&= -\frac{b^2}{4x^2} - \frac{1}{2}abc^2 \operatorname{csch}^{-1}(cx) - \frac{1}{4}b^2c^2 \operatorname{csch}^{-1}(cx)^2 + \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx))}{2x} -
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 100, normalized size = 1.15

$$\frac{2a^2 - 2abcx\sqrt{\frac{1}{c^2x^2} + 1} + 2abc^2x^2 \sinh^{-1}\left(\frac{1}{cx}\right) - 2b \operatorname{csch}^{-1}(cx) \left(bcx\sqrt{\frac{1}{c^2x^2} + 1} - 2a\right) + b^2(c^2x^2 + 2) \operatorname{csch}^{-1}(cx)}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])^2/x^3, x]

[Out] -1/4\*(2\*a^2 + b^2 - 2\*a\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x - 2\*b\*(-2\*a + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x)\*ArcCsch[c\*x] + b^2\*(2 + c^2\*x^2)\*ArcCsch[c\*x]^2 + 2\*a\*b\*c^2\*x^2\*ArcSinh[1/(c\*x)])/x^2

**fricas [B]** time = 0.67, size = 163, normalized size = 1.87

$$\frac{2abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - (b^2c^2x^2 + 2b^2) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx}\right)^2 - 2a^2 - b^2 - 2\left(abc^2x^2 - b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 2ab\right) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{c}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^3, x, algorithm="fricas")

[Out] 1/4\*(2\*a\*b\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - (b^2\*c^2\*x^2 + 2\*b^2)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 - 2\*a^2 - b^2 - 2\*(a\*b\*c^2\*x^2 - b^2\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 2\*a\*b)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/x^2

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^3, x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2/x^3, x)

**maple [F]** time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsch}(cx))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^2/x^3,x)

[Out] int((a+b\*arccsch(c\*x))^2/x^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} ab \left( \frac{\frac{2c^4x\sqrt{\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}+1\right)^{-1}} - c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}+1} + 1\right) + c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}+1} - 1\right)}{c} - \frac{4 \operatorname{arcsch}(cx)}{x^2} \right) - \frac{1}{2} b^2 \left( \frac{\log\left(\sqrt{c^2x^2+1}\right)}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^3,x, algorithm="maxima")

[Out] 1/4\*a\*b\*((2\*c^4\*x\*sqrt(1/(c^2\*x^2) + 1)/(c^2\*x^2\*(1/(c^2\*x^2) + 1) - 1) - c^3\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) + 1) + c^3\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) - 1))/c - 4\*arccsch(c\*x)/x^2) - 1/2\*b^2\*(log(sqrt(c^2\*x^2 + 1) + 1)^2/x^2 + 2\*integrate(-(c^2\*x^2\*log(c)^2 + (c^2\*x^2 + 1)\*log(x)^2 + log(c)^2 + 2\*(c^2\*x^2\*log(c) + log(c))\*log(x) - (2\*c^2\*x^2\*log(c) + 2\*(c^2\*x^2 + 1)\*log(x) + (c^2\*x^2\*(2\*log(c) - 1) + 2\*(c^2\*x^2 + 1)\*log(x) + 2\*log(c))\*sqrt(c^2\*x^2 + 1) + 2\*log(c))\*log(sqrt(c^2\*x^2 + 1) + 1) + (c^2\*x^2\*log(c)^2 + (c^2\*x^2 + 1)\*log(x)^2 + log(c)^2 + 2\*(c^2\*x^2\*log(c) + log(c))\*log(x))\*sqrt(c^2\*x^2 + 1))/(c^2\*x^5 + x^3 + (c^2\*x^5 + x^3)\*sqrt(c^2\*x^2 + 1)), x) - 1/2\*a^2/x^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))^2/x^3,x)

[Out] int((a + b\*asinh(1/(c\*x)))^2/x^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*\*2/x\*\*3,x)

[Out] Integral((a + b\*acsch(c\*x))\*\*2/x\*\*3, x)



$$3.22 \quad \int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x^4} dx$$

**Optimal.** Leaf size=100

$$\frac{2bc\sqrt{\frac{1}{c^2x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{9x^2} - \frac{4}{9}bc^3\sqrt{\frac{1}{c^2x^2} + 1} (a + b \operatorname{csch}^{-1}(cx)) - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{3x^3} + \frac{4b^2c^2}{9x} - \frac{2b^2}{27x^3}$$

[Out]  $-2/27*b^2/x^3 + 4/9*b^2*c^2/x - 1/3*(a + b*\operatorname{arccsch}(c*x))^2/x^3 - 4/9*b*c^3*(a + b*\operatorname{arccsch}(c*x))*(1 + 1/c^2/x^2)^{(1/2)} + 2/9*b*c*(a + b*\operatorname{arccsch}(c*x))*(1 + 1/c^2/x^2)^{(1/2)}/x^2$

**Rubi [A]** time = 0.11, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6286, 5446, 3310, 3296, 2637}

$$-\frac{4}{9}bc^3\sqrt{\frac{1}{c^2x^2} + 1} (a + b \operatorname{csch}^{-1}(cx)) + \frac{2bc\sqrt{\frac{1}{c^2x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{9x^2} - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{3x^3} + \frac{4b^2c^2}{9x} - \frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])^2/x^4, x]

[Out]  $(-2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) - (4*b*c^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*(a + b*\operatorname{ArcCsch}[c*x]))/9 + (2*b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*(a + b*\operatorname{ArcCsch}[c*x]))/(9*x^2) - (a + b*\operatorname{ArcCsch}[c*x])^2/(3*x^3)$

**Rule 2637**

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

**Rule 3296**

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[((c + d\*x)^m \* Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1) \* Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

**Rule 3310**

Int[((c\_.) + (d\_.)\*(x\_.))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

**Rule 5446**

Int[Cosh[(a\_.) + (b\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m \* Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1) \* Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

**Rule 6286**

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n \* Csch[x]^(m + 1) \* Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x^4} dx &= -\left(c^3 \operatorname{Subst}\left(\int (a + bx)^2 \cosh(x) \sinh^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\
&= -\frac{(a + b \operatorname{csch}^{-1}(cx))^2}{3x^3} + \frac{1}{3}(2bc^3) \operatorname{Subst}\left(\int (a + bx) \sinh^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= -\frac{2b^2}{27x^3} + \frac{2bc\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx))}{9x^2} - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{3x^3} - \frac{1}{9}(4bc^3) \operatorname{Subst}\left(\int (a + bx) \sinh^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= -\frac{2b^2}{27x^3} - \frac{4}{9}bc^3\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx)) + \frac{2bc\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx))}{9x^2} - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{3x^3} \\
&= -\frac{2b^2}{27x^3} + \frac{4b^2c^2}{9x} - \frac{4}{9}bc^3\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx)) + \frac{2bc\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx))}{9x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 106, normalized size = 1.06

$$\frac{-9a^2 + 6abcx\sqrt{\frac{1}{c^2x^2} + 1}(1 - 2c^2x^2) - 6b \operatorname{csch}^{-1}(cx)\left(3a + bcx\sqrt{\frac{1}{c^2x^2} + 1}(2c^2x^2 - 1)\right) + 2b^2(6c^2x^2 - 1) - 9b^2 \operatorname{csch}^{-1}(cx)^2}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])^2/x^4, x]

[Out] (-9\*a^2 + 6\*a\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(1 - 2\*c^2\*x^2) + 2\*b^2\*(-1 + 6\*c^2\*x^2) - 6\*b\*(3\*a + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-1 + 2\*c^2\*x^2))\*ArcCsch[c\*x] - 9\*b^2\*ArcCsch[c\*x]^2)/(27\*x^3)

**fricas [B]** time = 0.82, size = 178, normalized size = 1.78

$$\frac{12b^2c^2x^2 - 9b^2 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx}\right)^2 - 9a^2 - 2b^2 - 6\left(3ab + (2b^2c^3x^3 - b^2cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx}\right) - 6(2a + b \operatorname{arcsch}(cx))^2}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^4, x, algorithm="fricas")

[Out] 1/27\*(12\*b^2\*c^2\*x^2 - 9\*b^2\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 - 9\*a^2 - 2\*b^2 - 6\*(3\*a\*b + (2\*b^2\*c^3\*x^3 - b^2\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - 6\*(2\*a + b\*arccsch(c\*x))^2)/x^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^4, x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2/x^4, x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsch}(cx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsch(c*x))^2/x^4,x)`

[Out] `int((a+b*arccsch(c*x))^2/x^4,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{9} ab \left( \frac{c^4 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 3 c^4 \sqrt{\frac{1}{c^2 x^2} + 1}}{c} - \frac{3 \operatorname{arcsch}(cx)}{x^3} \right) - \frac{1}{3} b^2 \left( \frac{\log \left( \sqrt{c^2 x^2 + 1} + 1 \right)^2}{x^3} + 3 \int - \frac{3 c^2 x^2 \log(c)^2 + 3}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="maxima")`

[Out] `2/9*a*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3) - 1/3*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^3 + 3*integrate(-1/3*(3*c^2*x^2*log(c)^2 + 3*(c^2*x^2 + 1)*log(x)^2 + 3*log(c)^2 + 6*(c^2*x^2*log(c) + log(c))*log(x) - 2*(3*c^2*x^2*log(c) + 3*(c^2*x^2 + 1)*log(x) + (c^2*x^2*(3*log(c) - 1) + 3*(c^2*x^2 + 1)*log(x) + 3*log(c))*sqrt(c^2*x^2 + 1) + 3*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 3*(c^2*x^2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1)), x) - 1/3*a^2/x^3`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))^2/x^4,x)`

[Out] `int((a + b*asinh(1/(c*x)))^2/x^4, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))**2/x**4,x)`

[Out] `Integral((a + b*acsch(c*x))**2/x**4, x)`

$$3.23 \quad \int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x^5} dx$$

**Optimal.** Leaf size=132

$$\frac{3}{16} abc^4 \operatorname{csch}^{-1}(cx) + \frac{bc \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{8x^3} - \frac{3bc^3 \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{16x} - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{4x^4} + \frac{3}{32} b^2$$

[Out]  $-1/32*b^2/x^4 + 3/32*b^2*c^2/x^2 + 3/16*a*b*c^4*\operatorname{arccsch}(c*x) + 3/32*b^2*c^4*\operatorname{arccsch}(c*x)^2 - 1/4*(a + b*\operatorname{arccsch}(c*x))^2/x^4 + 1/8*b*c*(a + b*\operatorname{arccsch}(c*x))*(1 + 1/c^2/x^2)^{(1/2)}/x^3 - 3/16*b*c^3*(a + b*\operatorname{arccsch}(c*x))*(1 + 1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.12, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6286, 5446, 3310}

$$-\frac{3bc^3 \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{16x} + \frac{bc \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{8x^3} + \frac{3}{16} abc^4 \operatorname{csch}^{-1}(cx) - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{4x^4} + \frac{3}{32} b^2$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])^2/x^5, x]

[Out]  $-b^2/(32*x^4) + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcCsch[c*x])/16 + (3*b^2*c^4*ArcCsch[c*x]^2)/32 + (b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(8*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(16*x) - (a + b*ArcCsch[c*x])^2/(4*x^4)$

#### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

#### Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.))^(n\_)\*(x\_)^(m\_.), x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*CsSch[x]^(m + 1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{x^5} dx &= -\left(c^4 \operatorname{Subst}\left(\int (a + bx)^2 \cosh(x) \sinh^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\
&= -\frac{(a + b \operatorname{csch}^{-1}(cx))^2}{4x^4} + \frac{1}{2} (bc^4) \operatorname{Subst}\left(\int (a + bx) \sinh^4(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= -\frac{b^2}{32x^4} + \frac{bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx))}{8x^3} - \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{4x^4} - \frac{1}{8} (3bc^4) \operatorname{Subst}\left(\int (a + bx) \sinh^4(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\
&= -\frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx))}{8x^3} - \frac{3bc^3\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx))}{16x} \\
&= -\frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16} abc^4 \operatorname{csch}^{-1}(cx) + \frac{3}{32} b^2c^4 \operatorname{csch}^{-1}(cx)^2 + \frac{bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx))}{8x^3}
\end{aligned}$$

**Mathematica [A]** time = 0.18, size = 147, normalized size = 1.11

$$\frac{-8a^2 + 6abc^4x^4 \sinh^{-1}\left(\frac{1}{cx}\right) + 4abcx\sqrt{\frac{1}{c^2x^2} + 1} - 2b \operatorname{csch}^{-1}(cx) \left(8a + bcx\sqrt{\frac{1}{c^2x^2} + 1} (3c^2x^2 - 2)\right) - 6abc^3x^3\sqrt{\frac{1}{c^2x^2} + 1}}{32x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])^2/x^5, x]

[Out] (-8\*a^2 - b^2 + 4\*a\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x + 3\*b^2\*c^2\*x^2 - 6\*a\*b\*c^3\*Sqrt[1 + 1/(c^2\*x^2)]\*x^3 - 2\*b\*(8\*a + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-2 + 3\*c^2\*x^2))\*ArcCsch[c\*x] + b^2\*(-8 + 3\*c^4\*x^4)\*ArcCsch[c\*x]^2 + 6\*a\*b\*c^4\*x^4\*ArcSinh[1/(c\*x)])/(32\*x^4)

**fricas [A]** time = 0.83, size = 202, normalized size = 1.53

$$\frac{3b^2c^2x^2 + (3b^2c^4x^4 - 8b^2) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx}\right) - 8a^2 - b^2 + 2\left(3abc^4x^4 - 8ab - (3b^2c^3x^3 - 2b^2cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^5, x, algorithm="fricas")

[Out] 1/32\*(3\*b^2\*c^2\*x^2 + (3\*b^2\*c^4\*x^4 - 8\*b^2)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 - 8\*a^2 - b^2 + 2\*(3\*a\*b\*c^4\*x^4 - 8\*a\*b - (3\*b^2\*c^3\*x^3 - 2\*b^2\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - 2\*(3\*a\*b\*c^3\*x^3 - 2\*a\*b\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/x^4

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^5, x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2/x^5, x)

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^2/x^5,x)

[Out] int((a+b\*arccsch(c\*x))^2/x^5,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{32} ab \left( \frac{3c^5 \log\left(cx\sqrt{\frac{1}{c^2x^2} + 1} + 1\right) - 3c^5 \log\left(cx\sqrt{\frac{1}{c^2x^2} + 1} - 1\right) - \frac{2\left(3c^8x^3\left(\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 5c^6x\sqrt{\frac{1}{c^2x^2} + 1}\right)}{c^4x^4\left(\frac{1}{c^2x^2} + 1\right)^2 - 2c^2x^2\left(\frac{1}{c^2x^2} + 1\right) + 1}}{c} - \frac{16 \operatorname{arcsch}(cx)}{x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^2/x^5,x, algorithm="maxima")

[Out] 1/32\*a\*b\*((3\*c^5\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) + 1) - 3\*c^5\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) - 1) - 2\*(3\*c^8\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) - 5\*c^6\*x\*sqrt(1/(c^2\*x^2) + 1)))/(c^4\*x^4\*(1/(c^2\*x^2) + 1)^2 - 2\*c^2\*x^2\*(1/(c^2\*x^2) + 1) + 1))/c - 16\*arccsch(c\*x)/x^4 - 1/4\*b^2\*(log(sqrt(c^2\*x^2 + 1) + 1)^2/x^4 + 4\*integrate(-1/2\*(2\*c^2\*x^2\*log(c)^2 + 2\*(c^2\*x^2 + 1)\*log(x)^2 + 2\*log(c)^2 + 4\*(c^2\*x^2\*log(c) + log(c))\*log(x) - (4\*c^2\*x^2\*log(c) + 4\*(c^2\*x^2 + 1)\*log(x) + (c^2\*x^2\*(4\*log(c) - 1) + 4\*(c^2\*x^2 + 1)\*log(x) + 4\*log(c))\*sqrt(c^2\*x^2 + 1) + 4\*log(c))\*log(sqrt(c^2\*x^2 + 1) + 1) + 2\*(c^2\*x^2\*log(c))^2 + (c^2\*x^2 + 1)\*log(x)^2 + log(c)^2 + 2\*(c^2\*x^2\*log(c) + log(c))\*log(x))\*sqrt(c^2\*x^2 + 1))/(c^2\*x^7 + x^5 + (c^2\*x^7 + x^5)\*sqrt(c^2\*x^2 + 1)), x)) - 1/4\*a^2/x^4

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))^2/x^5,x)

[Out] int((a + b\*asinh(1/(c\*x)))^2/x^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*\*2/x\*\*5,x)

[Out] Integral((a + b\*acsch(c\*x))\*\*2/x\*\*5, x)

### 3.24 $\int x^3 (a + b \operatorname{csch}^{-1}(cx))^3 dx$

**Optimal.** Leaf size=195

$$\frac{b^2 \log\left(1 - e^{2\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{c^4} + \frac{b^2 x^2 (a + b \operatorname{csch}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^4} + \frac{bx^3 \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{4c}$$

[Out]  $\frac{1}{4} b^2 x^2 (a + b \operatorname{arccsch}(c x)) / c^2 - \frac{1}{2} b (a + b \operatorname{arccsch}(c x))^2 / c^4 + \frac{1}{4} x^4 (a + b \operatorname{arccsch}(c x))^3 + b^2 (a + b \operatorname{arccsch}(c x)) \ln\left(1 - \left(\frac{1}{c x} + \left(1 + \frac{1}{c^2 x^2}\right)^{1/2}\right)^2\right) / c^4 + \frac{1}{2} b^3 \operatorname{polylog}\left(2, \left(\frac{1}{c x} + \left(1 + \frac{1}{c^2 x^2}\right)^{1/2}\right)^2\right) / c^4 + \frac{1}{4} b^3 x (1 + \frac{1}{c^2 x^2})^{1/2} / c^3 - \frac{1}{2} b x (a + b \operatorname{arccsch}(c x))^2 (1 + \frac{1}{c^2 x^2})^{1/2} / c^3 + \frac{1}{4} b x^3 (a + b \operatorname{arccsch}(c x))^2 (1 + \frac{1}{c^2 x^2})^{1/2} / c$

**Rubi [A]** time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {6286, 5452, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$\frac{b^3 \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)}{2c^4} + \frac{b^2 x^2 (a + b \operatorname{csch}^{-1}(cx))}{4c^2} + \frac{b^2 \log\left(1 - e^{2\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{c^4} + \frac{bx^3 \sqrt{\frac{1}{c^2 x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))}{4c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3 (a + b \operatorname{ArcCsch}[c x])^3, x]$

[Out]  $(b^3 \operatorname{Sqrt}[1 + 1/(c^2 x^2)] x) / (4 c^3) + (b^2 x^2 (a + b \operatorname{ArcCsch}[c x])) / (4 c^2) - (b (a + b \operatorname{ArcCsch}[c x])^2) / (2 c^4) - (b \operatorname{Sqrt}[1 + 1/(c^2 x^2)] x (a + b \operatorname{ArcCsch}[c x])^2) / (2 c^3) + (b \operatorname{Sqrt}[1 + 1/(c^2 x^2)] x^3 (a + b \operatorname{ArcCsch}[c x])^2) / (4 c) + (x^4 (a + b \operatorname{ArcCsch}[c x])^3) / 4 + (b^2 (a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 - E^{(2 \operatorname{ArcCsch}[c x])}]) / c^4 + (b^3 \operatorname{PolyLog}[2, E^{(2 \operatorname{ArcCsch}[c x])}]) / (2 c^4)$

#### Rule 8

$\operatorname{Int}[a_, x\_Symbol] \rightarrow \operatorname{Simp}[a x, x] / ; \operatorname{FreeQ}[a, x]$

#### Rule 2190

$\operatorname{Int}[\frac{((F_)^{((g_.) * ((e_.) + (f_.) * (x_)))})^{(n_.) * ((c_.) + (d_.) * (x_))^{(m_.)}}}{((a_.) + (b_.) * ((F_)^{((g_.) * ((e_.) + (f_.) * (x_)))})^{(n_.)})}, x\_Symbol] \rightarrow \operatorname{Simp}[\frac{(c + d x)^m \operatorname{Log}[1 + (b (F^{(g(e + f x))))^n] / a]}{(b f g n \operatorname{Log}[F])}, x] - \operatorname{Dist}[\frac{(d m)}{(b f g n \operatorname{Log}[F])}, \operatorname{Int}[(c + d x)^{(m-1)} \operatorname{Log}[1 + (b (F^{(g(e + f x))))^n] / a], x], x] / ; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_.) * ((F_)^{((e_.) * ((c_.) + (d_.) * (x_)))})^{(n_.)}], x\_Symbol] \rightarrow \operatorname{Dist}[1 / (d e n \operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b x] / x, x], x, (F^{(e(c + d x))})^n], x] / ; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_.) * ((d_.) + (e_.) * (x_))^{(n_.)}] / (x_), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c e x^n)] / n, x] / ; \operatorname{FreeQ}\{c, d, e, n\}, x] \&\& \operatorname{EqQ}[c d, 1]$

#### Rule 3716

$\operatorname{Int}[\frac{((c_.) + (d_.) * (x_))^{(m_.)} \tan[(e_.) + \operatorname{Pi} * (k_.) + (\operatorname{Complex}[0, f z_]) * (f_.) * (x_)]}{(c_.) + (d_.) * (x_))^{(m_.)}}, x\_Symbol] \rightarrow -\operatorname{Simp}[(I * (c + d x)^{(m+1)}) / (d * (m+1)), x] + \operatorname{Dist}[2$

\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 2)\*(b\*Csc[e + f\*x])^(n - 2), x], x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)^m\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*m\*(c + d\*x)^(m - 1)\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

#### Rule 5452

Int[Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Csch[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csch[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)]\*(b\_.))^(n\_)\*(x\_)^m\_.), x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m + 1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rubi steps



$$\begin{aligned}
\int x^3 (a + b \operatorname{csch}^{-1}(cx))^3 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^3 \coth(x) \operatorname{csch}^4(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^4} \\
&= \frac{1}{4} x^4 (a + b \operatorname{csch}^{-1}(cx))^3 - \frac{(3b) \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{csch}^4(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{4c^4} \\
&= \frac{b^2 x^2 (a + b \operatorname{csch}^{-1}(cx))}{4c^2} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^3 (a + b \operatorname{csch}^{-1}(cx))^2}{4c} + \frac{1}{4} x^4 (a + b \operatorname{csch}^{-1}(cx))^3 \\
&= \frac{b^2 x^2 (a + b \operatorname{csch}^{-1}(cx))}{4c^2} - \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x (a + b \operatorname{csch}^{-1}(cx))^2}{2c^3} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x (a + b \operatorname{csch}^{-1}(cx))^3}{4c} \\
&= \frac{b^3 \sqrt{1 + \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{csch}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{4c} \\
&= \frac{b^3 \sqrt{1 + \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{csch}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{4c} \\
&= \frac{b^3 \sqrt{1 + \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{csch}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{4c} \\
&= \frac{b^3 \sqrt{1 + \frac{1}{c^2 x^2}} x}{4c^3} + \frac{b^2 x^2 (a + b \operatorname{csch}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{4c}
\end{aligned}$$

**Mathematica [A]** time = 1.02, size = 271, normalized size = 1.39

$$a^3 c^4 x^4 + b \operatorname{csch}^{-1}(cx) \left( cx \left( 3a^2 c^3 x^3 + 2ab \sqrt{\frac{1}{c^2 x^2} + 1} (c^2 x^2 - 2) + b^2 cx \right) + 4b^2 \log \left( 1 - e^{-2 \operatorname{csch}^{-1}(cx)} \right) \right) - 2a^2 b c x \sqrt{\frac{1}{c^2 x^2} + 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3\*(a + b\*ArcCsch[c\*x])^3,x]

[Out]  $(-2a^2 b c \sqrt{1 + 1/(c^2 x^2)} x + b^3 c \sqrt{1 + 1/(c^2 x^2)} x + a b^2 c^2 x^2 + a^2 b c^3 \sqrt{1 + 1/(c^2 x^2)} x^3 + a^3 c^4 x^4 + b^2 (3a c^4 x^4 + b (2 - 2c \sqrt{1 + 1/(c^2 x^2)} x + c^3 \sqrt{1 + 1/(c^2 x^2)} x^3)) \operatorname{ArcCsch}[c x]^2 + b^3 c^4 x^4 \operatorname{ArcCsch}[c x]^3 + b \operatorname{ArcCsch}[c x] (c x (b^2 c x + 3a^2 c^3 x^3 + 2a b \sqrt{1 + 1/(c^2 x^2)}) (-2 + c^2 x^2)) + 4b^2 \operatorname{Log}[1 - E^{-2 \operatorname{ArcCsch}[c x]}]) + 4a b^2 \operatorname{Log}[1/(c x)] - 2b^3 \operatorname{PolyLog}[2, E^{-2 \operatorname{ArcCsch}[c x]}]) / (4c^4)$

**fricas [F]** time = 0.63, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^3 x^3 \operatorname{arcsch}(cx)^3 + 3ab^2 x^3 \operatorname{arcsch}(cx)^2 + 3a^2 b x^3 \operatorname{arcsch}(cx) + a^3 x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))^3,x, algorithm="fricas")

[Out]  $\operatorname{integral}(b^3 x^3 \operatorname{arccsch}(c x)^3 + 3a b^2 x^3 \operatorname{arccsch}(c x)^2 + 3a^2 b x^3 \operatorname{arccsch}(c x) + a^3 x^3, x)$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3\*x^3, x)

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{arccsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsch(c\*x))^3,x)

[Out] int(x^3\*(a+b\*arccsch(c\*x))^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}b^3x^4\log(\sqrt{c^2x^2+1}+1)^3 - 12b^3c^2\int\frac{1}{4}x^5\log(x)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c)^2 + 12b^3c^2\int\frac{1}{4}x^5\log(\sqrt{c^2x^2+1}+1)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c)^2 + \frac{1}{4}a^3x^4 - 12b^3c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(x)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c) + 24b^3c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(x)\log(\sqrt{c^2x^2+1}+1)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c) - 12b^3c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(\sqrt{c^2x^2+1}+1)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c) - 12b^3c^2\int\frac{1}{4}x^5\log(x)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c) + 24b^3c^2\int\frac{1}{4}x^5\log(x)\log(\sqrt{c^2x^2+1}+1)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c) - 12b^3c^2\int\frac{1}{4}x^5\log(\sqrt{c^2x^2+1}+1)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c) + 24a^2b^2c^2\int\frac{1}{4}x^5\log(x)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c) - 24a^2b^2c^2\int\frac{1}{4}x^5\log(\sqrt{c^2x^2+1}+1)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)\log(c) - 4b^3c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(x)^3/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) + 12b^3c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(x)^2\log(\sqrt{c^2x^2+1}+1)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) - 12b^3c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(x)\log(\sqrt{c^2x^2+1}+1)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) - 4b^3c^2\int\frac{1}{4}x^5\log(x)^3/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) + 12b^3c^2\int\frac{1}{4}x^5\log(x)^2\log(\sqrt{c^2x^2+1}+1)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) - 12b^3c^2\int\frac{1}{4}x^5\log(x)\log(\sqrt{c^2x^2+1}+1)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) + 12a^2b^2c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(x)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) - 24a^2b^2c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(x)\log(\sqrt{c^2x^2+1}+1)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) + 12a^2b^2c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(\sqrt{c^2x^2+1}+1)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) - 3b^3c^2\int\frac{1}{4}\sqrt{c^2x^2+1}x^5\log(\sqrt{c^2x^2+1}+1)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) + 12a^2b^2c^2\int\frac{1}{4}x^5\log(x)^2/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x) - 24a^2b^2c^2\int\frac{1}{4}x^5\log(x)\log(\sqrt{c^2x^2+1}+1)/(\sqrt{c^2x^2+1}c^2x^2+c^2x^2+\sqrt{c^2x^2+1}+1), x)$

$$\begin{aligned}
& c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 12ab^2c^2 \int (1/4 \\
& x^5 \log(\sqrt{c^2x^2 + 1} + 1)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 12b^3 \int (1/4 x^3 \log(x) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c)^2 + 12b^3 \int (1/4 x^3 \log(\sqrt{c^2x^2 + 1} + 1) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c)^2 - 12b^3 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(x)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c) + 24b^3 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(x) * \log(\sqrt{c^2x^2 + 1} + 1) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c) - 12b^3 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(\sqrt{c^2x^2 + 1} + 1)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c) - 12b^3 \int (1/4 x^3 \log(x)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c) + 24b^3 \int (1/4 x^3 \log(x) * \log(\sqrt{c^2x^2 + 1} + 1) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c) - 12b^3 \int (1/4 x^3 \log(\sqrt{c^2x^2 + 1} + 1)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c) + 24ab^2 \int (1/4 x^3 \log(x) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c) - 24ab^2 \int (1/4 x^3 \log(\sqrt{c^2x^2 + 1} + 1) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) * \log(c) + 1/4 * (3x^4 \operatorname{arccsch}(cx) + (c^2x^3 * (1/(c^2x^2) + 1))^{3/2} - 3x * \sqrt{1/(c^2x^2) + 1}) / c^3) * a^2b - 4b^3 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(x)^3 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 12b^3 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(x)^2 * \log(\sqrt{c^2x^2 + 1} + 1) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 12b^3 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(x) * \log(\sqrt{c^2x^2 + 1} + 1)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 4b^3 \int (1/4 x^3 \log(x)^3 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 12b^3 \int (1/4 x^3 \log(x)^2 * \log(\sqrt{c^2x^2 + 1} + 1) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 12b^3 \int (1/4 x^3 \log(x) * \log(\sqrt{c^2x^2 + 1} + 1)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 12ab^2 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(x)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) - 24ab^2 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(x) * \log(\sqrt{c^2x^2 + 1} + 1) / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 12ab^2 \int (1/4 \sqrt{c^2x^2 + 1} x^3 \log(\sqrt{c^2x^2 + 1} + 1)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 12ab^2 \int (1/4 x^3 \log(\sqrt{c^2x^2 + 1} + 1)^2 / (\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1), x) + 1/12 * (6c^2x^2 - 3(c^2x^2 + 1)^2 + 4(c^2x^2 + 1)^{3/2}) - 12\sqrt{c^2x^2 + 1} + 6) * b^3 \log(c)^3 / c^4 + 1/6 * (3c^2x^2 - 2(c^2x^2 + 1)^{3/2} + 6\sqrt{c^2x^2 + 1} - 3\log(c^2x^2 + 1) + 3) * b^3 \log(c)^3 / c^4 - 1/2 * (c^2x^2 - 2\sqrt{c^2x^2 + 1} + 1) * b^3 \log(c)^3 / c^4 - 1/2 * b^3 * (2\sqrt{c^2x^2 + 1} - \log(c^2x^2 + 1)) * \log(c)^3 / c^4 + 1/4 * (6c^2x^2 - 3(c^2x^2 + 1)^2 + 4(c^2x^2 + 1)^{3/2} - 12\sqrt{c^2x^2 + 1} + 6) * b^3 \log(c)^2 * \log(x) / c^4 - 3/2 * (c^2x^2 - 2\sqrt{c^2x^2 + 1} + 1) * b^3 \log(c)^2 * \log(x) / c^4 - 1/4 * (6c^2x^2 - 3(c^2x^2 + 1)^2 + 4(c^2x^2 + 1)^{3/2} - 12\sqrt{c^2x^2 + 1} + 6) * b^3 \log(c)^2 * \log(\sqrt{c^2x^2 + 1} + 1) / c^4 + 3/2 * (c^2x^2 - 2\sqrt{c^2x^2 + 1} + 1) * b^3 \log(c)^2 * \log(\sqrt{c^2x^2 + 1} + 1) / c^4 - 1/4 * (6c^2x^2 - 3(c^2x^2 + 1)^2 + 4(c^2x^2 + 1)^{3/2} - 12\sqrt{c^2x^2 + 1} + 6) * ab^2 * \log(c)^2 / c^4 - 1/2 * (3c^2x^2 - 2(c^2x^2 + 1)^{3/2} + 6\sqrt{c^2x^2 + 1} - 3\log(c^2x^2 + 1) + 3) * ab^2 * \log(c)^2 / c^4 + 3/2 * (c^2x^2 - 2\sqrt{c^2x^2 + 1} + 1) * ab^2 * \log(c)^2 / c^4 - 1/48 * (18c^2x^2 - 9(c^2x^2 + 1)^2 + 16(c^2x^2 + 1)^{3/2} - 96\sqrt{c^2x^2 + 1} + 66\log(\sqrt{c^2x^2 + 1} + 1) - 30\log(\sqrt{c^2x^2 + 1} - 1) + 18) * b^3 \log(c)^2 / c^4 - 1/48 * (6c^2x^2 + 9(c^2x^2 + 1)^2 - 28(c^2x^2 + 1)^{3/2} + 132\sqrt{c^2x^2 + 1} - 132\log(\sqrt{c^2x^2 + 1} + 1) + 6) * b^3 \log(c)^2 / c^4 + 3/4 * (c
\end{aligned}$$

$$\begin{aligned} &^2*x^2 - 4*\sqrt{c^2*x^2 + 1} + 3*\log(\sqrt{c^2*x^2 + 1} + 1) - \log(\sqrt{c^2*x^2 + 1} - 1) + 1)*b^3*\log(c)^2/c^4 - 3/4*(c^2*x^2 - 6*\sqrt{c^2*x^2 + 1} + \\ &6*\log(\sqrt{c^2*x^2 + 1} + 1) + 1)*b^3*\log(c)^2/c^4 + 3/2*a*b^2*(2*\sqrt{c^2*x^2 + 1} - \log(c^2*x^2 + 1))*\log(c)^2/c^4 - 1/2*(6*c^2*x^2 - 3*(c^2*x^2 + 1) \\ &)^2 + 4*(c^2*x^2 + 1)^{(3/2)} - 12*\sqrt{c^2*x^2 + 1} + 6)*a*b^2*\log(c)*\log(x) /c^4 + 3*(c^2*x^2 - 2*\sqrt{c^2*x^2 + 1} + 1)*a*b^2*\log(c)*\log(x)/c^4 + 1/2* \\ &(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^{(3/2)} - 12*\sqrt{c^2*x^2 + 1} + 6)*a*b^2*\log(c)*\log(\sqrt{c^2*x^2 + 1} + 1)/c^4 - 3*(c^2*x^2 - 2*\sqrt{c^2*x^2 + 1} + 1)*a*b^2*\log(c)*\log(\sqrt{c^2*x^2 + 1} + 1)/c^4 + 1/24*(18*c^2* \\ &*x^2 - 9*(c^2*x^2 + 1)^2 + 16*(c^2*x^2 + 1)^{(3/2)} - 96*\sqrt{c^2*x^2 + 1} + 66*\log(\sqrt{c^2*x^2 + 1} + 1) - 30*\log(\sqrt{c^2*x^2 + 1} - 1) + 18)*a*b^2*\log(c)/c^4 + 1/24*(6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^2 + 1)^{(3/2)} + \\ &132*\sqrt{c^2*x^2 + 1} - 132*\log(\sqrt{c^2*x^2 + 1} + 1) + 6)*a*b^2*\log(c)/c^4 - 3/2*(c^2*x^2 - 4*\sqrt{c^2*x^2 + 1} + 3*\log(\sqrt{c^2*x^2 + 1} + 1) - \log(\sqrt{c^2*x^2 + 1} - 1) + 1)*a*b^2*\log(c)/c^4 + 3/2*(c^2*x^2 - 6*\sqrt{c^2*x^2 + 1} + 6*\log(\sqrt{c^2*x^2 + 1} + 1) + 1)*a*b^2*\log(c)/c^4 \end{aligned}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a + b\*asinh(1/(c\*x)))^3,x)

[Out] int(x^3\*(a + b\*asinh(1/(c\*x)))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{acsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsch(c\*x))\*\*3,x)

[Out] Integral(x\*\*3\*(a + b\*acsch(c\*x))\*\*3, x)

### 3.25 $\int x^2 (a + b \operatorname{csch}^{-1}(cx))^3 dx$

**Optimal.** Leaf size=194

$$\frac{b^2 \operatorname{Li}_2\left(-e^{\operatorname{csch}^{-1}(cx)}\right)(a + b \operatorname{csch}^{-1}(cx))}{c^3} + \frac{b^2 \operatorname{Li}_2\left(e^{\operatorname{csch}^{-1}(cx)}\right)(a + b \operatorname{csch}^{-1}(cx))}{c^3} + \frac{b^2 x (a + b \operatorname{csch}^{-1}(cx))}{c^2} - \frac{b \tanh^{-1}\left(\frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right)}{c^3} + \frac{b^3 \operatorname{PolyLog}\left(3, -e^{\operatorname{csch}^{-1}(cx)}\right)(a + b \operatorname{csch}^{-1}(cx))}{c^3} + \frac{b^3 \operatorname{PolyLog}\left(3, e^{\operatorname{csch}^{-1}(cx)}\right)(a + b \operatorname{csch}^{-1}(cx))}{c^3} + \frac{b^3 \operatorname{PolyLog}\left(3, \frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right)}{c^3}$$

[Out]  $b^2 x (a + b \operatorname{arccsch}(c x)) / c^2 + 1/3 x^3 (a + b \operatorname{arccsch}(c x))^3 - b (a + b \operatorname{arccsch}(c x))^2 \operatorname{arctanh}\left(\frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right) / c^3 + b^3 \operatorname{arctanh}\left(\frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right) / c^3 - b^2 (a + b \operatorname{arccsch}(c x)) \operatorname{polylog}\left(2, -1/c/x - (1 + 1/c^2/x^2)^{1/2}\right) / c^3 + b^2 (a + b \operatorname{arccsch}(c x)) \operatorname{polylog}\left(2, 1/c/x + (1 + 1/c^2/x^2)^{1/2}\right) / c^3 + b^3 \operatorname{polylog}\left(3, -1/c/x - (1 + 1/c^2/x^2)^{1/2}\right) / c^3 - b^3 \operatorname{polylog}\left(3, 1/c/x + (1 + 1/c^2/x^2)^{1/2}\right) / c^3 + 1/2 b^2 x^2 (a + b \operatorname{arccsch}(c x))^2 (1 + 1/c^2/x^2)^{1/2} / c$

**Rubi [A]** time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6286, 5452, 4186, 3770, 4182, 2531, 2282, 6589}

$$\frac{b^2 \operatorname{PolyLog}\left(2, -e^{\operatorname{csch}^{-1}(cx)}\right)(a + b \operatorname{csch}^{-1}(cx))}{c^3} + \frac{b^2 \operatorname{PolyLog}\left(2, e^{\operatorname{csch}^{-1}(cx)}\right)(a + b \operatorname{csch}^{-1}(cx))}{c^3} + \frac{b^3 \operatorname{PolyLog}\left(3, \frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right)}{c^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 (a + b \operatorname{ArcCsch}[c x])^3, x]$

[Out]  $(b^2 x (a + b \operatorname{ArcCsch}[c x])) / c^2 + (b \operatorname{Sqrt}[1 + 1/(c^2 x^2)]) x^2 (a + b \operatorname{ArcCsch}[c x])^2 / (2 c) + (x^3 (a + b \operatorname{ArcCsch}[c x])^3) / 3 - (b (a + b \operatorname{ArcCsch}[c x])^2 \operatorname{ArcTanh}[E^{\operatorname{ArcCsch}[c x]}]) / c^3 + (b^3 \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + 1/(c^2 x^2)]]) / c^3 - (b^2 (a + b \operatorname{ArcCsch}[c x]) \operatorname{PolyLog}[2, -E^{\operatorname{ArcCsch}[c x]}]) / c^3 + (b^2 (a + b \operatorname{ArcCsch}[c x]) \operatorname{PolyLog}[2, E^{\operatorname{ArcCsch}[c x]}]) / c^3 + (b^3 \operatorname{PolyLog}[3, -E^{\operatorname{ArcCsch}[c x]}]) / c^3 - (b^3 \operatorname{PolyLog}[3, E^{\operatorname{ArcCsch}[c x]}]) / c^3$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] := \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; \operatorname{FreeQ}[\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n] \&\& \operatorname{!MatchQ}[u, E^((c_)*((a_)+(b_)*x))*(F_)[v_] /; \operatorname{FreeQ}[\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_)*((F_)^((c_)*((a_)+(b_)*(x_))))^(n_)]*((f_)+(g_)*(x_))^(m_), x\_Symbol] := -\operatorname{Simp}[(f + g x)^m \operatorname{PolyLog}[2, -(e*(F^(c*(a + b*x))))^n] / (b*c*n*\operatorname{Log}[F]), x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f + g*x)^(m-1)*\operatorname{PolyLog}[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; \operatorname{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0]$

#### Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_)+(d_)*(x_)], x\_Symbol] := -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

#### Rule 4182

$\operatorname{Int}[\operatorname{csc}[(e_)+(Complex[0, fz_])*(f_)*(x_)]*((c_)+(d_)*(x_))^(m_), x\_Symbol] := \operatorname{Simp}[(-2*(c + d*x)^m \operatorname{ArcTanh}[E^(-(I*e) + f*fz*x)]] / (f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^(m-1)*\operatorname{Log}[1 - E^(-(I*e) + f*fz*x)], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^(m-1)*\operatorname{Log}[1 + E^(-(I*e) + f*fz*x)], x], x]$

$f*Fz*x)], x], x]) /; FreeQ[\{c, d, e, f, fz\}, x] \&\& IGtQ[m, 0]$

### Rule 4186

$Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x\_Symbol] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[\{b, c, d, e, f\}, x] \&\& GtQ[n, 1] \&\& NeQ[n, 2] \&\& GtQ[m, 1]$

### Rule 5452

$Int[Coth[(a_.) + (b_.)*(x_.)]^(p_.)*Csch[(a_.) + (b_.)*(x_.)]^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x\_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; FreeQ[\{a, b, c, d, n\}, x] \&\& EqQ[p, 1] \&\& GtQ[m, 0]$

### Rule 6286

$Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.), x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[\{a, b, c\}, x] \&\& IntegerQ[n] \&\& IntegerQ[m] \&\& (GtQ[n, 0] || LtQ[m, -1])$

### Rule 6589

$Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[\{a, b, c, d, e, n, p\}, x] \&\& EqQ[b*d, a*e]$

### Rubi steps

$$\begin{aligned} \int x^2 (a + b \operatorname{csch}^{-1}(cx))^3 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^3 \coth(x) \operatorname{csch}^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3} \\ &= \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx))^3 - \frac{b \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{csch}^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^3} \\ &= \frac{b^2 x (a + b \operatorname{csch}^{-1}(cx))}{c^2} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^2 (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx))^3 \\ &= \frac{b^2 x (a + b \operatorname{csch}^{-1}(cx))}{c^2} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^2 (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx))^3 \\ &= \frac{b^2 x (a + b \operatorname{csch}^{-1}(cx))}{c^2} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^2 (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx))^3 \\ &= \frac{b^2 x (a + b \operatorname{csch}^{-1}(cx))}{c^2} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^2 (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx))^3 \\ &= \frac{b^2 x (a + b \operatorname{csch}^{-1}(cx))}{c^2} + \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x^2 (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{3} x^3 (a + b \operatorname{csch}^{-1}(cx))^3 \end{aligned}$$

**Mathematica [B]** time = 7.52, size = 548, normalized size = 2.82

$$\frac{a^3 x^3}{3} + \frac{a^2 b x^2 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}}{2c} - \frac{a^2 b \log\left(x\left(\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1\right)\right)}{2c^3} + a^2 b x^3 \operatorname{csch}^{-1}(cx) + \frac{ab^2 \left(2c^3 x^3 \left(-\frac{4\operatorname{Li}_2\left(e^{-\operatorname{csch}^{-1}(cx)}\right)}{c^3 x^3} + 4\operatorname{csch}^{-1}(cx)\right)\right)}{2c^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*(a + b\*ArcCsch[c\*x])^3,x]

[Out] (a^3\*x^3)/3 + (a^2\*b\*x^2\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)]/(2\*c) + a^2\*b\*x^3\*ArcCsch[c\*x] - (a^2\*b\*Log[x\*(1 + Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)])])/(2\*c^3) + (a\*b^2\*(8\*PolyLog[2, -E^(-ArcCsch[c\*x])] + 2\*c^3\*x^3\*(-2 + 4\*ArcCsch[c\*x]^2 + 2\*Cosh[2\*ArcCsch[c\*x]] - (3\*ArcCsch[c\*x]\*Log[1 - E^(-ArcCsch[c\*x])]))/(c\*x) + (3\*ArcCsch[c\*x]\*Log[1 + E^(-ArcCsch[c\*x])]))/(c\*x) - (4\*PolyLog[2, E^(-ArcCsch[c\*x])])/(c^3\*x^3) + 2\*ArcCsch[c\*x]\*Sinh[2\*ArcCsch[c\*x]] + ArcCsch[c\*x]\*Log[1 - E^(-ArcCsch[c\*x])]\*Sinh[3\*ArcCsch[c\*x]] - ArcCsch[c\*x]\*Log[1 + E^(-ArcCsch[c\*x])]\*Sinh[3\*ArcCsch[c\*x]])))/(8\*c^3) + (b^3\*(24\*ArcCsch[c\*x]\*Coth[ArcCsch[c\*x]/2] - 4\*ArcCsch[c\*x]^3\*Coth[ArcCsch[c\*x]/2] + 6\*ArcCsch[c\*x]^2\*Csch[ArcCsch[c\*x]/2]^2 + (ArcCsch[c\*x]^3\*Csch[ArcCsch[c\*x]/2]^4)/(c\*x) + 24\*ArcCsch[c\*x]^2\*Log[1 - E^(-ArcCsch[c\*x])] - 24\*ArcCsch[c\*x]^2\*Log[1 + E^(-ArcCsch[c\*x])] - 48\*Log[Tanh[ArcCsch[c\*x]/2]] + 48\*ArcCsch[c\*x]\*PolyLog[2, -E^(-ArcCsch[c\*x])] - 48\*ArcCsch[c\*x]\*PolyLog[2, E^(-ArcCsch[c\*x])] + 48\*PolyLog[3, -E^(-ArcCsch[c\*x])] - 48\*PolyLog[3, E^(-ArcCsch[c\*x])] + 6\*ArcCsch[c\*x]^2\*Sech[ArcCsch[c\*x]/2]^2 + 16\*c^3\*x^3\*ArcCsch[c\*x]^3\*Sinh[ArcCsch[c\*x]/2]^4 - 24\*ArcCsch[c\*x]\*Tanh[ArcCsch[c\*x]/2] + 4\*ArcCsch[c\*x]^3\*Tanh[ArcCsch[c\*x]/2]))/(48\*c^3)

**fricas [F]** time = 1.12, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^3 x^2 \operatorname{arcsch}(cx)^3 + 3 a b^2 x^2 \operatorname{arcsch}(cx)^2 + 3 a^2 b x^2 \operatorname{arcsch}(cx) + a^3 x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x^2\*arccsch(c\*x)^3 + 3\*a\*b^2\*x^2\*arccsch(c\*x)^2 + 3\*a^2\*b\*x^2\*arccsch(c\*x) + a^3\*x^2, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^3 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3\*x^2, x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{arccsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))^3,x)

[Out] int(x^2\*(a+b\*arccsch(c\*x))^3,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))^3,x, algorithm="maxima")

[Out] 1/3\*b^3\*x^3\*log(sqrt(c^2\*x^2 + 1) + 1)^3 + 1/3\*a^3\*x^3 + 1/4\*(4\*x^3\*arccsch(c\*x) + (2\*sqrt(1/(c^2\*x^2) + 1)/(c^2\*(1/(c^2\*x^2) + 1) - c^2) - log(sqrt(1/(c^2\*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*a^2\*b - integrate(((b^3\*c^2\*log(c)^3 - 3\*a\*b^2\*c^2\*log(c)^2)\*x^4 + (b^3\*c^2\*x^4 + b^3\*x^2)\*log(x)^3 + (b^3\*log(c)^3 - 3\*a\*b^2\*log(c)^2)\*x^2 + 3\*((b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^4 + (b^3\*log(c) - a\*b^2)\*x^2)\*log(x)^2 + (3\*(b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^4 + 3\*(b^3\*log(c) - a\*b^2)\*x^2 + 3\*(b^3\*c^2\*x^4 + b^3\*x^2)\*log(x) + ((b^3\*c^2\*(3\*log(c) + 1) - 3\*a\*b^2\*c^2)\*x^4 + 3\*(b^3\*log(c) - a\*b^2)\*x^2 + 3\*(b^3\*c^2\*x^4 + b^3\*x^2)\*log(x))\*sqrt(c^2\*x^2 + 1))\*log(sqrt(c^2\*x^2 + 1) + 1)^2 + 3\*((b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^4 + (b^3\*log(c)^2 - 2\*a\*b^2\*log(c))\*x^2)\*log(x) - 3\*((b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^4 + (b^3\*log(c)^2 - 2\*a\*b^2\*log(c))\*x^2 + (b^3\*c^2\*x^4 + b^3\*x^2)\*log(x)^2 + 2\*((b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^4 + (b^3\*log(c) - a\*b^2)\*x^2)\*log(x) + ((b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^4 + (b^3\*log(c)^2 - 2\*a\*b^2\*log(c))\*x^2 + (b^3\*c^2\*x^4 + b^3\*x^2)\*log(x)^2 + 2\*((b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^4 + (b^3\*log(c) - a\*b^2)\*x^2)\*log(x))\*sqrt(c^2\*x^2 + 1))\*log(sqrt(c^2\*x^2 + 1) + 1) + ((b^3\*c^2\*log(c)^3 - 3\*a\*b^2\*c^2\*log(c)^2)\*x^4 + (b^3\*c^2\*x^4 + b^3\*x^2)\*log(x)^3 + (b^3\*log(c)^3 - 3\*a\*b^2\*log(c)^2)\*x^2 + 3\*((b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^4 + (b^3\*log(c) - a\*b^2)\*x^2)\*log(x)^2 + 3\*((b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^4 + (b^3\*log(c)^2 - 2\*a\*b^2\*log(c))\*x^2)\*log(x))\*sqrt(c^2\*x^2 + 1))/(c^2\*x^2 + (c^2\*x^2 + 1)^(3/2) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*asinh(1/(c\*x)))^3,x)

[Out] int(x^2\*(a + b\*asinh(1/(c\*x)))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{acsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))\*\*3,x)

[Out] Integral(x\*\*2\*(a + b\*acsch(c\*x))\*\*3, x)



### 3.26 $\int x \left( a + b \operatorname{csch}^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=117

$$\frac{3b^2 \log\left(1 - e^{2\operatorname{csch}^{-1}(cx)}\right) \left(a + b \operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{3bx \sqrt{\frac{1}{c^2 x^2} + 1} \left(a + b \operatorname{csch}^{-1}(cx)\right)^2}{2c} + \frac{3b \left(a + b \operatorname{csch}^{-1}(cx)\right)^2}{2c^2} + \frac{1}{2} x^2$$

[Out]  $3/2*b*(a+b*\operatorname{arccsch}(c*x))^2/c^2+1/2*x^2*(a+b*\operatorname{arccsch}(c*x))^3-3*b^2*(a+b*\operatorname{arccsch}(c*x))*\ln(1-(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)/c^2-3/2*b^3*\operatorname{polylog}(2,(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)/c^2+3/2*b*x*(a+b*\operatorname{arccsch}(c*x))^2*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.16, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$ , Rules used = {6286, 5452, 4184, 3716, 2190, 2279, 2391}

$$\frac{3b^3 \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)}{2c^2} - \frac{3b^2 \log\left(1 - e^{2\operatorname{csch}^{-1}(cx)}\right) \left(a + b \operatorname{csch}^{-1}(cx)\right)}{c^2} + \frac{3bx \sqrt{\frac{1}{c^2 x^2} + 1} \left(a + b \operatorname{csch}^{-1}(cx)\right)^2}{2c} + \frac{1}{2} x^2$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*(a + b*\operatorname{ArcCsch}[c*x])^3, x]$

[Out]  $(3*b*(a + b*\operatorname{ArcCsch}[c*x])^2)/(2*c^2) + (3*b*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*(a + b*\operatorname{ArcCsch}[c*x])^2)/(2*c) + (x^2*(a + b*\operatorname{ArcCsch}[c*x])^3)/2 - (3*b^2*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 - E^(2*\operatorname{ArcCsch}[c*x])])/c^2 - (3*b^3*\operatorname{PolyLog}[2, E^(2*\operatorname{ArcCsch}[c*x])])/ (2*c^2)$

#### Rule 2190

$\operatorname{Int}[(((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)*((c_) + (d_)*(x_))^\wedge(m_))/((a_) + (b_)*((F_)^\wedge((g_)*(e_) + (f_)*(x_)))^\wedge(n_)), x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^m * \operatorname{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a]] / (b*f*g*n * \operatorname{Log}[F]), x] - \operatorname{Dist}[(d*m) / (b*f*g*n * \operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m-1)} * \operatorname{Log}[1 + (b*(F^{g*(e + f*x)})^n)/a]], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_) + (b_)*((F_)^\wedge((e_)*((c_) + (d_)*(x_)))^\wedge(n_)]], x\_Symbol] \rightarrow \operatorname{Dist}[1/(d*e*n * \operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{e*(c + d*x)})^n], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_)*((d_) + (e_)*(x_)^{n_})]/(x_), x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x] \&\& \operatorname{EqQ}[c*d, 1]$

#### Rule 3716

$\operatorname{Int}[((c_) + (d_)*(x_))^\wedge(m_)*\tan[(e_) + \operatorname{Pi}*(k_) + (\operatorname{Complex}[0, fz_])*(f_)*(x_)], x\_Symbol] \rightarrow -\operatorname{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] + \operatorname{Dist}[2*I, \operatorname{Int}[(c + d*x)^m * E^{(2*(-I*e) + f*fz*x))} / (E^{(2*I*k*Pi)} * (1 + E^{(2*(-I*e) + f*fz*x)}) / E^{(2*I*k*Pi)})], x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \&\& \operatorname{IntegerQ}[4*k] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 4184

$\operatorname{Int}[\operatorname{csc}[(e_) + (f_)*(x_)]^\wedge 2 * ((c_) + (d_)*(x_))^\wedge(m_), x\_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m * \operatorname{Cot}[e + f*x]] / f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)} * \operatorname{Cot}[e + f*x]], x]$

t[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 5452

Int[Coth[(a\_.) + (b\_.)\*(x\_.)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_.)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Csch[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csch[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m + 1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rubi steps

$$\begin{aligned} \int x (a + b \operatorname{csch}^{-1}(cx))^3 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^3 \coth(x) \operatorname{csch}^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c^2} \\ &= \frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx))^3 - \frac{(3b) \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{csch}^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{2c^2} \\ &= \frac{3b\sqrt{1 + \frac{1}{c^2x^2}} x (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx))^3 - \frac{(3b^2) \operatorname{Subst}\left(\int (a + bx) \operatorname{csch}^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{2c^2} \\ &= \frac{3b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1 + \frac{1}{c^2x^2}} x (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx))^3 \\ &= \frac{3b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1 + \frac{1}{c^2x^2}} x (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx))^3 \\ &= \frac{3b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1 + \frac{1}{c^2x^2}} x (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx))^3 \\ &= \frac{3b (a + b \operatorname{csch}^{-1}(cx))^2}{2c^2} + \frac{3b\sqrt{1 + \frac{1}{c^2x^2}} x (a + b \operatorname{csch}^{-1}(cx))^2}{2c} + \frac{1}{2}x^2 (a + b \operatorname{csch}^{-1}(cx))^3 \end{aligned}$$

**Mathematica [A]** time = 0.47, size = 171, normalized size = 1.46

$$\frac{a \left( acx \left( acx + 3b\sqrt{\frac{1}{c^2x^2} + 1} \right) - 6b^2 \log\left(\frac{1}{cx}\right) \right) + 3b^2 \operatorname{csch}^{-1}(cx)^2 \left( ac^2x^2 + b \left( cx\sqrt{\frac{1}{c^2x^2} + 1} - 1 \right) \right) + 3b \operatorname{csch}^{-1}(cx) \left( acx^2 + b \left( cx\sqrt{\frac{1}{c^2x^2} + 1} - 1 \right) \right)}{2c^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x\*(a + b\*ArcCsch[c\*x])^3, x]

[Out] (3\*b^2\*(a\*c^2\*x^2 + b\*(-1 + c\*Sqrt[1 + 1/(c^2\*x^2)]\*x))\*ArcCsch[c\*x]^2 + b^3\*c^2\*x^2\*ArcCsch[c\*x]^3 + 3\*b\*ArcCsch[c\*x]\*(a\*c\*x\*(2\*b\*Sqrt[1 + 1/(c^2\*x^2)] + a\*c\*x) - 2\*b^2\*Log[1 - E^(-2\*ArcCsch[c\*x])]) + a\*(a\*c\*x\*(3\*b\*Sqrt[1 + 1/(c^2\*x^2)] + a\*c\*x) - 6\*b^2\*Log[1/(c\*x)]) + 3\*b^3\*PolyLog[2, E^(-2\*ArcCsch[c\*x])])/(2\*c^2)

**fricas [F]** time = 1.39, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(b^3 x \operatorname{arcsch}(cx)^3 + 3 ab^2 x \operatorname{arcsch}(cx)^2 + 3 a^2 b x \operatorname{arcsch}(cx) + a^3 x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*x\*arccsch(c\*x)^3 + 3\*a\*b^2\*x\*arccsch(c\*x)^2 + 3\*a^2\*b\*x\*arccsch(c\*x) + a^3\*x, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^3 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3\*x, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{arcsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))^3,x)

[Out] int(x\*(a+b\*arccsch(c\*x))^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 3/2*a*b^2*x^2*arccsch(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsch(c*x) + x*\sqrt{(1/(c^2*x^2) + 1)/c}) * a^2*b + 3*(x*\sqrt{(1/(c^2*x^2) + 1)} * arccsch(c*x)/c + \log(x)/c^2) * a*b^2 - 1/4*(24*c^2*\int(1/2*x^3*\log(x)/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1), x) * \log(c)^2 - 24*c^2*\int(1/2*x^3*\log(\sqrt{c^2*x^2 + 1}) + 1)/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) * \log(c)^2 - 2*x^2*\log(\sqrt{c^2*x^2 + 1} + 1)^3 + 24*c^2*\int(1/2*\sqrt{c^2*x^2 + 1}) * x^3*\log(x)^2/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) * \log(c) - 48*c^2*\int(1/2*\sqrt{c^2*x^2 + 1}) * x^3*\log(x) * \log(\sqrt{c^2*x^2 + 1} + 1)/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) * \log(c) + 24*c^2*\int(1/2*\sqrt{c^2*x^2 + 1}) * x^3*\log(\sqrt{c^2*x^2 + 1} + 1)^2/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) * \log(c) + 24*c^2*\int(1/2*x^3*\log(x)^2/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) * \log(c) - 48*c^2*\int(1/2*x^3*\log(x) * \log(\sqrt{c^2*x^2 + 1} + 1)/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) * \log(c) + 24*c^2*\int(1/2*x^3*\log(\sqrt{c^2*x^2 + 1} + 1)^2/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) * \log(c) + 8*c^2*\int(1/2*\sqrt{c^2*x^2 + 1}) * x^3*\log(x)^3/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) - 24*c^2*\int(1/2*\sqrt{c^2*x^2 + 1}) * x^3*\log(x)^2 * \log(\sqrt{c^2*x^2 + 1} + 1)/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) + 24*c^2*\int(1/2*\sqrt{c^2*x^2 + 1}) * x^3*\log(x) * \log(\sqrt{c^2*x^2 + 1} + 1)^2/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) + 8*c^2*\int(1/2*x^3*\log(x)^3/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) - 24*c^2*\int(1/2*x^3*\log(x)^2 * \log(\sqrt{c^2*x^2 + 1} + 1)/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) + 24*c^2*\int(1/2*x^3*\log(x) * \log(\sqrt{c^2*x^2 + 1} + 1)^2/(\sqrt{c^2*x^2 + 1}) * c^2*x^2 + c^2*x^2 + \sqrt{c^2*x^2 + 1} + 1, x) \end{aligned}$$

```
(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*c^2*
integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*
x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*
sqrt(c^2*x^2 + 1)*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*
x^2 + 1) + 1), x)*log(c)^2 + 24*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c
^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 24*integrate(1/2*x
*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2
*x^2 + 1) + 1), x)*log(c)^2 + 24*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^2
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) -
48*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*
integrate(1/2*x*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^
2 + 1) + 1), x)*log(c) - 48*integrate(1/2*x*log(x)*log(sqrt(c^2*x^2 + 1) +
1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)
+ 24*integrate(1/2*x*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 2*(c^2*x^2 - 2*sqrt(c^2*
x^2 + 1) + 1)*log(c)^3/c^2 + 2*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*log
(c)^3/c^2 + 2*(log(c^2*x^2 + 1) - 2*log(sqrt(c^2*x^2 + 1) + 1))*log(c)^3/c^
2 + 6*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2*log(x)/c^2 - 6*(c^2*x^2
- 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)/c^2 + 4*log(
c)^3*log(sqrt(c^2*x^2 + 1) + 1)/c^2 - 6*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)
^2/c^2 + 4*log(c)*log(sqrt(c^2*x^2 + 1) + 1)^3/c^2 - 3*(c^2*x^2 - 4*sqrt(c^
2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) + 1) - log(sqrt(c^2*x^2 + 1) - 1) + 1)
*log(c)^2/c^2 + 3*(c^2*x^2 - 6*sqrt(c^2*x^2 + 1) + 6*log(sqrt(c^2*x^2 + 1)
+ 1) + 1)*log(c)^2/c^2 + 8*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^3/(sqrt
(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*integrate
(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2
+ 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*sqrt
(c^2*x^2 + 1)*x*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 8*integrate(1/2*x*log(x)^3/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*integra
te(1/2*x*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c
^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*x*log(x)*log(sqrt(c^
2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
+ 1), x))*b^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*asinh(1/(c\*x)))^3,x)

[Out] int(x\*(a + b\*asinh(1/(c\*x)))^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{acsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))\*\*3,x)

[Out] Integral(x\*(a + b\*acsch(c\*x))\*\*3, x)

### 3.27 $\int (a + b \operatorname{csch}^{-1}(cx))^3 dx$

**Optimal.** Leaf size=120

$$\frac{6b^2 \operatorname{Li}_2\left(-e^{\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{c} - \frac{6b^2 \operatorname{Li}_2\left(e^{\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{c} + x (a + b \operatorname{csch}^{-1}(cx))^3 + \frac{6b \tanh^{-1}\left(\frac{1}{c/x + (1 + 1/c^2/x^2)^{1/2}}\right)}{c}$$

[Out]  $x*(a+b*\operatorname{arccsch}(c*x))^3+6*b*(a+b*\operatorname{arccsch}(c*x))^2*\operatorname{arctanh}(1/c/x+(1+1/c^2/x^2)^{1/2})/c+6*b^2*(a+b*\operatorname{arccsch}(c*x))*\operatorname{polylog}(2,-1/c/x-(1+1/c^2/x^2)^{1/2})/c-6*b^2*(a+b*\operatorname{arccsch}(c*x))*\operatorname{polylog}(2,1/c/x+(1+1/c^2/x^2)^{1/2})/c-6*b^3*\operatorname{polylog}(3,-1/c/x-(1+1/c^2/x^2)^{1/2})/c+6*b^3*\operatorname{polylog}(3,1/c/x+(1+1/c^2/x^2)^{1/2})/c$

**Rubi [A]** time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$ , Rules used = {6280, 5452, 4182, 2531, 2282, 6589}

$$\frac{6b^2 \operatorname{PolyLog}\left(2, -e^{\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{c} - \frac{6b^2 \operatorname{PolyLog}\left(2, e^{\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{c} + \frac{6b^3 \operatorname{PolyLog}\left(3, -e^{\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{c} + \frac{6b^3 \operatorname{PolyLog}\left(3, e^{\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))}{c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])^3, x]$

[Out]  $x*(a + b*\operatorname{ArcCsch}[c*x])^3 + (6*b*(a + b*\operatorname{ArcCsch}[c*x])^2*\operatorname{ArcTanh}[E^{\operatorname{ArcCsch}[c*x]}])/c + (6*b^2*(a + b*\operatorname{ArcCsch}[c*x])* \operatorname{PolyLog}[2, -E^{\operatorname{ArcCsch}[c*x]}])/c - (6*b^2*(a + b*\operatorname{ArcCsch}[c*x])* \operatorname{PolyLog}[2, E^{\operatorname{ArcCsch}[c*x]}])/c - (6*b^3*\operatorname{PolyLog}[3, -E^{\operatorname{ArcCsch}[c*x]}])/c + (6*b^3*\operatorname{PolyLog}[3, E^{\operatorname{ArcCsch}[c*x]}])/c$

#### Rule 2282

$\operatorname{Int}[u_, x\_Symbol] := \operatorname{With}[\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x]] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; \operatorname{FreeQ}[\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n]] \&\& \operatorname{!MatchQ}[u, E^{((c_)*((a_.) + (b_.)*x))}*(F_)] [v_] /; \operatorname{FreeQ}[\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_)*((F_)^{((c_)*((a_.) + (b_.)*(x_)))})^{(n_)}] * ((f_.) + (g_.)*(x_))^{(m_)}, x\_Symbol] := -\operatorname{Simp}[(f + g*x)^m * \operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n] / (b*c*n*\operatorname{Log}[F]), x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f + g*x)^{(m-1)} * \operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n], x], x] /; \operatorname{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0]$

#### Rule 4182

$\operatorname{Int}[\operatorname{csc}[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)] * ((c_.) + (d_.)*(x_))^{(m_)}, x\_Symbol] := \operatorname{Simp}[(-2*(c + d*x)^m * \operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}]) / (f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)} * \operatorname{Log}[1 - E^{-(I*e) + f*fz*x}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)} * \operatorname{Log}[1 + E^{-(I*e) + f*fz*x}], x], x]) /; \operatorname{FreeQ}[\{c, d, e, f, fz\}, x] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 5452

$\operatorname{Int}[\operatorname{Coth}[(a_.) + (b_.)*(x_)]^{(p_)} * \operatorname{Csch}[(a_.) + (b_.)*(x_)]^{(n_)} * ((c_.) + (d_.)*(x_))^{(m_)}, x\_Symbol] := -\operatorname{Simp}[(c + d*x)^m * \operatorname{Csch}[a + b*x]^n / (b*n), x] + \operatorname{Dist}[(d*m)/(b*n), \operatorname{Int}[(c + d*x)^{(m-1)} * \operatorname{Csch}[a + b*x]^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \&\& \operatorname{GtQ}[m, 0]$

$eQ[\{a, b, c, d, n\}, x] \ \&\& \ EqQ[p, 1] \ \&\& \ GtQ[m, 0]$

### Rule 6280

$\text{Int}[(a + \text{ArcCsch}[c \cdot x] \cdot b)^n, x\_Symbol] \rightarrow -\text{Dist}[c^{-1}, \text{Subst}[\text{Int}[(a + b \cdot x)^n \cdot \text{Csch}[x] \cdot \text{Coth}[x], x], x, \text{ArcCsch}[c \cdot x]], x] /;$   $\text{FreeQ}[\{a, b, c, n\}, x] \ \&\& \ \text{IGtQ}[n, 0]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n, (c + (a + b \cdot x)^p) / (d + e \cdot x)], x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c \cdot (a + b \cdot x)^p / (e \cdot x)], x] /;$   $\text{FreeQ}[\{a, b, c, d, e, n, p\}, x] \ \&\& \ EqQ[b \cdot d, a \cdot e]$

### Rubi steps

$$\begin{aligned} \int (a + b \operatorname{csch}^{-1}(cx))^3 dx &= -\frac{\text{Subst}\left(\int (a + bx)^3 \coth(x) \operatorname{csch}(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c} \\ &= x (a + b \operatorname{csch}^{-1}(cx))^3 - \frac{(3b) \text{Subst}\left(\int (a + bx)^2 \operatorname{csch}(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c} \\ &= x (a + b \operatorname{csch}^{-1}(cx))^3 + \frac{6b (a + b \operatorname{csch}^{-1}(cx))^2 \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} + \frac{(6b^2) \text{Subst}\left(\int (a + bx) \operatorname{csch}(x) dx, x, \operatorname{csch}^{-1}(cx)\right)}{c} \\ &= x (a + b \operatorname{csch}^{-1}(cx))^3 + \frac{6b (a + b \operatorname{csch}^{-1}(cx))^2 \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} + \frac{6b^2 (a + b \operatorname{csch}^{-1}(cx))}{c} \\ &= x (a + b \operatorname{csch}^{-1}(cx))^3 + \frac{6b (a + b \operatorname{csch}^{-1}(cx))^2 \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} + \frac{6b^2 (a + b \operatorname{csch}^{-1}(cx))}{c} \\ &= x (a + b \operatorname{csch}^{-1}(cx))^3 + \frac{6b (a + b \operatorname{csch}^{-1}(cx))^2 \tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c} + \frac{6b^2 (a + b \operatorname{csch}^{-1}(cx))}{c} \end{aligned}$$

**Mathematica [B]** time = 0.34, size = 246, normalized size = 2.05

$$a^3 x + \frac{3a^2 b \log\left(cx \left(\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1\right)\right)}{c} + 3a^2 b x \operatorname{csch}^{-1}(cx) + \frac{3ab^2 \left(-2\operatorname{Li}_2\left(-e^{-\operatorname{csch}^{-1}(cx)}\right) + 2\operatorname{Li}_2\left(e^{-\operatorname{csch}^{-1}(cx)}\right) + \operatorname{csch}^{-1}(cx)\right)}{c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])^3, x]

[Out]  $a^3 x + 3a^2 b x \operatorname{ArcCsch}[c x] + (3a^2 b \operatorname{Log}[c x (1 + \sqrt{(1 + c^2 x^2) / (c^2 x^2)})] / c + (3a b^2 (\operatorname{ArcCsch}[c x] (\operatorname{csch}^{-1}(cx) - 2 \operatorname{Log}[1 - E^{-\operatorname{ArcCsch}[c x]})] + 2 \operatorname{Log}[1 + E^{-\operatorname{ArcCsch}[c x]})]) - 2 \operatorname{PolyLog}[2, -E^{-\operatorname{ArcCsch}[c x]})] + 2 \operatorname{PolyLog}[2, E^{-\operatorname{ArcCsch}[c x]})]) / c + (b^3 (\operatorname{csch}^{-1}(cx)^3 - 3 \operatorname{ArcCsch}[c x]^2 \operatorname{Log}[1 - E^{-\operatorname{ArcCsch}[c x]})] + 3 \operatorname{ArcCsch}[c x]^2 \operatorname{Log}[1 + E^{-\operatorname{ArcCsch}[c x]})] - 6 \operatorname{ArcCsch}[c x] \operatorname{PolyLog}[2, -E^{-\operatorname{ArcCsch}[c x]})] + 6 \operatorname{ArcCsch}[c x] \operatorname{PolyLog}[2, E^{-\operatorname{ArcCsch}[c x]})] - 6 \operatorname{PolyLog}[3, -E^{-\operatorname{ArcCsch}[c x]})] + 6 \operatorname{PolyLog}[3, E^{-\operatorname{ArcCsch}[c x]})]) / c$

**fricas [F]** time = 2.95, size = 0, normalized size = 0.00

$$\text{integral}\left(b^3 \operatorname{arcsch}(cx)^3 + 3ab^2 \operatorname{arcsch}(cx)^2 + 3a^2 b \operatorname{arcsch}(cx) + a^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3,x, algorithm="fricas")

[Out] integral(b^3\*arccsch(c\*x)^3 + 3\*a\*b^2\*arccsch(c\*x)^2 + 3\*a^2\*b\*arccsch(c\*x) + a^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3, x)

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arcsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^3,x)

[Out] int((a+b\*arccsch(c\*x))^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b^3 x \log\left(\sqrt{c^2 x^2 + 1} + 1\right)^3 + a^3 x + \frac{3\left(2cx \operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} - 1\right)\right) a^2 b}{2c} - \int \frac{b^3 \log\left(\sqrt{c^2 x^2 + 1} + 1\right)^3}{2c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3,x, algorithm="maxima")

[Out] b^3\*x\*log(sqrt(c^2\*x^2 + 1) + 1)^3 + a^3\*x + 3/2\*(2\*c\*x\*arccsch(c\*x) + log(sqrt(1/(c^2\*x^2) + 1) + 1) - log(sqrt(1/(c^2\*x^2) + 1) - 1))\*a^2\*b/c - integrate((b^3\*log(c)^3 - 3\*a\*b^2\*log(c)^2 + (b^3\*c^2\*x^2 + b^3)\*log(x)^3 + (b^3\*c^2\*log(c)^3 - 3\*a\*b^2\*c^2\*log(c)^2)\*x^2 + 3\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2)\*log(x)^2 + 3\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2 + (b^3\*c^2\*x^2 + b^3)\*log(x) + sqrt(c^2\*x^2 + 1)\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*(log(c) + 1) - a\*b^2\*c^2)\*x^2 + (b^3\*c^2\*x^2 + b^3)\*log(x)))\*log(sqrt(c^2\*x^2 + 1) + 1)^2 + 3\*(b^3\*log(c)^2 - 2\*a\*b^2\*log(c) + (b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^2)\*log(x) - 3\*(b^3\*log(c)^2 - 2\*a\*b^2\*log(c) + (b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^2 + (b^3\*c^2\*x^2 + b^3)\*log(x)^2 + 2\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2)\*log(x) + (b^3\*log(c)^2 - 2\*a\*b^2\*log(c) + (b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^2 + (b^3\*c^2\*x^2 + b^3)\*log(x)^2 + 2\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2)\*log(x))\*sqrt(c^2\*x^2 + 1)\*log(sqrt(c^2\*x^2 + 1) + 1) + (b^3\*log(c)^3 - 3\*a\*b^2\*log(c)^2 + (b^3\*c^2\*x^2 + b^3)\*log(x)^3 + (b^3\*c^2\*log(c)^3 - 3\*a\*b^2\*c^2\*log(c)^2)\*x^2 + 3\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2)\*log(x)^2 + 3\*(b^3\*log(c)^2 - 2\*a\*b^2\*log(c) + (b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^2)\*log(x))\*sqrt(c^2\*x^2 + 1))/(c^2\*x^2 + (c^2\*x^2 + 1)^(3/2) + 1), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^3,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^3, x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \operatorname{acsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3, x)
```



$$3.28 \quad \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x} dx$$

**Optimal.** Leaf size=110

$$\frac{3}{2}b^2 \operatorname{Li}_3\left(e^{2\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx)) - \frac{3}{2}b \operatorname{Li}_2\left(e^{2\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))^2 + \frac{(a + b \operatorname{csch}^{-1}(cx))^4}{4b} - \log\left(1 - e^{2\operatorname{csch}^{-1}(cx)}\right)$$

```
[Out] 1/4*(a+b*arccsch(c*x))^4/b-(a+b*arccsch(c*x))^3*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)-3/2*b*(a+b*arccsch(c*x))^2*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+3/2*b^2*(a+b*arccsch(c*x))*polylog(3,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)-3/4*b^3*polylog(4,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

**Rubi [A]** time = 0.15, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6286, 3716, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}b^2 \operatorname{PolyLog}\left(3, e^{2\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx)) - \frac{3}{2}b \operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right) (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{3}{4}b^3 \operatorname{PolyLog}\left(4, e^{2\operatorname{csch}^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x, x]
```

```
[Out] (a + b*ArcCsch[c*x])^4/(4*b) - (a + b*ArcCsch[c*x])^3*Log[1 - E^(2*ArcCsch[c*x])] - (3*b*(a + b*ArcCsch[c*x])^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2 + (3*b^2*(a + b*ArcCsch[c*x])*PolyLog[3, E^(2*ArcCsch[c*x])])/2 - (3*b^3*PolyLog[4, E^(2*ArcCsch[c*x])])/4
```

**Rule 2190**

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

**Rule 2282**

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

**Rule 2531**

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)*(x_))^(m_), x_Symbol] := -Simp[(((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

**Rule 3716**

```
Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*e) + f*fz*x))/E^(2*I*k*Pi))))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
```

erQ[4\*k] && IGtQ[m, 0]

### Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))^n\_\*(x\_)^m\_., x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m + 1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^p\_]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_.))^m\_\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_.)))^p\_)], x\_Symbol] :> Simp[(e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x} dx &= -\operatorname{Subst}\left(\int (a + bx)^3 \operatorname{coth}(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\
 &= \frac{(a + b \operatorname{csch}^{-1}(cx))^4}{4b} + 2 \operatorname{Subst}\left(\int \frac{e^{2x}(a + bx)^3}{1 - e^{2x}} dx, x, \operatorname{csch}^{-1}(cx)\right) \\
 &= \frac{(a + b \operatorname{csch}^{-1}(cx))^4}{4b} - (a + b \operatorname{csch}^{-1}(cx))^3 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) + (3b) \operatorname{Subst}\left(\int (a + bx)^2 dx, x, \operatorname{csch}^{-1}(cx)\right) \\
 &= \frac{(a + b \operatorname{csch}^{-1}(cx))^4}{4b} - (a + b \operatorname{csch}^{-1}(cx))^3 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) - \frac{3}{2}b(a + b \operatorname{csch}^{-1}(cx))^2 \\
 &= \frac{(a + b \operatorname{csch}^{-1}(cx))^4}{4b} - (a + b \operatorname{csch}^{-1}(cx))^3 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) - \frac{3}{2}b(a + b \operatorname{csch}^{-1}(cx)) \\
 &= \frac{(a + b \operatorname{csch}^{-1}(cx))^4}{4b} - (a + b \operatorname{csch}^{-1}(cx))^3 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) - \frac{3}{2}b(a + b \operatorname{csch}^{-1}(cx)) \\
 &= \frac{(a + b \operatorname{csch}^{-1}(cx))^4}{4b} - (a + b \operatorname{csch}^{-1}(cx))^3 \log\left(1 - e^{2 \operatorname{csch}^{-1}(cx)}\right) - \frac{3}{2}b(a + b \operatorname{csch}^{-1}(cx))
 \end{aligned}$$

**Mathematica** [A] time = 0.23, size = 198, normalized size = 1.80

$$\frac{1}{4} \left( 4a^3 \log(cx) + 6a^2b \left( \operatorname{Li}_2\left(e^{-2 \operatorname{csch}^{-1}(cx)}\right) - \operatorname{csch}^{-1}(cx) \left( \operatorname{csch}^{-1}(cx) + 2 \log\left(1 - e^{-2 \operatorname{csch}^{-1}(cx)}\right)\right) \right) \right) + 2ab^2 \left( -6 \operatorname{csch}^{-1}(cx) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])^3/x, x]

[Out] (4\*a^3\*Log[c\*x] + 6\*a^2\*b\*(-(ArcCsch[c\*x]\*(ArcCsch[c\*x] + 2\*Log[1 - E^(-2\*ArcCsch[c\*x])])) + PolyLog[2, E^(-2\*ArcCsch[c\*x])]) + 2\*a\*b^2\*(2\*ArcCsch[c\*x]^2\*(ArcCsch[c\*x] - 3\*Log[1 - E^(2\*ArcCsch[c\*x])]) - 6\*ArcCsch[c\*x]\*PolyLog



$^3c^2\log(c) - a*b^2*c^2)*x^2)*\log(x))*\sqrt{c^2*x^2 + 1})*\log(\sqrt{c^2*x^2 + 1} + 1) + (b^3*\log(c)^3 - 3*a*b^2*\log(c)^2 + 3*a^2*b*\log(c) + (b^3*c^2*x^2 + b^3)*\log(x)^3 + (b^3*c^2*\log(c)^3 - 3*a*b^2*c^2*\log(c)^2 + 3*a^2*b*c^2*\log(c))*x^2 + 3*(b^3*\log(c) - a*b^2 + (b^3*c^2*\log(c) - a*b^2*c^2)*x^2)*\log(x)^2 + 3*(b^3*\log(c)^2 - 2*a*b^2*\log(c) + a^2*b + (b^3*c^2*\log(c)^2 - 2*a*b^2*c^2*\log(c) + a^2*b*c^2)*x^2)*\log(x))*\sqrt{c^2*x^2 + 1})/(c^2*x^3 + (c^2*x^3 + x)*\sqrt{c^2*x^2 + 1} + x), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))^3/x, x)

[Out] int((a + b\*asinh(1/(c\*x)))^3/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*\*3/x, x)

[Out] Integral((a + b\*acsch(c\*x))\*\*3/x, x)

$$3.29 \quad \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x^2} dx$$

**Optimal.** Leaf size=78

$$-\frac{6b^2(a + b \operatorname{csch}^{-1}(cx))}{x} + 3bc\sqrt{\frac{1}{c^2x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x} + 6b^3c\sqrt{\frac{1}{c^2x^2} + 1}$$

[Out]  $-6*b^2*(a+b*\operatorname{arccsch}(c*x))/x-(a+b*\operatorname{arccsch}(c*x))^3/x+6*b^3*c*(1+1/c^2/x^2)^(1/2)+3*b*c*(a+b*\operatorname{arccsch}(c*x))^2*(1+1/c^2/x^2)^(1/2)$

**Rubi [A]** time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6286, 3296, 2638}

$$-\frac{6b^2(a + b \operatorname{csch}^{-1}(cx))}{x} + 3bc\sqrt{\frac{1}{c^2x^2} + 1} (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x} + 6b^3c\sqrt{\frac{1}{c^2x^2} + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])^3/x^2,x]

[Out]  $6*b^3*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)] - (6*b^2*(a + b*\operatorname{ArcCsch}[c*x]))/x + 3*b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*(a + b*\operatorname{ArcCsch}[c*x])^2 - (a + b*\operatorname{ArcCsch}[c*x])^3/x$

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] := -Dist[(c^(m+1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m+1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

#### Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x^2} dx &= -\left(c \operatorname{Subst}\left(\int (a + bx)^3 \cosh(x) dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x} + (3bc) \operatorname{Subst}\left(\int (a + bx)^2 \sinh(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= 3bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x} - (6b^2c) \operatorname{Subst}\left(\int (a + b \operatorname{csch}^{-1}(cx))^2 dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= -\frac{6b^2(a + b \operatorname{csch}^{-1}(cx))}{x} + 3bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x} \\ &= 6b^3c\sqrt{1 + \frac{1}{c^2x^2}} - \frac{6b^2(a + b \operatorname{csch}^{-1}(cx))}{x} + 3bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx))^2 - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x} \end{aligned}$$

**Mathematica [A]** time = 0.25, size = 132, normalized size = 1.69

$$\frac{a^3 + 3bcsch^{-1}(cx) \left( a^2 - 2abcx\sqrt{\frac{1}{c^2x^2} + 1} + 2b^2 \right) - 3a^2bcx\sqrt{\frac{1}{c^2x^2} + 1} + 3b^2csch^{-1}(cx)^2 \left( a - bcx\sqrt{\frac{1}{c^2x^2} + 1} \right) + 6a}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])^3/x^2,x]

[Out] -((a^3 + 6\*a\*b^2 - 3\*a^2\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x - 6\*b^3\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x + 3\*b\*(a^2 + 2\*b^2 - 2\*a\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x)\*ArcCsch[c\*x] + 3\*b^2\*(a - b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x)\*ArcCsch[c\*x]^2 + b^3\*ArcCsch[c\*x]^3)/x)

**fricas [B]** time = 0.83, size = 222, normalized size = 2.85

$$\frac{b^3 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3 - 3(a^2b + 2b^3)cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + a^3 + 6ab^2 - 3\left(b^3cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - ab^2\right) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 3\left(\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^2,x, algorithm="fricas")

[Out] -(b^3\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^3 - 3\*(a^2\*b + 2\*b^3)\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + a^3 + 6\*a\*b^2 - 3\*(b^3\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - a\*b^2)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 - 3\*(2\*a\*b^2\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - a^2\*b - 2\*b^3)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/x

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3/x^2, x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^3/x^2,x)

[Out] int((a+b\*arccsch(c\*x))^3/x^2,x)

**maxima [A]** time = 0.35, size = 144, normalized size = 1.85

$$-\frac{b^3 \operatorname{arcsch}(cx)^3}{x} + 3 \left( c\sqrt{\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsch}(cx)}{x} \right) a^2 b + 6 \left( c\sqrt{\frac{1}{c^2x^2} + 1} \operatorname{arcsch}(cx) - \frac{1}{x} \right) a b^2 + 3 \left( c\sqrt{\frac{1}{c^2x^2} + 1} \operatorname{arcsch}(cx) - \frac{1}{x} \right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^2,x, algorithm="maxima")

```
[Out] -b^3*arccsch(c*x)^3/x + 3*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a^2*b
+ 6*(c*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x) - 1/x)*a*b^2 + 3*(c*sqrt(1/(c^2*x
^2) + 1)*arccsch(c*x)^2 + 2*c*sqrt(1/(c^2*x^2) + 1) - 2*arccsch(c*x)/x)*b^3
- 3*a*b^2*arccsch(c*x)^2/x - a^3/x
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^3/x^2, x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^3/x^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**2, x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**2, x)
```

$$3.30 \quad \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x^3} dx$$

**Optimal.** Leaf size=123

$$-\frac{3b^2(a + b \operatorname{csch}^{-1}(cx))}{4x^2} + \frac{3bc\sqrt{\frac{1}{c^2x^2} + 1}(a + b \operatorname{csch}^{-1}(cx))^2}{4x} - \frac{1}{4}c^2(a + b \operatorname{csch}^{-1}(cx))^3 - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{2x^2} + \frac{3b^3c\sqrt{\frac{1}{c^2x^2} + 1}}{8}$$

[Out]  $-3/8*b^3*c^2*\operatorname{arccsch}(c*x) - 3/4*b^2*(a+b*\operatorname{arccsch}(c*x))/x^2 - 1/4*c^2*(a+b*\operatorname{arccsch}(c*x))^3 - 1/2*(a+b*\operatorname{arccsch}(c*x))^3/x^2 + 3/8*b^3*c*(1+1/c^2/x^2)^{(1/2)}/x + 3/4*b*c*(a+b*\operatorname{arccsch}(c*x))^2*(1+1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6286, 5446, 3311, 32, 2635, 8}

$$-\frac{3b^2(a + b \operatorname{csch}^{-1}(cx))}{4x^2} + \frac{3bc\sqrt{\frac{1}{c^2x^2} + 1}(a + b \operatorname{csch}^{-1}(cx))^2}{4x} - \frac{1}{4}c^2(a + b \operatorname{csch}^{-1}(cx))^3 - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{2x^2} + \frac{3b^3c\sqrt{\frac{1}{c^2x^2} + 1}}{8}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])^3/x^3, x]

[Out]  $(3*b^3*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(8*x) - (3*b^3*c^2*\operatorname{ArcCsch}[c*x])/8 - (3*b^2*(a + b*\operatorname{ArcCsch}[c*x]))/(4*x^2) + (3*b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*(a + b*\operatorname{ArcCsch}[c*x])^2)/(4*x) - (c^2*(a + b*\operatorname{ArcCsch}[c*x])^3)/4 - (a + b*\operatorname{ArcCsch}[c*x])^3/(2*x^2)$

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*cos[c + d\*x])\*(b\*sin[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*sin[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*sin[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*cos[e + f\*x]\*(b\*sin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_), x\_Symbol] := Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]



## Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*(x\_.)^(m\_.), x\_Symbol] :> -Dist  
 [(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m + 1)\*Coth[x], x], x, Ar  
 cCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt  
 Q[n, 0] || LtQ[m, -1])

## Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x^3} dx &= -\left(c^2 \operatorname{Subst}\left(\int (a + bx)^3 \cosh(x) \sinh(x) dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{csch}^{-1}(cx))^3}{2x^2} + \frac{1}{2} (3bc^2) \operatorname{Subst}\left(\int (a + bx)^2 \sinh^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= -\frac{3b^2 (a + b \operatorname{csch}^{-1}(cx))}{4x^2} + \frac{3bc \sqrt{1 + \frac{1}{c^2 x^2}} (a + b \operatorname{csch}^{-1}(cx))^2}{4x} - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{2x^2} \\ &= \frac{3b^3 c \sqrt{1 + \frac{1}{c^2 x^2}}}{8x} - \frac{3b^2 (a + b \operatorname{csch}^{-1}(cx))}{4x^2} + \frac{3bc \sqrt{1 + \frac{1}{c^2 x^2}} (a + b \operatorname{csch}^{-1}(cx))^2}{4x} - \frac{1}{4} c \\ &= \frac{3b^3 c \sqrt{1 + \frac{1}{c^2 x^2}}}{8x} - \frac{3}{8} b^3 c^2 \operatorname{csch}^{-1}(cx) - \frac{3b^2 (a + b \operatorname{csch}^{-1}(cx))}{4x^2} + \frac{3bc \sqrt{1 + \frac{1}{c^2 x^2}} (a + b \operatorname{csch}^{-1}(cx))^2}{4x} \end{aligned}$$

**Mathematica [A]** time = 0.33, size = 182, normalized size = 1.48

$$\frac{4a^3 + 3bc^2x^2(2a^2 + b^2) \sinh^{-1}\left(\frac{1}{cx}\right) + 6b \operatorname{csch}^{-1}(cx) \left(2a^2 - 2abcx \sqrt{\frac{1}{c^2x^2} + 1} + b^2\right) - 6a^2bcx \sqrt{\frac{1}{c^2x^2} + 1} + 6b^2c}{8x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])^3/x^3,x]

[Out] -1/8\*(4\*a^3 + 6\*a\*b^2 - 6\*a^2\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x - 3\*b^3\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x + 6\*b\*(2\*a^2 + b^2 - 2\*a\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x)\*ArcCsch[c\*x] + 6\*b^2\*(-(b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x) + a\*(2 + c^2\*x^2))\*ArcCsch[c\*x]^2 + 2\*b^3\*(2 + c^2\*x^2)\*ArcCsch[c\*x]^3 + 3\*b\*(2\*a^2 + b^2)\*c^2\*x^2\*ArcSinh[1/(c\*x)]/x^2

**fricas [B]** time = 0.40, size = 267, normalized size = 2.17

$$\frac{2(b^3c^2x^2 + 2b^3) \log\left(\frac{cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx}\right)^3 - 3(2a^2b + b^3)cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} + 4a^3 + 6ab^2 + 6\left(ab^2c^2x^2 - b^3cx \sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^3,x, algorithm="fricas")

[Out] -1/8\*(2\*(b^3\*c^2\*x^2 + 2\*b^3)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^3 - 3\*(2\*a^2\*b + b^3)\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 4\*a^3 + 6\*a\*b^2 + 6\*(a\*b^2\*c^2\*x^2 - b^3\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 2\*a\*b^2)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 - 3\*(4\*a\*b^2\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - (2\*a^2\*b + b^3)\*c^2\*x^2 - 4\*a^2\*b - 2\*b^3)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/x^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3/x^3, x)

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^3/x^3,x)

[Out] int((a+b\*arccsch(c\*x))^3/x^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^3,x, algorithm="maxima")

[Out]  $\frac{3}{8}a^2b((2c^4x\sqrt{1/(c^2x^2)+1})/(c^2x^2(1/(c^2x^2)+1)-1) - c^3\log(cx\sqrt{1/(c^2x^2)+1}) + 1) + c^3\log(cx\sqrt{1/(c^2x^2)+1}) - 1)/c - 4\operatorname{arccsch}(cx)/x^2 - 1/2b^3\log(\sqrt{c^2x^2+1})^3/x^2 - 1/2a^3/x^2 - \operatorname{integrate}(1/2(2b^3\log(c)^3 - 6ab^2\log(c)^2 + 2(b^3c^2x^2 + b^3)\log(x)^3 + 2(b^3c^2\log(c)^3 - 3ab^2c^2\log(c)^2)x^2 + 6(b^3\log(c) - ab^2 + (b^3c^2\log(c) - ab^2c^2)x^2)\log(x)^2 + 3(2b^3\log(c) - 2ab^2 + 2(b^3c^2\log(c) - ab^2c^2)x^2 + 2(b^3c^2x^2 + b^3)\log(x) + \sqrt{c^2x^2+1})(2b^3\log(c) - 2ab^2 + (b^3c^2(2\log(c) - 1) - 2ab^2c^2)x^2 + 2(b^3c^2x^2 + b^3)\log(x)))\log(\sqrt{c^2x^2+1}) + 1)^2 + 6(b^3\log(c)^2 - 2ab^2\log(c) + (b^3c^2\log(c)^2 - 2ab^2c^2\log(c))x^2)\log(x) - 6(b^3\log(c)^2 - 2ab^2\log(c) + (b^3c^2\log(c)^2 - 2ab^2c^2\log(c))x^2 + (b^3c^2x^2 + b^3)\log(x)^2 + 2(b^3\log(c) - ab^2 + (b^3c^2\log(c) - ab^2c^2)x^2)\log(x) + (b^3\log(c)^2 - 2ab^2\log(c) + (b^3c^2\log(c)^2 - 2ab^2c^2\log(c))x^2 + (b^3c^2x^2 + b^3)\log(x)^2 + 2(b^3\log(c) - ab^2 + (b^3c^2\log(c) - ab^2c^2)x^2)\log(x))\sqrt{c^2x^2+1})\log(\sqrt{c^2x^2+1}) + 1) + 2(b^3\log(c)^3 - 3ab^2\log(c)^2 + (b^3c^2x^2 + b^3)\log(x)^3 + (b^3c^2\log(c)^3 - 3ab^2c^2\log(c)^2)x^2 + 3(b^3\log(c) - ab^2 + (b^3c^2\log(c) - ab^2c^2)x^2)\log(x)^2 + 3(b^3\log(c)^2 - 2ab^2\log(c) + (b^3c^2\log(c)^2 - 2ab^2c^2\log(c))x^2)\log(x))\sqrt{c^2x^2+1})/(c^2x^5 + x^3 + (c^2x^5 + x^3)\sqrt{c^2x^2+1}), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))^3/x^3,x)

[Out] int((a + b\*asinh(1/(c\*x)))^3/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**3, x)
```

$$3.31 \quad \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x^4} dx$$

**Optimal.** Leaf size=166

$$\frac{4b^2c^2(a + b \operatorname{csch}^{-1}(cx))}{3x} - \frac{2b^2(a + b \operatorname{csch}^{-1}(cx))}{9x^3} + \frac{bc\sqrt{\frac{1}{c^2x^2} + 1}(a + b \operatorname{csch}^{-1}(cx))^2}{3x^2} - \frac{2}{3}bc^3\sqrt{\frac{1}{c^2x^2} + 1}(a + b \operatorname{csch}^{-1}(cx))$$

[Out]  $2/27*b^3*c^3*(1+1/c^2/x^2)^(3/2)-2/9*b^2*(a+b*\operatorname{arccsch}(c*x))/x^3+4/3*b^2*c^2*(a+b*\operatorname{arccsch}(c*x))/x-1/3*(a+b*\operatorname{arccsch}(c*x))^3/x^3-14/9*b^3*c^3*(1+1/c^2/x^2)^(1/2)-2/3*b*c^3*(a+b*\operatorname{arccsch}(c*x))^2*(1+1/c^2/x^2)^(1/2)+1/3*b*c*(a+b*\operatorname{arccsch}(c*x))^2*(1+1/c^2/x^2)^(1/2)/x^2$

**Rubi [A]** time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6286, 5446, 3311, 3296, 2638, 2633}

$$\frac{4b^2c^2(a + b \operatorname{csch}^{-1}(cx))}{3x} - \frac{2b^2(a + b \operatorname{csch}^{-1}(cx))}{9x^3} - \frac{2}{3}bc^3\sqrt{\frac{1}{c^2x^2} + 1}(a + b \operatorname{csch}^{-1}(cx))^2 + \frac{bc\sqrt{\frac{1}{c^2x^2} + 1}(a + b \operatorname{csch}^{-1}(cx))}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])^3/x^4, x]

[Out]  $(-14*b^3*c^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/9 + (2*b^3*c^3*(1 + 1/(c^2*x^2))^(3/2))/27 - (2*b^2*(a + b*\operatorname{ArcCsch}[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*\operatorname{ArcCsch}[c*x]))/(3*x) - (2*b*c^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*(a + b*\operatorname{ArcCsch}[c*x])^2)/3 + (b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*(a + b*\operatorname{ArcCsch}[c*x])^2)/(3*x^2) - (a + b*\operatorname{ArcCsch}[c*x])^3/(3*x^3)$

#### Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

#### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sin[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x^4} dx &= -\left(c^3 \operatorname{Subst}\left(\int (a + bx)^3 \cosh(x) \sinh^2(x) dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3x^3} + (bc^3) \operatorname{Subst}\left(\int (a + bx)^2 \sinh^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= -\frac{2b^2(a + b \operatorname{csch}^{-1}(cx))}{9x^3} + \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx))^2}{3x^2} - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{3x^3} \\ &= -\frac{2b^2(a + b \operatorname{csch}^{-1}(cx))}{9x^3} - \frac{2}{3}bc^3\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx))^2 + \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}(a + b \operatorname{csch}^{-1}(cx))^3}{3x^3} \\ &= -\frac{2}{9}b^3c^3\sqrt{1 + \frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1 + \frac{1}{c^2x^2}\right)^{3/2} - \frac{2b^2(a + b \operatorname{csch}^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a + b \operatorname{csch}^{-1}(cx))^3}{27x^3} \\ &= -\frac{14}{9}b^3c^3\sqrt{1 + \frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1 + \frac{1}{c^2x^2}\right)^{3/2} - \frac{2b^2(a + b \operatorname{csch}^{-1}(cx))}{9x^3} + \frac{4b^2c^2(a + b \operatorname{csch}^{-1}(cx))^3}{27x^3} \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 200, normalized size = 1.20

$$\frac{-9a^3 + 3b \operatorname{csch}^{-1}(cx) \left(-9a^2 + 6abcx\sqrt{\frac{1}{c^2x^2} + 1} (1 - 2c^2x^2) + 2b^2(6c^2x^2 - 1)\right) + 9a^2bcx\sqrt{\frac{1}{c^2x^2} + 1} (1 - 2c^2x^2)}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^4, x]
```

```
[Out] (-9*a^3 + 2*b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 20*c^2*x^2) + 9*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 6*a*b^2*(-1 + 6*c^2*x^2) + 3*b*(-9*a^2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*c^2*x^2))*ArcCsch[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*x^2))*ArcCsch[c*x]^2 - 9*b^3*ArcCsch[c*x]^3)/(27*x^3)
```

**fricas [B]** time = 0.77, size = 301, normalized size = 1.81

$$\frac{36ab^2c^2x^2 - 9b^3 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - 9a^3 - 6ab^2 - 9\left(3ab^2 + (2b^3c^3x^3 - b^3cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^4,x, algorithm="fricas")

[Out] 
$$\frac{1}{27} \cdot (36 a^2 b^2 c^2 x^2 - 9 b^3 \log((c x \sqrt{(c^2 x^2 + 1)/(c^2 x^2)} + 1)/(c x))^3 - 9 a^3 - 6 a^2 b^2 - 9(3 a^2 b^2 + (2 b^3 c^3 x^3 - b^3 c x) \sqrt{(c^2 x^2 + 1)/(c^2 x^2)})) \log((c x \sqrt{(c^2 x^2 + 1)/(c^2 x^2)} + 1)/(c x))^2 + 3(12 b^3 c^2 x^2 - 9 a^2 b - 2 b^3 - 6(2 a b^2 c^3 x^3 - a b^2 c x) \sqrt{(c^2 x^2 + 1)/(c^2 x^2)})) \log((c x \sqrt{(c^2 x^2 + 1)/(c^2 x^2)} + 1)/(c x)) - (2(9 a^2 b + 20 b^3) c^3 x^3 - (9 a^2 b + 2 b^3) c x) \sqrt{(c^2 x^2 + 1)/(c^2 x^2)}) / x^3$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^4,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3/x^4, x)

**maple** [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsch}(cx))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^3/x^4,x)

[Out] int((a+b\*arccsch(c\*x))^3/x^4,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a^2 b \left( \frac{c^4 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} - 3 c^4 \sqrt{\frac{1}{c^2 x^2} + 1}}{c} - \frac{3 \operatorname{arcsch}(cx)}{x^3} \right) - \frac{b^3 \log(\sqrt{c^2 x^2 + 1} + 1)^3}{3 x^3} - \frac{a^3}{3 x^3} - \int \frac{b^3 \log(c)^3 - 3 a b^2 \log(c)^2 + (b^3 c^2 \log(c)^3 - 3 a^2 b^2 c^2 \log(c)^2) x^2 + 3(b^3 \log(c) - a b^2 + (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)^2 + (3 b^3 \log(c) - 3 a^2 b^2 + 3(b^3 c^2 \log(c) - a b^2 c^2) x^2 + 3(b^3 c^2 x^2 + b^3) \log(x)) \log(\sqrt{c^2 x^2 + 1} + 1)^2 + 3(b^3 \log(c)^2 - 2 a b^2 \log(c) + (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2) \log(x) - 3(b^3 \log(c)^2 - 2 a b^2 \log(c) + (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2 + (b^3 c^2 x^2 + b^3) \log(x))^2 + 2(b^3 \log(c) - a b^2 + (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x) + (b^3 \log(c)^2 - 2 a b^2 \log(c) + (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2 + (b^3 c^2 x^2 + b^3) \log(x))^2 + 2(b^3 \log(c) - a b^2 + (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)) \sqrt{c^2 x^2 + 1} \log(\sqrt{c^2 x^2 + 1} + 1) + (b^3 \log(c)^3 - 3 a^2 b^2 \log(c)^2 + (b^3 c^2 x^2 + b^3) \log(x))^3 + (b^3 c^2 \log(c)^3 - 3 a^2 b^2 c^2 \log(c)^2) x^2 + 3(b^3 \log(c) - a b^2 + (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)^2 + 3(b^3 \log(c)^2 - 2 a b^2 \log(c) + (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2) \log(x)}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^4,x, algorithm="maxima")

[Out] 
$$\frac{1}{3} a^2 b \left( (c^4 (1/(c^2 x^2) + 1)^{(3/2)} - 3 c^4 \sqrt{1/(c^2 x^2) + 1}) / c - 3 \operatorname{arcsch}(c x) / x^3 \right) - \frac{1}{3} b^3 \log(\sqrt{c^2 x^2 + 1} + 1)^3 / x^3 - \frac{1}{3} a^3 / x^3 - \int (b^3 \log(c)^3 - 3 a^2 b^2 \log(c)^2 + (b^3 c^2 \log(c)^3 - 3 a^2 b^2 c^2 \log(c)^2) x^2 + 3(b^3 \log(c) - a b^2 + (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)^2 + (3 b^3 \log(c) - 3 a^2 b^2 + 3(b^3 c^2 \log(c) - a b^2 c^2) x^2 + 3(b^3 c^2 x^2 + b^3) \log(x)) \log(\sqrt{c^2 x^2 + 1} + 1)^2 + 3(b^3 \log(c)^2 - 2 a b^2 \log(c) + (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2) \log(x) - 3(b^3 \log(c)^2 - 2 a b^2 \log(c) + (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2 + (b^3 c^2 x^2 + b^3) \log(x))^2 + 2(b^3 \log(c) - a b^2 + (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x) + (b^3 \log(c)^2 - 2 a b^2 \log(c) + (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2 + (b^3 c^2 x^2 + b^3) \log(x))^2 + 2(b^3 \log(c) - a b^2 + (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)) \sqrt{c^2 x^2 + 1} \log(\sqrt{c^2 x^2 + 1} + 1) + (b^3 \log(c)^3 - 3 a^2 b^2 \log(c)^2 + (b^3 c^2 x^2 + b^3) \log(x))^3 + (b^3 c^2 \log(c)^3 - 3 a^2 b^2 c^2 \log(c)^2) x^2 + 3(b^3 \log(c) - a b^2 + (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)^2 + 3(b^3 \log(c)^2 - 2 a b^2 \log(c) + (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2) \log(x)$$

$(x) \cdot \sqrt{c^2 x^2 + 1} / (c^2 x^6 + x^4 + (c^2 x^6 + x^4) \sqrt{c^2 x^2 + 1})$   
 $, x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))^3/x^4, x)`

[Out] `int((a + b*asinh(1/(c*x)))^3/x^4, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))**3/x**4, x)`

[Out] `Integral((a + b*acsch(c*x))**3/x**4, x)`

$$3.32 \quad \int \frac{(a+b\operatorname{csch}^{-1}(cx))^3}{x^5} dx$$

**Optimal.** Leaf size=204

$$\frac{9b^2c^2(a+b\operatorname{csch}^{-1}(cx))}{32x^2} - \frac{3b^2(a+b\operatorname{csch}^{-1}(cx))}{32x^4} + \frac{3}{32}c^4(a+b\operatorname{csch}^{-1}(cx))^3 + \frac{3bc\sqrt{\frac{1}{c^2x^2}+1}(a+b\operatorname{csch}^{-1}(cx))^2}{16x^3} - \frac{9b^3}{32x^5}$$

[Out] 45/256\*b^3\*c^4\*arccsch(c\*x)-3/32\*b^2\*(a+b\*arccsch(c\*x))/x^4+9/32\*b^2\*c^2\*(a+b\*arccsch(c\*x))/x^2+3/32\*c^4\*(a+b\*arccsch(c\*x))^3-1/4\*(a+b\*arccsch(c\*x))^3/x^4+3/128\*b^3\*c\*(1+1/c^2/x^2)^(1/2)/x^3-45/256\*b^3\*c^3\*(1+1/c^2/x^2)^(1/2)/x^3+16\*b\*c\*(a+b\*arccsch(c\*x))^2\*(1+1/c^2/x^2)^(1/2)/x^3-9/32\*b\*c^3\*(a+b\*arccsch(c\*x))^2\*(1+1/c^2/x^2)^(1/2)/x

**Rubi [A]** time = 0.18, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6286, 5446, 3311, 32, 2635, 8}

$$\frac{9b^2c^2(a+b\operatorname{csch}^{-1}(cx))}{32x^2} - \frac{3b^2(a+b\operatorname{csch}^{-1}(cx))}{32x^4} - \frac{9bc^3\sqrt{\frac{1}{c^2x^2}+1}(a+b\operatorname{csch}^{-1}(cx))^2}{32x} + \frac{3bc\sqrt{\frac{1}{c^2x^2}+1}(a+b\operatorname{csch}^{-1}(cx))}{16x^3} - \frac{9b^3}{32x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])^3/x^5, x]

[Out] (3\*b^3\*c\*Sqrt[1 + 1/(c^2\*x^2)])/(128\*x^3) - (45\*b^3\*c^3\*Sqrt[1 + 1/(c^2\*x^2)])/(256\*x) + (45\*b^3\*c^4\*ArcCsch[c\*x])/256 - (3\*b^2\*(a + b\*ArcCsch[c\*x]))/(32\*x^4) + (9\*b^2\*c^2\*(a + b\*ArcCsch[c\*x]))/(32\*x^2) + (3\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*(a + b\*ArcCsch[c\*x])^2)/(16\*x^3) - (9\*b\*c^3\*Sqrt[1 + 1/(c^2\*x^2)]\*(a + b\*ArcCsch[c\*x])^2)/(32\*x) + (3\*c^4\*(a + b\*ArcCsch[c\*x])^3)/32 - (a + b\*ArcCsch[c\*x])^3/(4\*x^4)

#### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*SIN[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*SIN[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*SIN[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*SIN[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*COS[e + f\*x]\*(b\*SIN[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 5446



```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 6286

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{x^5} dx &= -\left(c^4 \operatorname{Subst}\left(\int (a + bx)^3 \cosh(x) \sinh^3(x) dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{csch}^{-1}(cx))^3}{4x^4} + \frac{1}{4} (3bc^4) \operatorname{Subst}\left(\int (a + bx)^2 \sinh^4(x) dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= -\frac{3b^2 (a + b \operatorname{csch}^{-1}(cx))}{32x^4} + \frac{3bc\sqrt{1 + \frac{1}{c^2x^2}} (a + b \operatorname{csch}^{-1}(cx))^2}{16x^3} - \frac{(a + b \operatorname{csch}^{-1}(cx))^3}{4x^4} \\ &= \frac{3b^3c\sqrt{1 + \frac{1}{c^2x^2}}}{128x^3} - \frac{3b^2 (a + b \operatorname{csch}^{-1}(cx))}{32x^4} + \frac{9b^2c^2 (a + b \operatorname{csch}^{-1}(cx))}{32x^2} + \frac{3bc\sqrt{1 + \frac{1}{c^2x^2}}}{c^2} \\ &= \frac{3b^3c\sqrt{1 + \frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1 + \frac{1}{c^2x^2}}}{256x} - \frac{3b^2 (a + b \operatorname{csch}^{-1}(cx))}{32x^4} + \frac{9b^2c^2 (a + b \operatorname{csch}^{-1}(cx))}{32x^2} \\ &= \frac{3b^3c\sqrt{1 + \frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1 + \frac{1}{c^2x^2}}}{256x} + \frac{45}{256}b^3c^4\operatorname{csch}^{-1}(cx) - \frac{3b^2 (a + b \operatorname{csch}^{-1}(cx))}{32x^4} \end{aligned}$$

**Mathematica [A]** time = 0.37, size = 277, normalized size = 1.36

$$-64a^3 + 9bc^4x^4(8a^2 + 5b^2) \sinh^{-1}\left(\frac{1}{cx}\right) - 24b \operatorname{csch}^{-1}(cx) \left(8a^2 + 2abcx\sqrt{\frac{1}{c^2x^2} + 1} (3c^2x^2 - 2) + b^2(1 - 3c^2x^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^5, x]
```

```
[Out] (-64*a^3 - 24*a*b^2 + 48*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 24*b*(8*a^2 + b^2*(1 - 3*c^2*x^2) + 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + 3*c^2*x^2))*ArcCsch[c*x] + 24*b^2*(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(2 - 3*c^2*x^2) + a*(-8 + 3*c^4*x^4))*ArcCsch[c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^3 + 9*b*(8*a^2 + 5*b^2)*c^4*x^4*ArcSinh[1/(c*x)])/(256*x^4)
```

**fricas [A]** time = 0.98, size = 346, normalized size = 1.70

$$72ab^2c^2x^2 + 8(3b^3c^4x^4 - 8b^3) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx}\right) - 64a^3 - 24ab^2 + 24\left(3ab^2c^4x^4 - 8ab^2 - (3b^3c^3x^3 - 2b^3c^3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^5,x, algorithm="fricas")

[Out] 1/256\*(72\*a\*b^2\*c^2\*x^2 + 8\*(3\*b^3\*c^4\*x^4 - 8\*b^3)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^3 - 64\*a^3 - 24\*a\*b^2 + 24\*(3\*a\*b^2\*c^4\*x^4 - 8\*a\*b^2 - (3\*b^3\*c^3\*x^3 - 2\*b^3\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x))^2 + 3\*(3\*(8\*a^2\*b + 5\*b^3)\*c^4\*x^4 + 24\*b^3\*c^2\*x^2 - 64\*a^2\*b - 8\*b^3 - 16\*(3\*a\*b^2\*c^3\*x^3 - 2\*a\*b^2\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - 3\*(3\*(8\*a^2\*b + 5\*b^3)\*c^3\*x^3 - 2\*(8\*a^2\*b + b^3)\*c\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/x^4

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^5,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3/x^5, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arcsch}(cx))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))^3/x^5,x)

[Out] int((a+b\*arccsch(c\*x))^3/x^5,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))^3/x^5,x, algorithm="maxima")

[Out] 3/64\*a^2\*b\*((3\*c^5\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) + 1) - 3\*c^5\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) - 1) - 2\*(3\*c^8\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) - 5\*c^6\*x\*sqrt(1/(c^2\*x^2) + 1)))/(c^4\*x^4\*(1/(c^2\*x^2) + 1)^2 - 2\*c^2\*x^2\*(1/(c^2\*x^2) + 1) + 1))/c - 16\*arccsch(c\*x)/x^4 - 1/4\*b^3\*log(sqrt(c^2\*x^2 + 1) + 1)^3/x^4 - 1/4\*a^3/x^4 - integrate(1/4\*(4\*b^3\*log(c)^3 - 12\*a\*b^2\*log(c)^2 + 4\*(b^3\*c^2\*x^2 + b^3)\*log(x)^3 + 4\*(b^3\*c^2\*log(c)^3 - 3\*a\*b^2\*c^2\*log(c)^2)\*x^2 + 12\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2)\*log(x)^2 + 3\*(4\*b^3\*log(c) - 4\*a\*b^2 + 4\*(b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2 + 4\*(b^3\*c^2\*x^2 + b^3)\*log(x) + sqrt(c^2\*x^2 + 1)\*(4\*b^3\*log(c) - 4\*a\*b^2 + (b^3\*c^2\*(4\*log(c) - 1) - 4\*a\*b^2\*c^2)\*x^2 + 4\*(b^3\*c^2\*x^2 + b^3)\*log(x)))\*log(sqrt(c^2\*x^2 + 1) + 1)^2 + 12\*(b^3\*log(c)^2 - 2\*a\*b^2\*log(c) + (b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^2)\*log(x) - 12\*(b^3\*log(c)^2 - 2\*a\*b^2\*log(c) + (b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^2 + (b^3\*c^2\*x^2 + b^3)\*log(x)^2 + 2\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2)\*log(x) + (b^3\*log(c))^2 - 2\*a\*b^2\*log(c) + (b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^2 + (b^3\*c^2\*x^2 + b^3)\*log(x)^2 + 2\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2)\*log(x))\*sqrt(c^2\*x^2 + 1)\*log(sqrt(c^2\*x^2 + 1) + 1) + 4\*(b^3\*log(c)^3 - 3\*a\*b^2\*log(c)^2 + (b^3\*c^2\*x^2 + b^3)\*log(x)^3 + (b^3\*c^2\*log(c)^3 - 3\*a\*b^2\*c^2\*log(c)^2)\*x^2 + 3\*(b^3\*log(c) - a\*b^2 + (b^3\*c^2\*log(c) - a\*b^2\*c^2)\*x^2)\*log(x)^2 + 3\*(b^3\*log(c)^2 - 2\*a\*b^2\*log(c) + (b^3\*c^2\*log(c)^2 - 2\*a\*b^2\*c^2\*log(c))\*x^2)\*log(x))\*sqrt(c^2\*x^2 + 1))/(c^2\*x^7 + x^5 + (c^2\*x^7 + x^5)\*sqrt(c^2\*x^2 + 1)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))^3/x^5, x)

[Out] int((a + b\*asinh(1/(c\*x)))^3/x^5, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*\*3/x\*\*5, x)

[Out] Integral((a + b\*acsch(c\*x))\*\*3/x\*\*5, x)

$$3.33 \quad \int \frac{x}{a+b\operatorname{csch}^{-1}(cx)} dx$$

**Optimal.** Leaf size=15

$$\operatorname{Int}\left(\frac{x}{a+b\operatorname{csch}^{-1}(cx)}, x\right)$$

[Out] Unintegrable(x/(a+b\*arccsch(c\*x)), x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b\*ArcCsch[c\*x]), x]

[Out] Defer[Int][x/(a + b\*ArcCsch[c\*x]), x]

Rubi steps

$$\int \frac{x}{a+b\operatorname{csch}^{-1}(cx)} dx = \int \frac{x}{a+b\operatorname{csch}^{-1}(cx)} dx$$

**Mathematica [A]** time = 2.93, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[x/(a + b\*ArcCsch[c\*x]), x]

**fricas [A]** time = 0.88, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x}{b\operatorname{arcsch}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral(x/(b\*arccsch(c\*x) + a), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b\operatorname{arcsch}(cx)+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate(x/(b\*arccsch(c\*x) + a), x)

**maple [A]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\operatorname{arcsch}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a+b*arccsch(c*x)),x)`

[Out] `int(x/(a+b*arccsch(c*x)),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{b \operatorname{arcsch}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arccsch(c*x)),x, algorithm="maxima")`

[Out] `integrate(x/(b*arccsch(c*x) + a), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*asinh(1/(c*x))),x)`

[Out] `int(x/(a + b*asinh(1/(c*x))), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{acsch}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*acsch(c*x)),x)`

[Out] `Integral(x/(a + b*acsch(c*x)), x)`

$$3.34 \quad \int \frac{1}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Optimal. Leaf size=13

$$\operatorname{Int}\left(\frac{1}{a+b\operatorname{csch}^{-1}(cx)}, x\right)$$

[Out] Unintegrable(1/(a+b\*arccsch(c\*x)), x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])^(-1), x]

[Out] Defer[Int][(a + b\*ArcCsch[c\*x])^(-1), x]

Rubi steps

$$\int \frac{1}{a+b\operatorname{csch}^{-1}(cx)} dx = \int \frac{1}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Mathematica [A] time = 2.82, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])^(-1), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])^(-1), x]

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{b \operatorname{arcsch}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral(1/(b\*arccsch(c\*x) + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \operatorname{arcsch}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate(1/(b\*arccsch(c\*x) + a), x)

**maple** [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{arccsch}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b\*arccsch(c\*x)),x)

[Out] int(1/(a+b\*arccsch(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b \operatorname{arcsch}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] integrate(1/(b\*arccsch(c\*x) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b\*asinh(1/(c\*x))),x)

[Out] int(1/(a + b\*asinh(1/(c\*x))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{acsch}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*acsch(c\*x)),x)

[Out] Integral(1/(a + b\*acsch(c\*x)), x)

$$3.35 \quad \int \frac{1}{x(a+b\operatorname{csch}^{-1}(cx))} dx$$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{1}{x(a+b\operatorname{csch}^{-1}(cx))}, x\right)$$

[Out] Unintegrable(1/x/(a+b\*arccsch(c\*x)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{1}{x(a+b\operatorname{csch}^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + b\*ArcCsch[c\*x])), x]

[Out] Defer[Int][1/(x\*(a + b\*ArcCsch[c\*x])), x]

Rubi steps

$$\int \frac{1}{x(a+b\operatorname{csch}^{-1}(cx))} dx = \int \frac{1}{x(a+b\operatorname{csch}^{-1}(cx))} dx$$

Mathematica [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+b\operatorname{csch}^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + b\*ArcCsch[c\*x])), x]

[Out] Integrate[1/(x\*(a + b\*ArcCsch[c\*x])), x]

fricas [A] time = 0.90, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{bx \operatorname{arcsch}(cx) + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral(1/(b\*x\*arccsch(c\*x) + a\*x), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsch}(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate(1/((b\*arccsch(c\*x) + a)\*x), x)



**maple** [A] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{arccsch}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b\*arccsch(c\*x)),x)

[Out] int(1/x/(a+b\*arccsch(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arsch}(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] integrate(1/((b\*arccsch(c\*x) + a)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x\*(a + b\*asinh(1/(c\*x))))),x)

[Out] int(1/(x\*(a + b\*asinh(1/(c\*x))))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a + b \operatorname{acsch}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b\*acsch(c\*x)),x)

[Out] Integral(1/(x\*(a + b\*acsch(c\*x))), x)

$$3.36 \quad \int \frac{1}{x^2(a+b\operatorname{csch}^{-1}(cx))} dx$$

**Optimal.** Leaf size=46

$$\frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{b} - \frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{b}$$

[Out]  $-c \operatorname{Chi}(a/b + \operatorname{arccsch}(c*x)) * \cosh(a/b) / b + c * \operatorname{Shi}(a/b + \operatorname{arccsch}(c*x)) * \sinh(a/b) / b$

**Rubi [A]** time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {6286, 3303, 3298, 3301}

$$\frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{b} - \frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(x^2*(a + b*\operatorname{ArcCsch}[c*x])), x]$

[Out]  $-((c*\operatorname{Cosh}[a/b]*\operatorname{CoshIntegral}[a/b + \operatorname{ArcCsch}[c*x]])/b) + (c*\operatorname{Sinh}[a/b]*\operatorname{SinhIntegral}[a/b + \operatorname{ArcCsch}[c*x]])/b$

#### Rule 3298

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

#### Rule 3301

$\operatorname{Int}[\sin[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

#### Rule 3303

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[d*e - c*f, 0]$

#### Rule 6286

$\operatorname{Int}[(a_.) + \operatorname{ArcCsch}[(c_.)*(x_)]*(b_.))^n*(x_)^m, x\_Symbol] \rightarrow -\operatorname{Dist}[(c^{(m+1)})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Csch}[x]^{(m+1)}*\operatorname{Coth}[x], x], x, \operatorname{ArcCsch}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \ \&\& \operatorname{IntegerQ}[n] \ \&\& \operatorname{IntegerQ}[m] \ \&\& (\operatorname{GtQ}[n, 0] \ \|\ \operatorname{LtQ}[m, -1])$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(a+b\operatorname{csch}^{-1}(cx))} dx &= -\left(c \operatorname{Subst}\left(\int \frac{\cosh(x)}{a+bx} dx, x, \operatorname{csch}^{-1}(cx)\right)\right) \\ &= -\left(\left(c \cosh\left(\frac{a}{b}\right)\right) \operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx} dx, x, \operatorname{csch}^{-1}(cx)\right)\right) + \left(c \sinh\left(\frac{a}{b}\right)\right) \operatorname{Subst}\left(\int \frac{1}{a+bx} dx, x, \operatorname{csch}^{-1}(cx)\right) \\ &= -\frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{b} + \frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 44, normalized size = 0.96

$$\frac{c \left( \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right) - \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right) \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(a + b\*ArcCsch[c\*x])), x]

[Out] -((c\*(Cosh[a/b]\*CoshIntegral[a/b + ArcCsch[c\*x]] - Sinh[a/b]\*SinhIntegral[a/b + ArcCsch[c\*x]]))/b)

**fricas [F]** time = 1.59, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{bx^2 \operatorname{arcsch}(cx) + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral(1/(b\*x^2\*arccsch(c\*x) + a\*x^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsch}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate(1/((b\*arccsch(c\*x) + a)\*x^2), x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{arccsch}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b\*arccsch(c\*x)), x)

[Out] int(1/x^2/(a+b\*arccsch(c\*x)), x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsch}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b\*arccsch(c\*x)), x, algorithm="maxima")

[Out] integrate(1/((b\*arccsch(c\*x) + a)\*x^2), x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^2*(a + b*asinh(1/(c*x))))),x)`

[Out] `int(1/(x^2*(a + b*asinh(1/(c*x))))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b \operatorname{acsch}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(a+b*acsch(c*x)),x)`

[Out] `Integral(1/(x**2*(a + b*acsch(c*x))), x)`

$$3.37 \quad \int \frac{1}{x^3(a+b\operatorname{csch}^{-1}(cx))} dx$$

**Optimal.** Leaf size=63

$$\frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2\operatorname{csch}^{-1}(cx)\right)}{2b} - \frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2\operatorname{csch}^{-1}(cx)\right)}{2b}$$

[Out]  $-1/2*c^2*\cosh(2*a/b)*\operatorname{Shi}(2*a/b+2*\operatorname{arccsch}(c*x))/b+1/2*c^2*\operatorname{Chi}(2*a/b+2*\operatorname{arccsch}(c*x))*\sinh(2*a/b)/b$

**Rubi [A]** time = 0.14, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6286, 5448, 12, 3303, 3298, 3301}

$$\frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2\operatorname{csch}^{-1}(cx)\right)}{2b} - \frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2\operatorname{csch}^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/(x^3*(a + b*\operatorname{ArcCsch}[c*x])), x]$

[Out]  $(c^2*\operatorname{CoshIntegral}[(2*a)/b + 2*\operatorname{ArcCsch}[c*x]]*\operatorname{Sinh}[(2*a)/b])/(2*b) - (c^2*\operatorname{Cosh}[(2*a)/b]*\operatorname{SinhIntegral}[(2*a)/b + 2*\operatorname{ArcCsch}[c*x]])/(2*b)$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 3298

$\operatorname{Int}[\sin[(e_*) + (\operatorname{Complex}[0, fz_*])*(f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[(I*\operatorname{SinhIntegral}[(c*f*fz)/d + f*fz*x])/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \operatorname{EqQ}[d*e - c*f*fz*I, 0]$

#### Rule 3301

$\operatorname{Int}[\sin[(e_*) + (\operatorname{Complex}[0, fz_*])*(f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[(c*f*fz)/d + f*fz*x]/d, x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \operatorname{EqQ}[d*(e - \operatorname{Pi}/2) - c*f*fz*I, 0]$

#### Rule 3303

$\operatorname{Int}[\sin[(e_*) + (f_*)*(x_)]/((c_*) + (d_*)*(x_)), x\_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Sin}[(c*f)/d + f*x]/(c + d*x), x], x] + \operatorname{Dist}[\operatorname{Sin}[(d*e - c*f)/d], \operatorname{Int}[\operatorname{Cos}[(c*f)/d + f*x]/(c + d*x), x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \operatorname{NeQ}[d*e - c*f, 0]$

#### Rule 5448

$\operatorname{Int}[\operatorname{Cosh}[(a_*) + (b_*)*(x_)]^{(p_*)}*((c_*) + (d_*)*(x_))^{(m_*)}*\operatorname{Sinh}[(a_*) + (b_*)*(x_)]^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigReduce}[(c + d*x)^m, \operatorname{Sinh}[a + b*x]^n*\operatorname{Cosh}[a + b*x]^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \ \&\& \ \operatorname{IGtQ}[n, 0] \ \& \ \operatorname{IGtQ}[p, 0]$

#### Rule 6286

$\operatorname{Int}[(a_*) + \operatorname{ArcCsch}[(c_*)*(x_)]*(b_*)^{(n_*)}*(x_)^{(m_*)}, x\_Symbol] \rightarrow -\operatorname{Dist}[(c^{(m+1)})^{(-1)}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Csch}[x]^{(m+1)}*\operatorname{Coth}[x], x], x], \operatorname{Ar}$

`cSch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt Q[n, 0] || LtQ[m, -1])`

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 (a + b \operatorname{csch}^{-1}(cx))} dx &= - \left( c^2 \operatorname{Subst} \left( \int \frac{\cosh(x) \sinh(x)}{a + bx} dx, x, \operatorname{csch}^{-1}(cx) \right) \right) \\
 &= - \left( c^2 \operatorname{Subst} \left( \int \frac{\sinh(2x)}{2(a + bx)} dx, x, \operatorname{csch}^{-1}(cx) \right) \right) \\
 &= - \left( \frac{1}{2} c^2 \operatorname{Subst} \left( \int \frac{\sinh(2x)}{a + bx} dx, x, \operatorname{csch}^{-1}(cx) \right) \right) \\
 &= - \left( \frac{1}{2} \left( c^2 \cosh \left( \frac{2a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\sinh \left( \frac{2a}{b} + 2x \right)}{a + bx} dx, x, \operatorname{csch}^{-1}(cx) \right) \right) + \frac{1}{2} \left( c^2 \sinh \left( \frac{2a}{b} \right) \right) \\
 &= \frac{c^2 \operatorname{Chi} \left( \frac{2a}{b} + 2 \operatorname{csch}^{-1}(cx) \right) \sinh \left( \frac{2a}{b} \right)}{2b} - \frac{c^2 \cosh \left( \frac{2a}{b} \right) \operatorname{Shi} \left( \frac{2a}{b} + 2 \operatorname{csch}^{-1}(cx) \right)}{2b}
 \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 56, normalized size = 0.89

$$\frac{c^2 \left( \sinh \left( \frac{2a}{b} \right) \operatorname{Chi} \left( \frac{2a}{b} + 2 \operatorname{csch}^{-1}(cx) \right) - \cosh \left( \frac{2a}{b} \right) \operatorname{Shi} \left( \frac{2a}{b} + 2 \operatorname{csch}^{-1}(cx) \right) \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(a + b\*ArcCsch[c\*x])),x]

[Out] (c^2\*(CoshIntegral[(2\*a)/b + 2\*ArcCsch[c\*x]]\*Sinh[(2\*a)/b] - Cosh[(2\*a)/b]\*SinhIntegral[(2\*a)/b + 2\*ArcCsch[c\*x]])/(2\*b)

**fricas [F]** time = 2.15, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{1}{bx^3 \operatorname{arcsch}(cx) + ax^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*x^3\*arccsch(c\*x) + a\*x^3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsch}(cx) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate(1/((b\*arccsch(c\*x) + a)\*x^3), x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{arccsch}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(a+b*arccsch(c*x)),x)`

[Out] `int(1/x^3/(a+b*arccsch(c*x)),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsch}(cx) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="maxima")`

[Out] `integrate(1/((b*arccsch(c*x) + a)*x^3), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^3*(a + b*asinh(1/(c*x))))),x)`

[Out] `int(1/(x^3*(a + b*asinh(1/(c*x))))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (a + b \operatorname{acsch}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(a+b*acsch(c*x)),x)`

[Out] `Integral(1/(x**3*(a + b*acsch(c*x))), x)`

$$3.38 \quad \int \frac{1}{x^4(a+b\operatorname{csch}^{-1}(cx))} dx$$

**Optimal.** Leaf size=117

$$\frac{c^3 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{4b} - \frac{c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b} + 3\operatorname{csch}^{-1}(cx)\right)}{4b} - \frac{c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{4b} + \frac{c^3 \sinh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3a}{b} + 3\operatorname{csch}^{-1}(cx)\right)}{4b}$$

[Out] 1/4\*c^3\*Chi(a/b+arccsch(c\*x))\*cosh(a/b)/b-1/4\*c^3\*Chi(3\*a/b+3\*arccsch(c\*x))\*cosh(3\*a/b)/b-1/4\*c^3\*Shi(a/b+arccsch(c\*x))\*sinh(a/b)/b+1/4\*c^3\*Shi(3\*a/b+3\*arccsch(c\*x))\*sinh(3\*a/b)/b

**Rubi [A]** time = 0.24, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {6286, 5448, 3303, 3298, 3301}

$$\frac{c^3 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{4b} - \frac{c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b} + 3\operatorname{csch}^{-1}(cx)\right)}{4b} - \frac{c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{csch}^{-1}(cx)\right)}{4b} + \frac{c^3 \sinh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3a}{b} + 3\operatorname{csch}^{-1}(cx)\right)}{4b}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4\*(a + b\*ArcCsch[c\*x])),x]

[Out] (c^3\*Cosh[a/b]\*CoshIntegral[a/b + ArcCsch[c\*x]])/(4\*b) - (c^3\*Cosh[(3\*a)/b]\*CoshIntegral[(3\*a)/b + 3\*ArcCsch[c\*x]])/(4\*b) - (c^3\*Sinh[a/b]\*SinhIntegral[a/b + ArcCsch[c\*x]])/(4\*b) + (c^3\*Sinh[(3\*a)/b]\*SinhIntegral[(3\*a)/b + 3\*ArcCsch[c\*x]])/(4\*b)

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

#### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 5448

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sinh[a + b\*x]^(n)\*Cosh[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 6286

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^m, x\_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b\*x)^n\*Csch[x]^(m + 1)\*Coth[x], x], x, ArcCsch[c\*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt



Q[n, 0] || LtQ[m, -1])

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4 (a + b \operatorname{csch}^{-1}(cx))} dx &= - \left( c^3 \operatorname{Subst} \left( \int \frac{\cosh(x) \sinh^2(x)}{a + bx} dx, x, \operatorname{csch}^{-1}(cx) \right) \right) \\
 &= - \left( c^3 \operatorname{Subst} \left( \int \left( -\frac{\cosh(x)}{4(a + bx)} + \frac{\cosh(3x)}{4(a + bx)} \right) dx, x, \operatorname{csch}^{-1}(cx) \right) \right) \\
 &= \frac{1}{4} c^3 \operatorname{Subst} \left( \int \frac{\cosh(x)}{a + bx} dx, x, \operatorname{csch}^{-1}(cx) \right) - \frac{1}{4} c^3 \operatorname{Subst} \left( \int \frac{\cosh(3x)}{a + bx} dx, x, \operatorname{csch}^{-1}(cx) \right) \\
 &= \frac{1}{4} \left( c^3 \cosh \left( \frac{a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\cosh \left( \frac{a}{b} + x \right)}{a + bx} dx, x, \operatorname{csch}^{-1}(cx) \right) - \frac{1}{4} \left( c^3 \cosh \left( \frac{3a}{b} \right) \right) \operatorname{Subst} \left( \int \frac{\cosh \left( \frac{3a}{b} + x \right)}{a + bx} dx, x, \operatorname{csch}^{-1}(cx) \right) \\
 &= \frac{c^3 \cosh \left( \frac{a}{b} \right) \operatorname{Chi} \left( \frac{a}{b} + \operatorname{csch}^{-1}(cx) \right)}{4b} - \frac{c^3 \cosh \left( \frac{3a}{b} \right) \operatorname{Chi} \left( \frac{3a}{b} + 3 \operatorname{csch}^{-1}(cx) \right)}{4b} - \frac{c^3 \operatorname{Shi} \left( \frac{a}{b} + \operatorname{csch}^{-1}(cx) \right)}{4b} + \frac{c^3 \operatorname{Shi} \left( \frac{3a}{b} + 3 \operatorname{csch}^{-1}(cx) \right)}{4b}
 \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 91, normalized size = 0.78

$$\frac{c^3 \left( -\cosh \left( \frac{a}{b} \right) \operatorname{Chi} \left( \frac{a}{b} + \operatorname{csch}^{-1}(cx) \right) + \cosh \left( \frac{3a}{b} \right) \operatorname{Chi} \left( 3 \left( \frac{a}{b} + \operatorname{csch}^{-1}(cx) \right) \right) + \sinh \left( \frac{a}{b} \right) \operatorname{Shi} \left( \frac{a}{b} + \operatorname{csch}^{-1}(cx) \right) - \sinh \left( \frac{3a}{b} \right) \operatorname{Shi} \left( \frac{3a}{b} + 3 \operatorname{csch}^{-1}(cx) \right) \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(a + b\*ArcCsch[c\*x])),x]

[Out] -1/4\*(c^3\*(-(Cosh[a/b]\*CoshIntegral[a/b + ArcCsch[c\*x]]) + Cosh[(3\*a)/b]\*CoshIntegral[3\*(a/b + ArcCsch[c\*x]]) + Sinh[a/b]\*SinhIntegral[a/b + ArcCsch[c\*x]] - Sinh[(3\*a)/b]\*SinhIntegral[3\*(a/b + ArcCsch[c\*x])]))/b

**fricas [F]** time = 2.14, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{1}{bx^4 \operatorname{arcsch}(cx) + ax^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] integral(1/(b\*x^4\*arccsch(c\*x) + a\*x^4), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsch}(cx) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate(1/((b\*arccsch(c\*x) + a)\*x^4), x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \operatorname{arcsch}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(a+b*arccsch(c*x)),x)`

[Out] `int(1/x^4/(a+b*arccsch(c*x)),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \operatorname{arcsch}(cx) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="maxima")`

[Out] `integrate(1/((b*arccsch(c*x) + a)*x^4), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4*(a + b*asinh(1/(c*x))))),x)`

[Out] `int(1/(x^4*(a + b*asinh(1/(c*x))))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (a + b \operatorname{acsch}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(a+b*acsch(c*x)),x)`

[Out] `Integral(1/(x**4*(a + b*acsch(c*x))), x)`

### 3.39 $\int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^3 dx$

**Optimal.** Leaf size=19

$$\operatorname{Int}\left((dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^3, x\right)$$

[Out] Unintegrable((d\*x)^m\*(a+b\*arccsch(c\*x))^3, x)

**Rubi [A]** time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcCsch[c\*x])^3, x]

[Out] Defer[Int][(d\*x)^m\*(a + b\*ArcCsch[c\*x])^3, x]

Rubi steps

$$\int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^3 dx = \int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^3 dx$$

**Mathematica [A]** time = 6.87, size = 0, normalized size = 0.00

$$\int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcCsch[c\*x])^3, x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcCsch[c\*x])^3, x]

**fricas [A]** time = 2.69, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b^3 \operatorname{arcsch}(cx)^3 + 3ab^2 \operatorname{arcsch}(cx)^2 + 3a^2b \operatorname{arcsch}(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsch(c\*x))^3,x, algorithm="fricas")

[Out] integral((b^3\*arccsch(c\*x)^3 + 3\*a\*b^2\*arccsch(c\*x)^2 + 3\*a^2\*b\*arccsch(c\*x) + a^3)\*(d\*x)^m, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsch(c\*x))^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^3\*(d\*x)^m, x)

**maple [A]** time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^m \left( a + b \operatorname{arccsch}(cx) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(a+b*arccsch(c*x))^3,x)`

[Out] `int((d*x)^m*(a+b*arccsch(c*x))^3,x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="maxima")`

[Out] 
$$b^3 d^m x^m \log(\sqrt{c^2 x^2 + 1} + 1)^3 / (m + 1) + (d x)^{m+1} a^3 / (d(m+1)) - \text{integrate}((3((b^3 d^m (m+1) \log(c) - a b^2 d^m (m+1) - (a b^2 c^2 d^m (m+1) - (d^m (m+1) \log(c) + d^m) b^3 c^2) x^2 + (b^3 c^2 d^m (m+1) x^2 + b^3 d^m (m+1)) \log(x)) \sqrt{c^2 x^2 + 1} x^m + (b^3 d^m (m+1) \log(c) - a b^2 d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c) - a b^2 c^2 d^m (m+1)) x^2 + (b^3 c^2 d^m (m+1) x^2 + b^3 d^m (m+1)) \log(x)) x^m) \log(\sqrt{c^2 x^2 + 1} + 1)^2 + (b^3 d^m (m+1) \log(c)^3 - 3 a b^2 d^m (m+1) \log(c)^2 + 3 a^2 b d^m (m+1) \log(c) + (b^3 c^2 d^m (m+1) x^2 + b^3 d^m (m+1)) \log(x)^3 + (b^3 c^2 d^m (m+1) \log(c)^3 - 3 a b^2 c^2 d^m (m+1) \log(c)^2 + 3 a^2 b c^2 d^m (m+1) \log(c)) x^2 + 3 (b^3 d^m (m+1) \log(c) - a b^2 d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c) - a b^2 c^2 d^m (m+1)) x^2) \log(x)^2 + 3 (b^3 d^m (m+1) \log(c)^2 - 2 a b^2 d^m (m+1) \log(c) + a^2 b d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c)^2 - 2 a b^2 c^2 d^m (m+1) \log(c) + a^2 b c^2 d^m (m+1)) x^2) \log(x)) \sqrt{c^2 x^2 + 1} x^m + (b^3 d^m (m+1) \log(c)^3 - 3 a b^2 d^m (m+1) \log(c)^2 + 3 a^2 b d^m (m+1) \log(c) + (b^3 c^2 d^m (m+1) x^2 + b^3 d^m (m+1)) \log(x)^3 + (b^3 c^2 d^m (m+1) \log(c)^3 - 3 a b^2 c^2 d^m (m+1) \log(c)^2 + 3 a^2 b c^2 d^m (m+1) \log(c)) x^2 + 3 (b^3 d^m (m+1) \log(c) - a b^2 d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c) - a b^2 c^2 d^m (m+1)) x^2) \log(x)^2 + 3 (b^3 d^m (m+1) \log(c)^2 - 2 a b^2 d^m (m+1) \log(c) + a^2 b d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c)^2 - 2 a b^2 c^2 d^m (m+1) \log(c) + a^2 b c^2 d^m (m+1)) x^2) \log(x)) x^m - 3 ((b^3 d^m (m+1) \log(c)^2 - 2 a b^2 d^m (m+1) \log(c) + a^2 b d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c)^2 - 2 a b^2 c^2 d^m (m+1) \log(c) + a^2 b c^2 d^m (m+1)) x^2 + (b^3 c^2 d^m (m+1) x^2 + b^3 d^m (m+1)) \log(x)^2 + 2 (b^3 d^m (m+1) \log(c) - a b^2 d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c) - a b^2 c^2 d^m (m+1)) x^2) \log(x)) \sqrt{c^2 x^2 + 1} x^m + (b^3 d^m (m+1) \log(c)^2 - 2 a b^2 d^m (m+1) \log(c) + a^2 b d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c)^2 - 2 a b^2 c^2 d^m (m+1) \log(c) + a^2 b c^2 d^m (m+1)) x^2 + (b^3 c^2 d^m (m+1) x^2 + b^3 d^m (m+1)) \log(x)^2 + 2 (b^3 d^m (m+1) \log(c) - a b^2 d^m (m+1) + (b^3 c^2 d^m (m+1) \log(c) - a b^2 c^2 d^m (m+1)) x^2) \log(x)) x^m) \log(\sqrt{c^2 x^2 + 1} + 1) / (c^2 (m+1) x^2 + (c^2 (m+1) x^2 + m + 1) \sqrt{c^2 x^2 + 1} + m + 1), x)$$

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (dx)^m \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(a + b*asinh(1/(c*x)))^3,x)`

[Out] `int((d*x)^m*(a + b*asinh(1/(c*x)))^3, x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{acsch}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*acsch(c*x))**3, x)
```

### 3.40 $\int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^2 dx$

**Optimal.** Leaf size=19

$$\operatorname{Int}\left((dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^2, x\right)$$

[Out] Unintegrable((d\*x)^m\*(a+b\*arccsch(c\*x))^2,x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m\*(a + b\*ArcCsch[c\*x])^2,x]

[Out] Defer[Int] [(d\*x)^m\*(a + b\*ArcCsch[c\*x])^2, x]

Rubi steps

$$\int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^2 dx = \int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^2 dx$$

**Mathematica [A]** time = 4.49, size = 0, normalized size = 0.00

$$\int (dx)^m \left( a + b \operatorname{csch}^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m\*(a + b\*ArcCsch[c\*x])^2,x]

[Out] Integrate[(d\*x)^m\*(a + b\*ArcCsch[c\*x])^2, x]

**fricas [A]** time = 0.78, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b^2 \operatorname{arcsch}(cx)^2 + 2ab \operatorname{arcsch}(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsch(c\*x))^2,x, algorithm="fricas")

[Out] integral((b^2\*arccsch(c\*x)^2 + 2\*a\*b\*arccsch(c\*x) + a^2)\*(d\*x)^m, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsch(c\*x))^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)^2\*(d\*x)^m, x)

**maple [A]** time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^m \left( a + b \operatorname{arcsch}(cx) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arccsch(c\*x))^2,x)

[Out] int((d\*x)^m\*(a+b\*arccsch(c\*x))^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b^2 d^m x x^m \log\left(\sqrt{c^2 x^2 + 1} + 1\right)^2}{m + 1} + \frac{(dx)^{m+1} a^2}{d(m + 1)} \int - \frac{(b^2 d^m (m + 1) \log(c)^2 - 2 a b d^m (m + 1) \log(c) + (b^2 c^2 d^m (m + 1) \log(c) - a^2 d^m (m + 1) \log(c) + (b^2 c^2 d^m (m + 1) \log(c) - a^2 d^m (m + 1) \log(c)) x^2 + (b^2 c^2 d^m (m + 1) x^2 + b^2 d^m (m + 1)) \log(x)^2 + 2 (b^2 d^m (m + 1) \log(c) - a b d^m (m + 1) + (b^2 c^2 d^m (m + 1) \log(c) - a b c^2 d^m (m + 1)) x^2) \log(x)) \sqrt{c^2 x^2 + 1} x^m + (b^2 d^m (m + 1) \log(c)^2 - 2 a b d^m (m + 1) \log(c) + (b^2 c^2 d^m (m + 1) \log(c)^2 - 2 a b c^2 d^m (m + 1) \log(c)) x^2 + (b^2 c^2 d^m (m + 1) x^2 + b^2 d^m (m + 1)) \log(x)^2 + 2 (b^2 d^m (m + 1) \log(c) - a b d^m (m + 1) + (b^2 c^2 d^m (m + 1) \log(c) - a b c^2 d^m (m + 1)) x^2) \log(x)) x^m - 2 ((b^2 d^m (m + 1) \log(c) - a b d^m (m + 1) - (a b c^2 d^m (m + 1) - (d^m (m + 1) \log(c) + d^m) b^2 c^2) x^2 + (b^2 c^2 d^m (m + 1) x^2 + b^2 d^m (m + 1)) \log(x)) \sqrt{c^2 x^2 + 1} x^m + (b^2 d^m (m + 1) \log(c) - a b d^m (m + 1) + (b^2 c^2 d^m (m + 1) \log(c) - a b c^2 d^m (m + 1)) x^2 + (b^2 c^2 d^m (m + 1) x^2 + b^2 d^m (m + 1)) \log(x)) x^m) \log(\sqrt{c^2 x^2 + 1} + 1)}{(c^2 (m + 1) x^2 + (c^2 (m + 1) x^2 + m + 1) \sqrt{c^2 x^2 + 1} + m + 1), x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsch(c\*x))^2,x, algorithm="maxima")

[Out] b^2\*d^m\*x\*x^m\*log(sqrt(c^2\*x^2 + 1) + 1)^2/(m + 1) + (d\*x)^(m + 1)\*a^2/(d\*(m + 1)) - integrate(-((b^2\*d^m\*(m + 1)\*log(c)^2 - 2\*a\*b\*d^m\*(m + 1)\*log(c) + (b^2\*c^2\*d^m\*(m + 1)\*log(c)^2 - 2\*a\*b\*c^2\*d^m\*(m + 1)\*log(c))\*x^2 + (b^2\*c^2\*d^m\*(m + 1)\*x^2 + b^2\*d^m\*(m + 1))\*log(x)^2 + 2\*(b^2\*d^m\*(m + 1)\*log(c) - a\*b\*d^m\*(m + 1) + (b^2\*c^2\*d^m\*(m + 1)\*log(c) - a\*b\*c^2\*d^m\*(m + 1))\*x^2)\*log(x))\*sqrt(c^2\*x^2 + 1)\*x^m + (b^2\*d^m\*(m + 1)\*log(c)^2 - 2\*a\*b\*d^m\*(m + 1)\*log(c) + (b^2\*c^2\*d^m\*(m + 1)\*log(c)^2 - 2\*a\*b\*c^2\*d^m\*(m + 1)\*log(c))\*x^2 + (b^2\*c^2\*d^m\*(m + 1)\*x^2 + b^2\*d^m\*(m + 1))\*log(x)^2 + 2\*(b^2\*d^m\*(m + 1)\*log(c) - a\*b\*d^m\*(m + 1) + (b^2\*c^2\*d^m\*(m + 1)\*log(c) - a\*b\*c^2\*d^m\*(m + 1))\*x^2)\*log(x))\*x^m - 2\*((b^2\*d^m\*(m + 1)\*log(c) - a\*b\*d^m\*(m + 1) - (a\*b\*c^2\*d^m\*(m + 1) - (d^m\*(m + 1)\*log(c) + d^m)\*b^2\*c^2)\*x^2 + (b^2\*c^2\*d^m\*(m + 1)\*x^2 + b^2\*d^m\*(m + 1))\*log(x))\*sqrt(c^2\*x^2 + 1)\*x^m + (b^2\*d^m\*(m + 1)\*log(c) - a\*b\*d^m\*(m + 1) + (b^2\*c^2\*d^m\*(m + 1)\*log(c) - a\*b\*c^2\*d^m\*(m + 1))\*x^2 + (b^2\*c^2\*d^m\*(m + 1)\*x^2 + b^2\*d^m\*(m + 1))\*log(x))\*x^m)\*log(sqrt(c^2\*x^2 + 1) + 1)/(c^2\*(m + 1)\*x^2 + (c^2\*(m + 1)\*x^2 + m + 1)\*sqrt(c^2\*x^2 + 1) + m + 1), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (dx)^m \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a + b\*asinh(1/(c\*x)))^2,x)

[Out] int((d\*x)^m\*(a + b\*asinh(1/(c\*x)))^2, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{acsch}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m\*(a+b\*acsch(c\*x))\*\*2,x)

[Out] Integral((d\*x)\*\*m\*(a + b\*acsch(c\*x))\*\*2, x)

### 3.41 $\int (dx)^m (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=67

$$\frac{(dx)^{m+1} (a + b \operatorname{csch}^{-1}(cx))}{d(m+1)} + \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; -\frac{1}{c^2 x^2}\right)}{cm(m+1)}$$

[Out]  $(d*x)^{(1+m)}*(a+b*\operatorname{arccsch}(c*x))/d/(1+m)+b*(d*x)^m*\operatorname{hypergeom}([1/2, -1/2*m], [1-1/2*m], -1/c^2/x^2)/c/m/(1+m)$

**Rubi [A]** time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {6284, 339, 364}

$$\frac{(dx)^{m+1} (a + b \operatorname{csch}^{-1}(cx))}{d(m+1)} + \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; -\frac{1}{c^2 x^2}\right)}{cm(m+1)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*x)^m*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $((d*x)^{(1+m)}*(a + b*\operatorname{ArcCsch}[c*x]))/(d*(1+m)) + (b*(d*x)^m*\operatorname{Hypergeometric2F1}[1/2, -m/2, 1 - m/2, -(1/(c^2*x^2))])/(c*m*(1+m))$

#### Rule 339

$\operatorname{Int}[(c_*)*(x_*)^m*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow -\operatorname{Dist}[(c*x)^{(m+1)}*(1/x)^{(m+1)}/c, \operatorname{Subst}[\operatorname{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x], x] /;$   $\operatorname{FreeQ}\{a, b, c, m, p\}, x \&\& \operatorname{ILtQ}[n, 0] \&\& \operatorname{!RationalQ}[m]$

#### Rule 364

$\operatorname{Int}[(c_*)*(x_*)^m*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a^p*(c*x)^{(m+1)}*\operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n+1, -(b*x^n)/a])]/(c*(m+1)), x] /;$   $\operatorname{FreeQ}\{a, b, c, m, n, p\}, x \&\& \operatorname{!IGtQ}[p, 0] \&\& (\operatorname{ILtQ}[p, 0] \operatorname{||} \operatorname{GtQ}[a, 0])$

#### Rule 6284

$\operatorname{Int}[(a_*) + \operatorname{ArcCsch}[(c_*)*(x_*)]*(b_*)]*((d_*)*(x_*)^m), x\_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcCsch}[c*x])]/(d*(m+1)), x] + \operatorname{Dist}[(b*d)/(c*(m+1)), \operatorname{Int}[(d*x)^{(m-1)}/\operatorname{Sqrt}[1 + 1/(c^2*x^2)], x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, m\}, x \&\& \operatorname{NeQ}[m, -1]$

#### Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{(dx)^{1+m} (a + b \operatorname{csch}^{-1}(cx))}{d(1+m)} + \frac{(bd) \int \frac{(dx)^{-1+m} dx}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{c(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \operatorname{csch}^{-1}(cx))}{d(1+m)} - \frac{\left(b \left(\frac{1}{x}\right)^m (dx)^m\right) \operatorname{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{c(1+m)} \\ &= \frac{(dx)^{1+m} (a + b \operatorname{csch}^{-1}(cx))}{d(1+m)} + \frac{b(dx)^m {}_2F_1\left(\frac{1}{2}, -\frac{m}{2}; 1 - \frac{m}{2}; -\frac{1}{c^2 x^2}\right)}{cm(1+m)} \end{aligned}$$



**Mathematica [A]** time = 0.24, size = 81, normalized size = 1.21

$$\frac{x(dx)^m \left( (m+1)(a + b \operatorname{csch}^{-1}(cx)) + \frac{bcx \sqrt{\frac{1}{c^2 x^2} + 1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; -c^2 x^2\right)}{\sqrt{c^2 x^2 + 1}} \right)}{(m+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^m\*(a + b\*ArcCsch[c\*x]), x]

[Out] (x\*(d\*x)^m\*((1 + m)\*(a + b\*ArcCsch[c\*x]) + (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2\*x^2)]/Sqrt[1 + c^2\*x^2]))/(1 + m)^2

**fricas [F]** time = 0.83, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((b \operatorname{arcsch}(cx) + a)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)\*(d\*x)^m, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcsch}(cx) + a)(dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*(d\*x)^m, x)

**maple [F]** time = 0.05, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{arccsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m\*(a+b\*arccsch(c\*x)), x)

[Out] int((d\*x)^m\*(a+b\*arccsch(c\*x)), x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\left( c^2 d^m \int \frac{x^2 x^m}{c^2(m+1)x^2 + (c^2(m+1)x^2 + m+1)\sqrt{c^2 x^2 + 1} + m+1} dx - \frac{d^m x x^m \log(x) - d^m x x^m \log\left(\sqrt{c^2 x^2 + 1}\right)}{m+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m\*(a+b\*arccsch(c\*x)), x, algorithm="maxima")

[Out] (c^2\*d^m\*integrate(x^2\*x^m/(c^2\*(m + 1)\*x^2 + (c^2\*(m + 1)\*x^2 + m + 1)\*sqrt(c^2\*x^2 + 1) + m + 1), x) - (d^m\*x\*x^m\*log(x) - d^m\*x\*x^m\*log(sqrt(c^2\*x^2 + 1) + 1))/(m + 1) - integrate((c^2\*d^m\*(m + 1)\*x^2\*log(c) + d^m\*(m + 1)\*log(c) - d^m)\*x^m/(c^2\*(m + 1)\*x^2 + m + 1), x))\*b + (d\*x)^(m + 1)\*a/(d\*(m + 1))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (dx)^m \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x)^m*(a + b*asinh(1/(c*x))),x)`

[Out] `int((d*x)^m*(a + b*asinh(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x)**m*(a+b*acsch(c*x)),x)`

[Out] `Integral((d*x)**m*(a + b*acsch(c*x)), x)`

$$3.42 \quad \int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)}, x\right)$$

[Out] Unintegrable((d\*x)^m/(a+b\*arccsch(c\*x)), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcCsch[c\*x]), x]

[Out] Defer[Int][(d\*x)^m/(a + b\*ArcCsch[c\*x]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)} dx = \int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Mathematica [A] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\operatorname{csch}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcCsch[c\*x]), x]

fricas [A] time = 2.98, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(dx)^m}{b\operatorname{arcsch}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral((d\*x)^m/(b\*arccsch(c\*x) + a), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b\operatorname{arcsch}(cx)+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arccsch(c\*x) + a), x)

**maple** [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{arccsch}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arccsch(c\*x)), x)

[Out] int((d\*x)^m/(a+b\*arccsch(c\*x)), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{b \operatorname{arcsch}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsch(c\*x)), x, algorithm="maxima")

[Out] integrate((d\*x)^m/(b\*arccsch(c\*x) + a), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(dx)^m}{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a + b\*asinh(1/(c\*x))), x)

[Out] int((d\*x)^m/(a + b\*asinh(1/(c\*x))), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b \operatorname{acsch}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*acsch(c\*x)), x)

[Out] Integral((d\*x)\*\*m/(a + b\*acsch(c\*x)), x)

$$3.43 \quad \int \frac{(dx)^m}{(a+b\operatorname{csch}^{-1}(cx))^2} dx$$

**Optimal.** Leaf size=19

$$\operatorname{Int}\left(\frac{(dx)^m}{(a+b\operatorname{csch}^{-1}(cx))^2}, x\right)$$

[Out] Unintegrable((d\*x)^m/(a+b\*arccsch(c\*x))^2, x)

**Rubi [A]** time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(dx)^m}{(a+b\operatorname{csch}^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d\*x)^m/(a + b\*ArcCsch[c\*x])^2, x]

[Out] Defer[Int] [(d\*x)^m/(a + b\*ArcCsch[c\*x])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{(a+b\operatorname{csch}^{-1}(cx))^2} dx = \int \frac{(dx)^m}{(a+b\operatorname{csch}^{-1}(cx))^2} dx$$

**Mathematica [A]** time = 1.90, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+b\operatorname{csch}^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d\*x)^m/(a + b\*ArcCsch[c\*x])^2, x]

[Out] Integrate[(d\*x)^m/(a + b\*ArcCsch[c\*x])^2, x]

**fricas [A]** time = 1.36, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(dx)^m}{b^2 \operatorname{arcsch}(cx)^2 + 2ab \operatorname{arcsch}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsch(c\*x))^2, x, algorithm="fricas")

[Out] integral((d\*x)^m/(b^2\*arccsch(c\*x)^2 + 2\*a\*b\*arccsch(c\*x) + a^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b \operatorname{arcsch}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsch(c\*x))^2, x, algorithm="giac")

[Out] integrate((d\*x)^m/(b\*arccsch(c\*x) + a)^2, x)

**maple** [A] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b \operatorname{arccsch}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a+b\*arccsch(c\*x))^2,x)

[Out] int((d\*x)^m/(a+b\*arccsch(c\*x))^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(c^2 d^m x^3 + d^m x) \sqrt{c^2 x^2 + 1} x^m + (c^2 d^m x^3 + d^m x) x^m}{(b^2 c^2 \log(c) - abc^2) x^2 + b^2 \log(c) - ab + (b^2 c^2 x^2 + b^2) \log(x) - (b^2 c^2 x^2 + \sqrt{c^2 x^2 + 1} b^2 + b^2) \log(\sqrt{c^2 x^2 + 1} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^m/(a+b\*arccsch(c\*x))^2,x, algorithm="maxima")

[Out] -((c^2\*d^m\*x^3 + d^m\*x)\*sqrt(c^2\*x^2 + 1)\*x^m + (c^2\*d^m\*x^3 + d^m\*x)\*x^m)/((b^2\*c^2\*log(c) - a\*b\*c^2)\*x^2 + b^2\*log(c) - a\*b + (b^2\*c^2\*x^2 + b^2)\*log(x) - (b^2\*c^2\*x^2 + sqrt(c^2\*x^2 + 1)\*b^2 + b^2)\*log(sqrt(c^2\*x^2 + 1) + 1) + sqrt(c^2\*x^2 + 1)\*(b^2\*log(c) + b^2\*log(x) - a\*b)) - integrate(-((c^2\*d^m\*(m + 3)\*x^2 + d^m\*(m + 1))\*(c^2\*x^2 + 1)\*x^m + (c^4\*d^m\*(m + 2)\*x^4 + c^2\*d^m\*(3\*m + 5)\*x^2 + 2\*d^m\*(m + 1))\*sqrt(c^2\*x^2 + 1)\*x^m + (c^4\*d^m\*(m + 1)\*x^4 + 2\*c^2\*d^m\*(m + 1)\*x^2 + d^m\*(m + 1))\*x^m)/((b^2\*c^4\*log(c) - a\*b\*c^4)\*x^4 + 2\*(b^2\*c^2\*log(c) - a\*b\*c^2)\*x^2 + b^2\*log(c) + (c^2\*x^2 + 1)\*(b^2\*log(c) + b^2\*log(x) - a\*b) - a\*b + (b^2\*c^4\*x^4 + 2\*b^2\*c^2\*x^2 + b^2)\*log(x) - (b^2\*c^4\*x^4 + 2\*b^2\*c^2\*x^2 + (c^2\*x^2 + 1)\*b^2 + b^2 + 2\*(b^2\*c^2\*x^2 + b^2)\*sqrt(c^2\*x^2 + 1))\*log(sqrt(c^2\*x^2 + 1) + 1) + 2\*sqrt(c^2\*x^2 + 1)\*((b^2\*c^2\*log(c) - a\*b\*c^2)\*x^2 + b^2\*log(c) - a\*b + (b^2\*c^2\*x^2 + b^2)\*log(x))), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(dx)^m}{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^m/(a + b\*asinh(1/(c\*x)))^2,x)

[Out] int((d\*x)^m/(a + b\*asinh(1/(c\*x)))^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b \operatorname{acsch}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*m/(a+b\*acsch(c\*x))\*\*2,x)

[Out] Integral((d\*x)\*\*m/(a + b\*acsch(c\*x))\*\*2, x)

### 3.44 $\int (d + ex)^3 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=167

$$\frac{(d + ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} + \frac{bde^2 x^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{2c} + \frac{be^3 x^3 \sqrt{\frac{1}{c^2 x^2} + 1}}{12c} + \frac{bex \sqrt{\frac{1}{c^2 x^2} + 1} (9c^2 d^2 - e^2)}{6c^3} + \frac{bd(2c^2 d^2 - e^2)}{6c^3}$$

[Out]  $-1/4*b*d^4*\operatorname{arccsch}(c*x)/e+1/4*(e*x+d)^4*(a+b*\operatorname{arccsch}(c*x))/e+1/2*b*d*(2*c^2*d^2-e^2)*\operatorname{arctanh}((1+1/c^2/x^2)^{(1/2)})/c^3+1/6*b*e*(9*c^2*d^2-e^2)*x*(1+1/c^2/x^2)^{(1/2)}/c^3+1/2*b*d*e^2*x^2*(1+1/c^2/x^2)^{(1/2)}/c+1/12*b*e^3*x^3*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.38, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {6290, 1568, 1475, 1807, 844, 215, 266, 63, 208}

$$\frac{(d + ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} + \frac{bex \sqrt{\frac{1}{c^2 x^2} + 1} (9c^2 d^2 - e^2)}{6c^3} + \frac{bd(2c^2 d^2 - e^2) \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{2c^3} + \frac{bde^2 x^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{2c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e*x)^3*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $(b*e*(9*c^2*d^2 - e^2)*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*d*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x^2)/(2*c) + (b*e^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x^3)/(12*c) - (b*d^4*\operatorname{ArcCsch}[c*x])/(4*e) + ((d + e*x)^4*(a + b*\operatorname{ArcCsch}[c*x]))/(4*e) + (b*d*(2*c^2*d^2 - e^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + 1/(c^2*x^2)]])/(2*c^3)$

#### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b]$

#### Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_. + (b_.)*(x_.)^2], x\_Symbol] := \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

#### Rule 266

$\operatorname{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n]-1)}*(a + b*x)^p, x], x, x^n], x] /; \operatorname{FreeQ}[\{a, b, m, n, p\}, x] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

#### Rule 844

$\operatorname{Int}[(d_. + (e_.)*(x_.))^{(m_.)}*((f_.) + (g_.)*(x_.))*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x\_Symbol] := \operatorname{Dist}[g/e, \operatorname{Int}[(d + e*x)^{(m+1)}*(a + c*x^2)^p, x], x] + \operatorname{Dist}[(e*f - d*g)/e, \operatorname{Int}[(d + e*x)^m*(a + c*x^2)^p, x], x] /; \operatorname{FreeQ}[\{a, c, d, e, f, g, m, p\}, x] \&\& \operatorname{NeQ}[c*d^2 + a*e^2, 0] \&\& \operatorname{!IGtQ}[m, 0]$

Rule 1475

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^(q*(a + c*x^2)^p, x)], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1568

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^(mn_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])
```

Rule 1807

```
Int[(Pq_)*((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)), x_Symbol] := With[{Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, Simp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6290

```
Int[((a_) + ArcCsch[(c_)*(x_)])*(b_))*((d_) + (e_)*(x_)^(m_)), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps



$$\begin{aligned}
\int (d+ex)^3 (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{(d+ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} + \frac{b \int \frac{(d+ex)^4}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{4ce} \\
&= \frac{(d+ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} + \frac{b \int \frac{\left(e + \frac{d}{x}\right)^4 x^2}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{4ce} \\
&= \frac{(d+ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} - \frac{b \operatorname{Subst}\left(\int \frac{(e+dx)^4}{x^4 \sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{4ce} \\
&= \frac{be^3 \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{12c} + \frac{(d+ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} + \frac{b \operatorname{Subst}\left(\int \frac{-12de^3 - 2e^2(9d^2 - e^2)}{x^3} dx, x, \frac{1}{x}\right)}{12c} \\
&= \frac{bde^2 \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{2c} + \frac{be^3 \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{12c} + \frac{(d+ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} - \frac{bd^4 c^3}{12c^3} \\
&= \frac{be(9c^2 d^2 - e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{2c} + \frac{be^3 \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{12c} + \frac{(d+ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} - \frac{bd^4 c^3}{12c^3} \\
&= \frac{be(9c^2 d^2 - e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{2c} + \frac{be^3 \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{12c} + \frac{(d+ex)^4 (a + b \operatorname{csch}^{-1}(cx))}{4e} - \frac{bd^4 c^3}{12c^3} \\
&= \frac{be(9c^2 d^2 - e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{2c} + \frac{be^3 \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{12c} - \frac{bd^4 c^3}{12c^3} \\
&= \frac{be(9c^2 d^2 - e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x}{6c^3} + \frac{bde^2 \sqrt{1 + \frac{1}{c^2 x^2}} x^2}{2c} + \frac{be^3 \sqrt{1 + \frac{1}{c^2 x^2}} x^3}{12c} - \frac{bd^4 c^3}{12c^3}
\end{aligned}$$

**Mathematica [A]** time = 0.29, size = 165, normalized size = 0.99

$$\frac{3ac^3x(4d^3 + 6d^2ex + 4de^2x^2 + e^3x^3) + 3bc^3x \operatorname{csch}^{-1}(cx)(4d^3 + 6d^2ex + 4de^2x^2 + e^3x^3) + bex\sqrt{\frac{1}{c^2x^2} + 1}(c^2(12c^3 - (9c^2d^2 - e^2)x - 2de^2x^2 - be^3x^3))}{12c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(a + b\*ArcCsch[c\*x]), x]

[Out] (3\*a\*c^3\*x\*(4\*d^3 + 6\*d^2\*e\*x + 4\*d\*e^2\*x^2 + e^3\*x^3) + b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-2\*e^2 + c^2\*(18\*d^2 + 6\*d\*e\*x + e^2\*x^2)) + 3\*b\*c^3\*x\*(4\*d^3 + 6\*d^2\*e\*x + 4\*d\*e^2\*x^2 + e^3\*x^3)\*ArcCsch[c\*x] + 6\*b\*d\*(2\*c^2\*d^2 - e^2)\*Log[(1 + Sqrt[1 + 1/(c^2\*x^2)])\*x])/(12\*c^3)

**fricas** [B] time = 1.16, size = 419, normalized size = 2.51

$$3ac^3e^3x^4 + 12ac^3de^2x^3 + 18ac^3d^2ex^2 + 12ac^3d^3x + 3(4bc^3d^3 + 6bc^3d^2e + 4bc^3de^2 + bc^3e^3) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 1/12\*(3\*a\*c^3\*e^3\*x^4 + 12\*a\*c^3\*d\*e^2\*x^3 + 18\*a\*c^3\*d^2\*e\*x^2 + 12\*a\*c^3\*d^3\*x + 3\*(4\*b\*c^3\*d^3 + 6\*b\*c^3\*d^2\*e + 4\*b\*c^3\*d\*e^2 + b\*c^3\*e^3)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - 6\*(2\*b\*c^2\*d^3 - b\*d\*e^2)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) - 3\*(4\*b\*c^3\*d^3 + 6\*b\*c^3\*d^2\*e + 4\*b\*c^3\*d\*e^2 + b\*c^3\*e^3)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + 3\*(b\*c^3\*e^3\*x^4 + 4\*b\*c^3\*d\*e^2\*x^3 + 6\*b\*c^3\*d^2\*e\*x^2 + 4\*b\*c^3\*d^3\*x - 4\*b\*c^3\*d^3 - 6\*b\*c^3\*d^2\*e - 4\*b\*c^3\*d\*e^2 - b\*c^3\*e^3)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (b\*c^2\*e^3\*x^3 + 6\*b\*c^2\*d\*e^2\*x^2 + 2\*(9\*b\*c^2\*d^2\*e - b\*e^3)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^3

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^3(b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(b\*arccsch(c\*x) + a), x)

**maple** [A] time = 0.06, size = 269, normalized size = 1.61

$$\frac{(cx+cd)^4 a}{4c^3e} + \frac{b \left( \frac{e^3 \operatorname{arcsch}(cx)c^4 x^4}{4} + e^2 \operatorname{arcsch}(cx)c^4 x^3 d + \frac{3e \operatorname{arcsch}(cx)c^4 x^2 d^2}{2} + \operatorname{arcsch}(cx)c^4 x d^3 + \frac{\operatorname{arcsch}(cx)c^4 d^4}{4e} + \frac{\sqrt{c^2 x^2 + 1} \left( -3c^4 d^4 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right) + 12c^2 \right)}{4e} \right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c\*(1/4\*(c\*e\*x+c\*d)^4\*a/c^3/e+b/c^3\*(1/4\*e^3\*arccsch(c\*x)\*c^4\*x^4+e^2\*arccsch(c\*x)\*c^4\*x^3\*d+3/2\*e\*arccsch(c\*x)\*c^4\*x^2\*d^2+arccsch(c\*x)\*c^4\*x\*d^3+1/4/e\*arccsch(c\*x)\*c^4\*d^4+1/12/e\*(c^2\*x^2+1)^(1/2)\*(-3\*c^4\*d^4\*arctanh(1/(c^2\*x^2+1)^(1/2))+12\*c^3\*d^3\*e\*arcsinh(c\*x)+e^4\*c^2\*x^2\*(c^2\*x^2+1)^(1/2)+6\*c^2\*d\*e^3\*x\*(c^2\*x^2+1)^(1/2)+18\*c^2\*d^2\*e^2\*(c^2\*x^2+1)^(1/2)-6\*c\*d\*e^3\*arcsinh(c\*x)-2\*e^4\*(c^2\*x^2+1)^(1/2))/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x)

**maxima** [A] time = 0.32, size = 261, normalized size = 1.56

$$\frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2 \operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2} + 1}}{c}\right)bd^2e + \frac{1}{4}\left(4x^3 \operatorname{arcsch}(cx) + \frac{2\sqrt{\frac{1}{c^2x^2} + 1} \log\left(\sqrt{\frac{1}{c^2x^2} + 1}\right)}{c^2\left(\frac{1}{c^2x^2} + 1\right) - c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1}\right)}{c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{4}a^3e^3x^4 + a^2de^2x^3 + \frac{3}{2}a^2d^2e^2x^2 + \frac{3}{2}(x^2\operatorname{arccsch}(cx) + x\sqrt{\frac{1}{c^2x^2} + 1})/c * b^2d^2e + \frac{1}{4}(4x^3\operatorname{arccsch}(cx) + (2\sqrt{\frac{1}{c^2x^2} + 1})/(c^2(\frac{1}{c^2x^2} + 1) - c^2) - \log(\sqrt{\frac{1}{c^2x^2} + 1} + 1)/c^2 + \log(\sqrt{\frac{1}{c^2x^2} + 1} - 1)/c^2)/c * b^2de^2 + \frac{1}{12}(3x^4\operatorname{arccsch}(cx) + (c^2x^3(\frac{1}{c^2x^2} + 1)^{3/2} - 3x\sqrt{\frac{1}{c^2x^2} + 1}))/c^3 * b^2e^3 + a^2d^3x + \frac{1}{2}(2cx\operatorname{arccsch}(cx) + \log(\sqrt{\frac{1}{c^2x^2} + 1} + 1) - \log(\sqrt{\frac{1}{c^2x^2} + 1} - 1)) * b^2d^3/c$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))\*(d + e\*x)^3,x)

[Out] int((a + b\*asinh(1/(c\*x)))\*(d + e\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) (d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(a+b\*acsch(c\*x)),x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x)\*\*3, x)

### 3.45 $\int (d + ex)^2 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=122

$$\frac{(d + ex)^3 (a + b \operatorname{csch}^{-1}(cx))}{3e} + \frac{bdex \sqrt{\frac{1}{c^2 x^2} + 1}}{c} + \frac{be^2 x^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{6c} + \frac{b(6c^2 d^2 - e^2) \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{6c^3} - \frac{bd^3 \operatorname{csch}^{-1}(cx)}{3e}$$

[Out]  $-1/3*b*d^3*\operatorname{arccsch}(c*x)/e+1/3*(e*x+d)^3*(a+b*\operatorname{arccsch}(c*x))/e+1/6*b*(6*c^2*d^2-e^2)*\operatorname{arctanh}((1+1/c^2/x^2)^{(1/2)})/c^3+b*d*e*x*(1+1/c^2/x^2)^{(1/2)}/c+1/6*b*e^2*x^2*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.26, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {6290, 1568, 1475, 1807, 844, 215, 266, 63, 208}

$$\frac{(d + ex)^3 (a + b \operatorname{csch}^{-1}(cx))}{3e} + \frac{b(6c^2 d^2 - e^2) \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{6c^3} + \frac{bdex \sqrt{\frac{1}{c^2 x^2} + 1}}{c} + \frac{be^2 x^2 \sqrt{\frac{1}{c^2 x^2} + 1}}{6c} - \frac{bd^3 \operatorname{csch}^{-1}(cx)}{3e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e*x)^2*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $(b*d*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x)/c + (b*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x^2)/(6*c) - (b*d^3*\operatorname{ArcCsch}[c*x])/(3*e) + ((d + e*x)^3*(a + b*\operatorname{ArcCsch}[c*x]))/(3*e) + (b*(6*c^2*d^2 - e^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + 1/(c^2*x^2)]])/(6*c^3)$

#### Rule 63

$\operatorname{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_)^n), x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\operatorname{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{NegQ}[a/b]$

#### Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_.) + (b_.)*(x_)^2], x\_Symbol] \rightarrow \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b\}, x \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

#### Rule 266

$\operatorname{Int}[(x_)^m*((a_.) + (b_.)*(x_)^n)^p, x\_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n]-1)}*(a + b*x)^p, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, m, n, p\}, x \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

#### Rule 844

$\operatorname{Int}[(d_.) + (e_.)*(x_)^m*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^p, x\_Symbol] \rightarrow \operatorname{Dist}[g/e, \operatorname{Int}[(d + e*x)^{(m+1)}*(a + c*x^2)^p, x], x] + \operatorname{Dist}[(e*f - d*g)/e, \operatorname{Int}[(d + e*x)^m*(a + c*x^2)^p, x], x] /; \operatorname{FreeQ}\{a, c, d, e, f, g, m, p\}, x \&\& \operatorname{NeQ}[c*d^2 + a*e^2, 0] \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 1475

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 1568

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^(mn_))^(q_)*((a_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])
```

### Rule 1807

```
Int[(Pq_)*((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)), x_Symbol] := With[{Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, Simp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

### Rule 6290

```
Int[((a_) + ArcCsch[(c_)*(x_)])*(b_)*((d_) + (e_)*(x_)^(m_)), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int (d+ex)^2 (a+b\operatorname{csch}^{-1}(cx)) dx &= \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{b \int \frac{(d+ex)^3}{\sqrt{1+\frac{1}{c^2x^2}} x^2} dx}{3ce} \\
&= \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{b \int \frac{\left(\frac{e+d}{x}\right)^3 x}{\sqrt{1+\frac{1}{c^2x^2}}} dx}{3ce} \\
&= \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e} - \frac{b \operatorname{Subst}\left(\int \frac{(e+dx)^3}{x^3 \sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{3ce} \\
&= \frac{be^2 \sqrt{1+\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{b \operatorname{Subst}\left(\int \frac{-6de^2 - e\left(6d^2 - \frac{e^2}{c^2}\right)x -}{x^2 \sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{6ce} \\
&= \frac{bde \sqrt{1+\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1+\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e} - \frac{b \operatorname{Subst}\left(\int \frac{-6de^2 - e\left(6d^2 - \frac{e^2}{c^2}\right)x -}{x^2 \sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{6ce} \\
&= \frac{bde \sqrt{1+\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1+\frac{1}{c^2x^2}} x^2}{6c} + \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e} - \frac{(bd^3) \operatorname{Subst}\left(\int \frac{-6de^2 - e\left(6d^2 - \frac{e^2}{c^2}\right)x -}{x^2 \sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{6ce} \\
&= \frac{bde \sqrt{1+\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1+\frac{1}{c^2x^2}} x^2}{6c} - \frac{bd^3 \operatorname{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e} \\
&= \frac{bde \sqrt{1+\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1+\frac{1}{c^2x^2}} x^2}{6c} - \frac{bd^3 \operatorname{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e} \\
&= \frac{bde \sqrt{1+\frac{1}{c^2x^2}} x}{c} + \frac{be^2 \sqrt{1+\frac{1}{c^2x^2}} x^2}{6c} - \frac{bd^3 \operatorname{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 (a+b\operatorname{csch}^{-1}(cx))}{3e}
\end{aligned}$$

**Mathematica [A]** time = 0.19, size = 122, normalized size = 1.00

$$\frac{c^2x(2ac(3d^2+3dex+e^2x^2)+be\sqrt{\frac{1}{c^2x^2}+1}(6d+ex))+2bc^3x\operatorname{csch}^{-1}(cx)(3d^2+3dex+e^2x^2)+b(6c^2d^2-e^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1}{c^3}\right)}{6c^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(a + b\*ArcCsch[c\*x]), x]

[Out] (c^2\*x\*(b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*(6\*d + e\*x) + 2\*a\*c\*(3\*d^2 + 3\*d\*e\*x + e^2\*x^2)) + 2\*b\*c^3\*x\*(3\*d^2 + 3\*d\*e\*x + e^2\*x^2)\*ArcCsch[c\*x] + b\*(6\*c^2\*d^2 - e^2)\*Log[(1 + Sqrt[1 + 1/(c^2\*x^2)])\*x])/(6\*c^3)

**fricas [B]** time = 1.00, size = 328, normalized size = 2.69

$$2ac^3e^2x^3 + 6ac^3dex^2 + 6ac^3d^2x + 2(3bc^3d^2 + 3bc^3de + bc^3e^2)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - (6bc^2d^2 - be^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out]  $\frac{1}{6}*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*\log(c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} - c*x + 1) - (6*b*c^2*d^2 - b*e^2)*\log(c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} - c*x) - 2*(3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*\log(c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} - c*x - 1) + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)) + (b*c^2*e^2*x^2 + 6*b*c^2*d*e*x)*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)})/c^3$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^2 (b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(b\*arccsch(c\*x) + a), x)

**maple** [A] time = 0.05, size = 204, normalized size = 1.67

$$\frac{(cxe+cd)^3 a}{3c^2 e} + \frac{b \left( \frac{e^2 \operatorname{arcsch}(cx) c^3 x^3}{3} + e \operatorname{arcsch}(cx) c^3 x^2 d + \operatorname{arcsch}(cx) c^3 x d^2 + \frac{\operatorname{arcsch}(cx) c^3 d^3}{3e} + \frac{\sqrt{c^2 x^2 + 1} \left( -2c^3 d^3 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right) + 6c^2 d^2 e \operatorname{arcsinh}(cx) + e^3 c \right)}{6e \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} cx} \right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(a+b\*arccsch(c\*x)),x)

[Out]  $\frac{1}{c}*(\frac{1}{3}*(c*e*x+c*d)^3*a/c^2/e+b/c^2*(\frac{1}{3}*e^2*\operatorname{arccsch}(c*x)*c^3*x^3+e*\operatorname{arccsch}(c*x)*c^3*x^2*d+\operatorname{arccsch}(c*x)*c^3*x*d^2+1/3/e*\operatorname{arccsch}(c*x)*c^3*d^3+1/6/e*(c^2*x^2+1)^{(1/2)}*(-2*c^3*d^3*\operatorname{arctanh}(1/(c^2*x^2+1)^{(1/2)}))+6*c^2*d^2*e*\operatorname{arcsinh}(c*x)+e^3*c*x*(c^2*x^2+1)^{(1/2)}+6*c*d*e^2*(c^2*x^2+1)^{(1/2)}-e^3*\operatorname{arcsinh}(c*x))/(c^2*x^2+1)/c^2/x^2)^{(1/2)}/c/x)$

**maxima** [A] time = 0.34, size = 192, normalized size = 1.57

$$\frac{1}{3} a e^2 x^3 + a d e x^2 + \left( x^2 \operatorname{arcsch}(cx) + \frac{x \sqrt{\frac{1}{c^2 x^2} + 1}}{c} \right) b d e + \frac{1}{12} \left( 4 x^3 \operatorname{arcsch}(cx) + \frac{2 \sqrt{\frac{1}{c^2 x^2} + 1} - \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right)}{c^2 \left(\frac{1}{c^2 x^2} + 1\right) - c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right)}{c^2} + \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right) \right) / c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{3} a e^2 x^3 + a d e x^2 + (x^2 \operatorname{arccsch}(c*x) + x \sqrt{1/(c^2*x^2) + 1})/c * b d e + 1/12*(4*x^3*\operatorname{arccsch}(c*x) + (2*\sqrt{1/(c^2*x^2) + 1})/(c^2*(1/(c^2*x^2) + 1) - c^2) - \log(\sqrt{1/(c^2*x^2) + 1} + 1)/c^2 + \log(\sqrt{1/(c^2*x^2) + 1} - 1)/c^2)/c * b e^2 + a d^2 x + 1/2*(2*c*x*\operatorname{arccsch}(c*x) + \log(\sqrt{1/(c^2*x^2) + 1} + 1) - \log(\sqrt{1/(c^2*x^2) + 1} - 1))*b*d^2/c$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left( a + b \operatorname{asinh}\left(\frac{1}{c x}\right) \right) (d + e x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) (d + ex)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x)**2, x)
```



### 3.46 $\int (d + ex) (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=81

$$\frac{(d + ex)^2 (a + b \operatorname{csch}^{-1}(cx))}{2e} + \frac{bd \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{c} + \frac{bex \sqrt{\frac{1}{c^2 x^2} + 1}}{2c} - \frac{bd^2 \operatorname{csch}^{-1}(cx)}{2e}$$

[Out]  $-1/2*b*d^2*\operatorname{arccsch}(c*x)/e+1/2*(e*x+d)^2*(a+b*\operatorname{arccsch}(c*x))/e+b*d*\operatorname{arctanh}((1+1/c^2/x^2)^{(1/2)})/c+1/2*b*e*x*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.16, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {6290, 1568, 1396, 1807, 844, 215, 266, 63, 208}

$$\frac{(d + ex)^2 (a + b \operatorname{csch}^{-1}(cx))}{2e} + \frac{bd \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{c} + \frac{bex \sqrt{\frac{1}{c^2 x^2} + 1}}{2c} - \frac{bd^2 \operatorname{csch}^{-1}(cx)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(a + b\*ArcCsch[c\*x]), x]

[Out]  $(b*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x)/(2*c) - (b*d^2*\operatorname{ArcCsch}[c*x])/(2*e) + ((d + e*x)^2*(a + b*\operatorname{ArcCsch}[c*x]))/(2*e) + (b*d*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + 1/(c^2*x^2)]])/c$

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 844

Int[((d\_.) + (e\_.)\*(x\_)^(m\_))\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

#### Rule 1396

Int[((d\_) + (e\_.)\*(x\_)^(n\_))^(q\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> -Subst[Int[(d + e/x^n)^q\*(a + c/x^(2\*n))^p/x^2, x], x, 1/x] /; FreeQ

[{a, c, d, e, p, q}, x] && EqQ[n2, 2\*n] && ILtQ[n, 0]

### Rule 1568

Int[(x\_)^(m\_)\*((d\_) + (e\_)\*(x\_)^(mn\_))^(q\_)\*((a\_) + (c\_)\*(x\_)^(n2\_))^(p\_), x\_Symbol] := Int[x^(m + mn\*q)\*(e + d/x^mn)^q\*(a + c\*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2\*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])

### Rule 1807

Int[(Pq\_)\*((c\_)\*(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^2)^(p\_)), x\_Symbol] := With[{Q = PolynomialQuotient[Pq, c\*x, x], R = PolynomialRemainder[Pq, c\*x, x]}, Simp[(R\*(c\*x)^(m + 1)\*(a + b\*x^2)^(p + 1))/(a\*c\*(m + 1)), x] + Dist[1/(a\*c\*(m + 1)), Int[(c\*x)^(m + 1)\*(a + b\*x^2)^p\*ExpandToSum[a\*c\*(m + 1)\*Q - b\*R\*(m + 2\*p + 3)\*x, x], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ[m, -1] && (IntegerQ[2\*p] || NeQ[Expon[Pq, x], 1])

### Rule 6290

Int[((a\_) + ArcCsch[(c\_)\*(x\_)])\*(b\_)\*((d\_) + (e\_)\*(x\_)^(m\_)), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCsch[c\*x]))/(e\*(m + 1)), x] + Dist[b/(c\*e\*(m + 1)), Int[(d + e\*x)^(m + 1)/(x^2\*Sqrt[1 + 1/(c^2\*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int (d+ex)(a+b\operatorname{csch}^{-1}(cx)) dx &= \frac{(d+ex)^2(a+b\operatorname{csch}^{-1}(cx))}{2e} + \frac{b \int \frac{(d+ex)^2}{\sqrt{1+\frac{1}{c^2x^2}} x^2} dx}{2ce} \\
&= \frac{(d+ex)^2(a+b\operatorname{csch}^{-1}(cx))}{2e} + \frac{b \int \frac{\left(\frac{e+d}{x}\right)^2}{\sqrt{1+\frac{1}{c^2x^2}}} dx}{2ce} \\
&= \frac{(d+ex)^2(a+b\operatorname{csch}^{-1}(cx))}{2e} - \frac{b \operatorname{Subst}\left(\int \frac{(e+dx)^2}{x^2 \sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2ce} \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}} x}{2c} + \frac{(d+ex)^2(a+b\operatorname{csch}^{-1}(cx))}{2e} + \frac{b \operatorname{Subst}\left(\int \frac{-2de-d^2x}{x \sqrt{1+\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{2ce} \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}} x}{2c} + \frac{(d+ex)^2(a+b\operatorname{csch}^{-1}(cx))}{2e} - \frac{(bd) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1+\frac{x^2}{c^2}}} dx, x\right)}{c} \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}} x}{2c} - \frac{bd^2 \operatorname{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\operatorname{csch}^{-1}(cx))}{2e} - \frac{(bd) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1+\frac{x^2}{c^2}}} dx, x\right)}{c} \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}} x}{2c} - \frac{bd^2 \operatorname{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\operatorname{csch}^{-1}(cx))}{2e} - (bcd) \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1+\frac{x^2}{c^2}}} dx, x\right) \\
&= \frac{be\sqrt{1+\frac{1}{c^2x^2}} x}{2c} - \frac{bd^2 \operatorname{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2(a+b\operatorname{csch}^{-1}(cx))}{2e} + \frac{bd \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2+1}}\right)}{2c}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 99, normalized size = 1.22

$$adx + \frac{1}{2}aex^2 + \frac{bdx\sqrt{\frac{1}{c^2x^2}+1} \sinh^{-1}(cx)}{\sqrt{c^2x^2+1}} + \frac{bex\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2c} + bdx\operatorname{csch}^{-1}(cx) + \frac{1}{2}bex^2\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(a + b\*ArcCsch[c\*x]), x]

[Out] a\*d\*x + (a\*e\*x^2)/2 + (b\*e\*x\*sqrt((1 + c^2\*x^2)/(c^2\*x^2)))/(2\*c) + b\*d\*x\*ArcCsch[c\*x] + (b\*e\*x^2\*ArcCsch[c\*x])/2 + (b\*d\*sqrt(1 + 1/(c^2\*x^2))\*x\*ArcSinh[c\*x])/sqrt(1 + c^2\*x^2)

**fricas [B]** time = 1.70, size = 207, normalized size = 2.56

$$\frac{acex^2 + 2acdx + bex\sqrt{\frac{c^2x^2+1}{c^2x^2}} - 2bd \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (2bcd + bce) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - (2bcd + bce) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + cx + 1\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out]  $\frac{1}{2}(a*c*e*x^2 + 2*a*c*d*x + b*e*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}) - 2*b*d*\log(c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} - c*x) + (2*b*c*d + b*c*e)*\log(c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} - c*x + 1) - (2*b*c*d + b*c*e)*\log(c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} - c*x - 1) + (b*c*e*x^2 + 2*b*c*d*x - 2*b*c*d - b*c*e)*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)))/c$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)(b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="giac")`

[Out] `integrate((e*x + d)*(b*arccsch(c*x) + a), x)`

**maple** [A] time = 0.05, size = 115, normalized size = 1.42

$$\frac{a\left(\frac{1}{2}c^2x^2e+c^2dx\right)}{c} + \frac{b\left(\frac{\operatorname{arccsch}(cx)c^2x^2e}{2} + \operatorname{arccsch}(cx)c^2xd + \frac{\sqrt{c^2x^2+1}\left(e\sqrt{c^2x^2+1}+2cd\operatorname{arcsinh}(cx)\right)}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(a+b*arccsch(c*x)),x)`

[Out]  $\frac{1}{c}\left(\frac{a}{c}\left(\frac{1}{2}c^2x^2e+c^2d*x\right) + \frac{b}{c}\left(\frac{1}{2}\operatorname{arccsch}(c*x)*c^2x^2e + \operatorname{arccsch}(c*x)*c^2x*d + \frac{1}{2}\left(\frac{c^2x^2+1}{c^2/x^2}\right)^{(1/2)}/c/x*(c^2x^2+1)^{(1/2)}*(e*(c^2x^2+1)^{(1/2)}+2*c*d*\operatorname{arcsinh}(c*x))\right)\right)$

**maxima** [A] time = 0.39, size = 87, normalized size = 1.07

$$\frac{1}{2}aex^2 + \frac{1}{2}\left(x^2 \operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2} + 1}}{c}\right)be + adx + \frac{\left(2cx \operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2x^2} + 1} - 1\right)\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}a*e*x^2 + \frac{1}{2}(x^2*\operatorname{arccsch}(c*x) + x*\sqrt{1/(c^2*x^2) + 1}/c)*b*e + a*d*x + \frac{1}{2}(2*c*x*\operatorname{arccsch}(c*x) + \log(\sqrt{1/(c^2*x^2) + 1} + 1) - \log(\sqrt{1/(c^2*x^2) + 1} - 1))*b*d/c$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right) (d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))*(d + e*x),x)`

[Out] `int((a + b*asinh(1/(c*x)))*(d + e*x), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx))(d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(a+b*acsch(c*x)),x)`

[Out] `Integral((a + b*acsch(c*x))*(d + e*x), x)`

### 3.47 $\int (a + b \operatorname{csch}^{-1}(cx)) dx$

Optimal. Leaf size=30

$$ax + \frac{b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{c} + b \operatorname{csch}^{-1}(cx)$$

[Out] a\*x+b\*x\*arccsch(c\*x)+b\*arctanh((1+1/c^2/x^2)^(1/2))/c

**Rubi [A]** time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6278, 266, 63, 208}

$$ax + \frac{b \tanh^{-1}\left(\sqrt{\frac{1}{c^2 x^2} + 1}\right)}{c} + b \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b\*ArcCsch[c\*x], x]

[Out] a\*x + b\*x\*ArcCsch[c\*x] + (b\*ArcTanh[Sqrt[1 + 1/(c^2\*x^2)]])/c

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 6278

Int[ArcCsch[(c\_.)\*(x\_)], x\_Symbol] := Simp[x\*ArcCsch[c\*x], x] + Dist[1/c, Int[1/(x\*Sqrt[1 + 1/(c^2\*x^2)]), x], x] /; FreeQ[c, x]

#### Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{csch}^{-1}(cx)) dx &= ax + b \int \operatorname{csch}^{-1}(cx) dx \\
&= ax + bx \operatorname{csch}^{-1}(cx) + \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{c} \\
&= ax + bx \operatorname{csch}^{-1}(cx) - \frac{b \operatorname{Subst}\left(\int \frac{1}{x \sqrt{1 + \frac{x}{c^2}}} dx, x, \frac{1}{x^2}\right)}{2c} \\
&= ax + bx \operatorname{csch}^{-1}(cx) - (bc) \operatorname{Subst}\left(\int \frac{1}{-c^2 + c^2 x^2} dx, x, \sqrt{1 + \frac{1}{c^2 x^2}}\right) \\
&= ax + bx \operatorname{csch}^{-1}(cx) + \frac{b \tanh^{-1}\left(\sqrt{1 + \frac{1}{c^2 x^2}}\right)}{c}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 44, normalized size = 1.47

$$ax + \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1} \sinh^{-1}(cx)}{\sqrt{c^2 x^2 + 1}} + bx \operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b\*ArcCsch[c\*x], x]

[Out] a\*x + b\*x\*ArcCsch[c\*x] + (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*ArcSinh[c\*x])/Sqrt[1 + c^2\*x^2]

**fricas** [B] time = 0.78, size = 143, normalized size = 4.77

$$\frac{acx + bc \log\left(cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx + 1\right) - bc \log\left(cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx - 1\right) - b \log\left(cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx\right) + (bcx - bc) \log\left(\frac{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx + 1}{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} - cx - 1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsch(c\*x), x, algorithm="fricas")

[Out] (a\*c\*x + b\*c\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - b\*c\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) - b\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) + (b\*c\*x - b\*c)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/c

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int b \operatorname{arcsch}(cx) + a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b\*arccsch(c\*x), x, algorithm="giac")

[Out] integrate(b\*arccsch(c\*x) + a, x)

**maple** [A] time = 0.05, size = 36, normalized size = 1.20

$$ax + bx \operatorname{arccsch}(cx) + \frac{b \ln\left(cx + cx \sqrt{1 + \frac{1}{c^2 x^2}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arccsch(c*x),x)`

[Out] `a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))`

**maxima** [A] time = 0.31, size = 49, normalized size = 1.63

$$ax + \frac{\left(2cx \operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2x^2} + 1} - 1\right)\right)b}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arccsch(c*x),x, algorithm="maxima")`

[Out] `a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b/c`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int a + b \operatorname{asinh}\left(\frac{1}{cx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + b*asinh(1/(c*x)),x)`

[Out] `int(a + b*asinh(1/(c*x)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*acsch(c*x),x)`

[Out] `Integral(a + b*acsch(c*x), x)`

$$3.48 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex} dx$$

**Optimal.** Leaf size=215

$$\frac{(a + b\operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{(e - \sqrt{c^2d^2 + e^2})e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{(a + b\operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{(\sqrt{c^2d^2 + e^2} + e)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} - \log\left(1 - e^{2\operatorname{csch}^{-1}(cx)}\right)$$

[Out]  $-(a+b*\operatorname{arccsch}(c*x))*\ln(1-(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)/e+(a+b*\operatorname{arccsch}(c*x))*\ln(1-(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(e-(c^2*d^2+e^2)^{(1/2)})/c/d)/e+(a+b*\operatorname{arccsch}(c*x))*\ln(1-(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(e+(c^2*d^2+e^2)^{(1/2)})/c/d)/e-1/2*b*\operatorname{polylog}(2,(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)/e+b*\operatorname{polylog}(2,(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(e-(c^2*d^2+e^2)^{(1/2)})/c/d)/e+b*\operatorname{polylog}(2,(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(e+(c^2*d^2+e^2)^{(1/2)})/c/d)/e$

**Rubi [A]** time = 0.39, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {6289, 2518}

$$\frac{b\operatorname{PolyLog}\left(2, \frac{(e - \sqrt{c^2d^2 + e^2})e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2, \frac{(\sqrt{c^2d^2 + e^2} + e)e^{\operatorname{csch}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right)}{2e} + (a + b\operatorname{csch}^{-1}(cx))$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcCsch[c*x])/(d + e*x), x]`

[Out]  $((a + b*\operatorname{ArcCsch}[c*x])*Log[1 - ((e - \operatorname{Sqrt}[c^2*d^2 + e^2])*E^{\operatorname{ArcCsch}[c*x]})/(c*d)])/e + ((a + b*\operatorname{ArcCsch}[c*x])*Log[1 - ((e + \operatorname{Sqrt}[c^2*d^2 + e^2])*E^{\operatorname{ArcCsch}[c*x]})/(c*d)])/e - ((a + b*\operatorname{ArcCsch}[c*x])*Log[1 - E^{(2*\operatorname{ArcCsch}[c*x])}])/e + (b*\operatorname{PolyLog}[2, ((e - \operatorname{Sqrt}[c^2*d^2 + e^2])*E^{\operatorname{ArcCsch}[c*x]})/(c*d)])/e + (b*\operatorname{PolyLog}[2, ((e + \operatorname{Sqrt}[c^2*d^2 + e^2])*E^{\operatorname{ArcCsch}[c*x]})/(c*d)])/e - (b*\operatorname{PolyLog}[2, E^{(2*\operatorname{ArcCsch}[c*x])}])/e)$

**Rule 2518**

`Int[Log[v]*(u_), x_Symbol] := With[{w = DerivativeDivides[v, u*(1 - v), x]}, Simp[w*PolyLog[2, 1 - v], x] /; !FalseQ[w]]`

**Rule 6289**

`Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[((a + b*ArcCsch[c*x])*Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e, x] + (Dist[b/(c*e), Int[Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)]]/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] + Dist[b/(c*e), Int[Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)]]/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] - Dist[b/(c*e), Int[Log[1 - E^(2*ArcCsch[c*x])]]/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] + Simp[((a + b*ArcCsch[c*x])*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e, x] - Simp[((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e, x] /; FreeQ[{a, b, c, d, e}, x]`

**Rubi steps**



$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{d + ex} dx = \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{(e - \sqrt{c^2 d^2 + e^2}) e^{\operatorname{csch}^{-1}(cx)}}{cd} \right)}{e} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{(e + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{csch}^{-1}(cx)}}{cd} \right)}{e}$$

$$= \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{(e - \sqrt{c^2 d^2 + e^2}) e^{\operatorname{csch}^{-1}(cx)}}{cd} \right)}{e} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{(e + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{csch}^{-1}(cx)}}{cd} \right)}{e}$$

**Mathematica** [C] time = 0.66, size = 506, normalized size = 2.35

$$\frac{a \log(d + ex)}{e} + \frac{b \left( 8 \operatorname{Li}_2 \left( \frac{(e - \sqrt{c^2 d^2 + e^2}) e^{\operatorname{csch}^{-1}(cx)}}{cd} \right) + 8 \operatorname{Li}_2 \left( \frac{(e + \sqrt{c^2 d^2 + e^2}) e^{\operatorname{csch}^{-1}(cx)}}{cd} \right) + 8 \operatorname{csch}^{-1}(cx) \log \left( \frac{(\sqrt{c^2 d^2 + e^2} - e) e^{\operatorname{csch}^{-1}(cx)}}{cd} \right) \right)}{e}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x), x]

[Out] (a\*Log[d + e\*x])/e + (b\*(Pi^2 - (4\*I)\*Pi\*ArcCsch[c\*x] - 8\*ArcCsch[c\*x]^2 - 32\*ArcSin[Sqrt[1 + (I\*e)/(c\*d)]]/Sqrt[2]]\*ArcTan[((I\*c\*d + e)\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[c^2\*d^2 + e^2]] - 8\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])] + (4\*I)\*Pi\*Log[1 + ((-e + Sqrt[c^2\*d^2 + e^2])\*E^ArcCsch[c\*x])/(c\*d)] + 8\*ArcCsch[c\*x]\*Log[1 + ((-e + Sqrt[c^2\*d^2 + e^2])\*E^ArcCsch[c\*x])/(c\*d)] + (16\*I)\*ArcSin[Sqrt[1 + (I\*e)/(c\*d)]]/Sqrt[2]]\*Log[1 + ((-e + Sqrt[c^2\*d^2 + e^2])\*E^ArcCsch[c\*x])/(c\*d)] + (4\*I)\*Pi\*Log[1 - ((e + Sqrt[c^2\*d^2 + e^2])\*E^ArcCsch[c\*x])/(c\*d)] + 8\*ArcCsch[c\*x]\*Log[1 - ((e + Sqrt[c^2\*d^2 + e^2])\*E^ArcCsch[c\*x])/(c\*d)] - (16\*I)\*ArcSin[Sqrt[1 + (I\*e)/(c\*d)]]/Sqrt[2]]\*Log[1 - ((e + Sqrt[c^2\*d^2 + e^2])\*E^ArcCsch[c\*x])/(c\*d)] - (4\*I)\*Pi\*Log[e + d/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, ((e - Sqrt[c^2\*d^2 + e^2])\*E^ArcCsch[c\*x])/(c\*d)] + 8\*PolyLog[2, ((e + Sqrt[c^2\*d^2 + e^2])\*E^ArcCsch[c\*x])/(c\*d))])/(8\*e)

**fricas** [F] time = 0.68, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \operatorname{arcsch}(cx) + a}{ex + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d), x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e\*x + d), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x + d), x)

**maple** [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsch(c*x))/(e*x+d),x)`

[Out] `int((a+b*arccsch(c*x))/(e*x+d),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{ex + d} dx + \frac{a \log(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="maxima")`

[Out] `b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x + d), x) + a*log(e*x + d)/e`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))/(d + e*x),x)`

[Out] `int((a + b*asinh(1/(c*x)))/(d + e*x), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))/(e*x+d),x)`

[Out] `Integral((a + b*acsch(c*x))/(d + e*x), x)`

$$3.49 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^2} dx$$

Optimal. Leaf size=98

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} + \frac{b \tanh^{-1}\left(\frac{c^2 d - \frac{e}{x}}{c\sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{c^2 d^2 + e^2}}\right)}{d\sqrt{c^2 d^2 + e^2}} + \frac{b\operatorname{csch}^{-1}(cx)}{de}$$

[Out] b\*arccsch(c\*x)/d/e+(-a-b\*arccsch(c\*x))/e/(e\*x+d)+b\*arctanh((c^2\*d-e/x)/c/(c^2\*d^2+e^2)^(1/2)/(1+1/c^2/x^2)^(1/2))/d/(c^2\*d^2+e^2)^(1/2)

**Rubi [A]** time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {6290, 1568, 1475, 844, 215, 725, 206}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)} + \frac{b \tanh^{-1}\left(\frac{c^2 d - \frac{e}{x}}{c\sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{c^2 d^2 + e^2}}\right)}{d\sqrt{c^2 d^2 + e^2}} + \frac{b\operatorname{csch}^{-1}(cx)}{de}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/(d + e\*x)^2, x]

[Out] (b\*ArcCsch[c\*x])/(d\*e) - (a + b\*ArcCsch[c\*x])/(e\*(d + e\*x)) + (b\*ArcTanh[(c^2\*d - e/x)/(c\*sqrt[c^2\*d^2 + e^2]\*sqrt[1 + 1/(c^2\*x^2)])])/(d\*sqrt[c^2\*d^2 + e^2])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

#### Rule 1475

Int[(x\_)^(m\_.)\*((a\_) + (c\_.)\*(x\_)^(n2\_.))^(p\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(d + e\*x)^q\*(a + c\*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&

EqQ[n2, 2\*n] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 1568

Int[(x\_)^(m\_)\*((d\_) + (e\_)\*(x\_)^(mn\_))^(q\_)\*((a\_) + (c\_)\*(x\_)^(n2\_))^(p\_), x\_Symbol] := Int[x^(m + mn\*q)\*(e + d/x^mn)^q\*(a + c\*x^n2)^p, x] /; FreeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2\*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])

### Rule 6290

Int[((a\_) + ArcCsch[(c\_)\*(x\_)])\*(b\_)\*((d\_) + (e\_)\*(x\_))^(m\_), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCsch[c\*x]))/(e\*(m + 1)), x] + Dist[b/(c\*e\*(m + 1)), Int[(d + e\*x)^(m + 1)/(x^2\*Sqrt[1 + 1/(c^2\*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex)^2} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{e(d + ex)} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^{2(d+ex)}} dx}{ce} \\
 &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{e(d + ex)} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} \left(e + \frac{d}{x}\right) x^3} dx}{ce} \\
 &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{e(d + ex)} + \frac{b \operatorname{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{ce} \\
 &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{e(d + ex)} - \frac{b \operatorname{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{cd} + \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{cde} \\
 &= \frac{b \operatorname{csch}^{-1}(cx)}{de} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e(d + ex)} + \frac{b \operatorname{Subst}\left(\int \frac{1}{d^2 + \frac{e^2}{c^2} - x^2} dx, x, \frac{d - \frac{e}{c^2 x}}{\sqrt{1 + \frac{1}{c^2 x^2}}}\right)}{cd} \\
 &= \frac{b \operatorname{csch}^{-1}(cx)}{de} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e(d + ex)} + \frac{b \tanh^{-1}\left(\frac{c^2 d - \frac{e}{x}}{c \sqrt{c^2 d^2 + e^2} \sqrt{1 + \frac{1}{c^2 x^2}}}\right)}{d \sqrt{c^2 d^2 + e^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 134, normalized size = 1.37

$$\frac{a}{e(d + ex)} - \frac{b \log\left(cx \left(\sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{c^2 d^2 + e^2} - cd\right) + e\right)}{d \sqrt{c^2 d^2 + e^2}} + \frac{b \log(d + ex)}{d \sqrt{c^2 d^2 + e^2}} + \frac{b \sinh^{-1}\left(\frac{1}{cx}\right)}{de} - \frac{b \operatorname{csch}^{-1}(cx)}{e(d + ex)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x)^2, x]

[Out] -(a/(e\*(d + e\*x))) - (b\*ArcCsch[c\*x])/(e\*(d + e\*x)) + (b\*ArcSinh[1/(c\*x)])/(d\*e) + (b\*Log[d + e\*x])/(d\*Sqrt[c^2\*d^2 + e^2]) - (b\*Log[e + c\*(-(c\*d) + Sqrt[c^2\*d^2 + e^2]\*Sqrt[1 + 1/(c^2\*x^2)]]\*x))/(d\*Sqrt[c^2\*d^2 + e^2])

**fricas** [B] time = 0.61, size = 354, normalized size = 3.61

$$ac^2d^3 + ade^2 - \sqrt{c^2d^2 + e^2} (be^2x + bde) \log \left( -\frac{c^3d^2x - cde + (c^3d^2 + ce^2)x \sqrt{\frac{c^2x^2 + 1}{c^2x^2}} + \left( c^2dx \sqrt{\frac{c^2x^2 + 1}{c^2x^2}} + c^2dx - e \right) \sqrt{c^2d^2 + e^2}}{ex + d} \right) - (bc^2d^2 + e^2) \sqrt{c^2d^2 + e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^2,x, algorithm="fricas")

[Out]  $-(a*c^2*d^3 + a*d*e^2 - \text{sqrt}(c^2*d^2 + e^2)*(b*e^2*x + b*d*e))*\log(-(c^3*d^2*x - c*d*e + (c^3*d^2 + c*e^2)*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) + (c^2*d*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) + c^2*d*x - e)*\text{sqrt}(c^2*d^2 + e^2))/(e*x + d)) - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3)*x)*\log(c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3)*x)*\log(c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^2*d^3 + b*d*e^2)*\log((c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^2 + d*e^4)*x)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x + d)^2, x)

**maple** [B] time = 0.09, size = 208, normalized size = 2.12

$$\frac{ca}{(cxe + cd)e} - \frac{cb \operatorname{arcsch}(cx)}{(cxe + cd)e} + \frac{b\sqrt{c^2x^2 + 1} \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2 + 1}}\right)}{ce\sqrt{\frac{c^2x^2 + 1}{c^2x^2}} xd} - \frac{b\sqrt{c^2x^2 + 1} \ln\left(\frac{2\sqrt{c^2x^2 + 1} \sqrt{\frac{c^2d^2 + e^2}{e^2}} e^{-2c^2dx + 2e}}{cxe + cd}\right)}{ce\sqrt{\frac{c^2x^2 + 1}{c^2x^2}} xd\sqrt{\frac{c^2d^2 + e^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x+d)^2,x)

[Out]  $-c*a/(c*e*x+c*d)/e - c*b/(c*e*x+c*d)/e*\operatorname{arcsch}(c*x) + 1/c*b/e*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/d*\operatorname{arctanh}(1/(c^2*x^2+1)^{(1/2)}) - 1/c*b/e*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/d/((c^2*d^2+e^2)/e^2)^{(1/2)}*\ln(2*((c^2*x^2+1)^{(1/2)}*((c^2*d^2+e^2)/e^2)^{(1/2)}*e^{-c^2*d*x+e})/(c*e*x+c*d))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( 2c^2 \int \frac{x}{c^2e^2x^3 + c^2dex^2 + e^2x + de + (c^2e^2x^3 + c^2dex^2 + e^2x + de)\sqrt{c^2x^2 + 1}} dx + \frac{ic(\log(icx + 1) - \log(-icx + 1))}{c^2d^2 + e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^2,x, algorithm="maxima")

[Out]  $-1/2*(2*c^2*\integrate(x/(c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e + (c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*\text{sqrt}(c^2*x^2 + 1)), x) + I*c*(\log(I*c*x + 1) - \log(-I*c*x + 1))/(c^2*d^2 + e^2) - 2*e*\log(e*x + d)/(c^2*d^3 + d*e^2)$

- (2\*c^2\*d^3\*log(c) + 2\*d\*e^2\*log(c) - 2\*(c^2\*d^2\*e + e^3)\*x\*log(x) + (c^2\*d^2\*e\*x + c^2\*d^3)\*log(c^2\*x^2 + 1) - 2\*(c^2\*d^3 + d\*e^2)\*log(sqrt(c^2\*x^2 + 1) + 1))/(c^2\*d^4\*e + d^2\*e^3 + (c^2\*d^3\*e^2 + d\*e^4)\*x))\*b - a/(e^2\*x + d\*e)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(d + e\*x)^2,x)

[Out] int((a + b\*asinh(1/(c\*x)))/(d + e\*x)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/(e\*x+d)\*\*2,x)

[Out] Integral((a + b\*acsch(c\*x))/(d + e\*x)\*\*2, x)

$$3.50 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3} dx$$

Optimal. Leaf size=163

$$\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{bce\sqrt{\frac{1}{c^2x^2}+1}}{2d(c^2d^2+e^2)\left(\frac{d}{x}+e\right)} + \frac{b(2c^2d^2+e^2)\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{2d^2(c^2d^2+e^2)^{3/2}} + \frac{b\operatorname{csch}^{-1}(cx)}{2d^2e}$$

[Out]  $1/2*b*\operatorname{arccsch}(c*x)/d^2/e+1/2*(-a-b*\operatorname{arccsch}(c*x))/e/(e*x+d)^2+1/2*b*(2*c^2*d^2+e^2)*\operatorname{arctanh}((c^2*d-e/x)/c/(c^2*d^2+e^2)^{(1/2)/(1+1/c^2/x^2)^{(1/2)})/d^2/(c^2*d^2+e^2)^{(3/2)}-1/2*b*c*e*(1+1/c^2/x^2)^{(1/2)}/d/(c^2*d^2+e^2)/(e+d/x)$

**Rubi [A]** time = 0.29, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6290, 1568, 1475, 1651, 844, 215, 725, 206}

$$\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{bce\sqrt{\frac{1}{c^2x^2}+1}}{2d(c^2d^2+e^2)\left(\frac{d}{x}+e\right)} + \frac{b(2c^2d^2+e^2)\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{2d^2(c^2d^2+e^2)^{3/2}} + \frac{b\operatorname{csch}^{-1}(cx)}{2d^2e}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/(d + e\*x)^3, x]

[Out]  $-(b*c*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(2*d*(c^2*d^2 + e^2)*(e + d/x)) + (b*\operatorname{ArcCsch}[c*x])/(2*d^2*e) - (a + b*\operatorname{ArcCsch}[c*x])/(2*e*(d + e*x)^2) + (b*(2*c^2*d^2 + e^2)*\operatorname{ArcTanh}[(c^2*d - e/x)/(c*\operatorname{Sqrt}[c^2*d^2 + e^2]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])])/(2*d^2*(c^2*d^2 + e^2)^{(3/2)})$

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 725

Int[1/(((d\_) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :> -Subst[Int[1/(c\*d^2 + a\*e^2 - x^2), x], x, (a\*e - c\*d\*x)/Sqrt[a + c\*x^2]] /; FreeQ[{a, c, d, e}, x]

#### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

#### Rule 1475

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_
_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 1568

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] := Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] || !IntegerQ[p])
```

### Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :=>
With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

### Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex)^3} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex)^2} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^2 (d + ex)^2} dx}{2ce} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex)^2} - \frac{b \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} \left(e + \frac{d}{x}\right)^2 x^4} dx}{2ce} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex)^2} + \frac{b \operatorname{Subst} \left( \int \frac{x^2}{(e + dx)^2 \sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2ce} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{2d(c^2 d^2 + e^2) \left(e + \frac{d}{x}\right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex)^2} - \frac{(bc) \operatorname{Subst} \left( \int \frac{e - \left(d + \frac{e^2}{c^2 d}\right)x}{(e + dx) \sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2e(c^2 d^2 + e^2)} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{2d(c^2 d^2 + e^2) \left(e + \frac{d}{x}\right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex)^2} + \frac{b \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2cd^2 e} - \frac{(bc(2 + \frac{e^2}{c^2 d^2})) \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2(c^2 d^2 + e^2)} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{2d(c^2 d^2 + e^2) \left(e + \frac{d}{x}\right)} + \frac{b \operatorname{csch}^{-1}(cx)}{2d^2 e} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex)^2} + \frac{(bc(2 + \frac{e^2}{c^2 d^2})) \operatorname{Subst} \left( \int \frac{1}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{2(c^2 d^2 + e^2)} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{2d(c^2 d^2 + e^2) \left(e + \frac{d}{x}\right)} + \frac{b \operatorname{csch}^{-1}(cx)}{2d^2 e} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex)^2} + \frac{b(2c^2 d^2 + e^2) \operatorname{tanh}^{-1} \left( \frac{e + \frac{d}{x}}{\sqrt{1 + \frac{x^2}{c^2}}} \right)}{2d^2(c^2 d^2 + e^2)}
\end{aligned}$$

**Mathematica [A]** time = 0.48, size = 204, normalized size = 1.25

$$\frac{1}{2} \left( -\frac{a}{e(d + ex)^2} - \frac{bcex \sqrt{\frac{1}{c^2 x^2} + 1}}{d(c^2 d^2 + e^2)(d + ex)} - \frac{b(2c^2 d^2 + e^2) \log \left( cx \left( \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{c^2 d^2 + e^2} - cd \right) + e \right)}{d^2(c^2 d^2 + e^2)^{3/2}} + \frac{b(2c^2 d^2 + e^2) \operatorname{tanh}^{-1} \left( \frac{e + \frac{d}{x}}{\sqrt{1 + \frac{x^2}{c^2}}} \right)}{d^2(c^2 d^2 + e^2)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x)^3, x]

[Out] (-a/(e\*(d + e\*x)^2)) - (b\*c\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x)/(d\*(c^2\*d^2 + e^2)\*(d + e\*x)) - (b\*ArcCsch[c\*x])/(e\*(d + e\*x)^2) + (b\*ArcSinh[1/(c\*x)])/(d^2\*e) + (b\*(2\*c^2\*d^2 + e^2)\*Log[d + e\*x])/(d^2\*(c^2\*d^2 + e^2)^(3/2)) - (b\*(2\*c^2\*d^2 + e^2)\*Log[e + c\*(-c\*d) + Sqrt[c^2\*d^2 + e^2]\*Sqrt[1 + 1/(c^2\*x^2)]])/d^2\*(c^2\*d^2 + e^2)^(3/2))/2

**fricas [B]** time = 1.34, size = 745, normalized size = 4.57

$$ac^4 d^6 + bc^3 d^5 e + 2ac^2 d^4 e^2 + bcd^3 e^3 + ad^2 e^4 + (bc^3 d^3 e^3 + bcde^5)x^2 - (2bc^2 d^4 e + bd^2 e^3 + (2bc^2 d^2 e^3 + be^5)x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^4*d^6 + b*c^3*d^5*e + 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a*d^2*e^4 +
(b*c^3*d^3*e^3 + b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e + b*d^2*e^3 + (2*b*c^2*d^
2*e^3 + b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 + b*d*e^4)*x)*sqrt(c^2*d^2 + e^2)*l
og(-(c^3*d^2*x - c*d*e + (c^3*d^2 + c*e^2)*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ (c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + c^2*d*x - e)*sqrt(c^2*d^2 + e^2)
)/(e*x + d)) + 2*(b*c^3*d^4*e^2 + b*c*d^2*e^4)*x - (b*c^4*d^6 + 2*b*c^2*d^4
*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 + 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4
*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)
) - c*x + 1) + (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 +
2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*
x)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^4*d^6 + 2*b*c^2*
d^4*e^2 + b*d^2*e^4)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (
(b*c^3*d^3*e^3 + b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 + b*c*d^2*e^4)*x)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3
+ 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*
x)
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^3, x)
```

```
maple [B] time = 0.07, size = 963, normalized size = 5.91
```

$$\frac{c^2 a}{2(cxe + cd)^2 e} - \frac{c^2 b \operatorname{arcsch}(cx)}{2(cxe + cd)^2 e} + \frac{c^2 b \sqrt{c^2 x^2 + 1} \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right)}{2\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} (c^2 d^2 + e^2)(cxe + cd)} + \frac{c^2 b \sqrt{c^2 x^2 + 1} d \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right)}{2e\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} x (c^2 d^2 + e^2)(cxe + cd)} - \frac{c^2 b \sqrt{c^2 x^2 + 1}}{\sqrt{c^2 x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^3,x)
```

```
[Out] -1/2*c^2*a/(c*e*x+c*d)^2/e-1/2*c^2*b/(c*e*x+c*d)^2/e*arccsch(c*x)+1/2*c^2*b
*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*ar
ctanh(1/(c^2*x^2+1)^(1/2))+1/2*c^2*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x
^2)^(1/2)/x*d/(c^2*d^2+e^2)/(c*e*x+c*d)*arctanh(1/(c^2*x^2+1)^(1/2))-c^2*b*
(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/((c^2*d^2+e^2)/e^2)^(1/2)/(c^
2*d^2+e^2)/(c*e*x+c*d)*ln(2*((c^2*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-
c^2*d*x+e)/(c*e*x+c*d))-c^2*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/
2)/x*d/((c^2*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*ln(2*((c^2*x^2+1)
)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-c^2*d*x+e)/(c*e*x+c*d))-1/2*c^2*b*e/((c
^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(c^2*d^2+e^2)/(c*e*x+c*d)-1/2*b*e/((c^2*x^2+1)
/c^2/x^2)^(1/2)/x/d/(c^2*d^2+e^2)/(c*e*x+c*d)+1/2*b*e^2*(c^2*x^2+1)^(1/2)/(
(c^2*x^2+1)/c^2/x^2)^(1/2)/d^2/(c^2*d^2+e^2)/(c*e*x+c*d)*arctanh(1/(c^2*x^2
+1)^(1/2))+1/2*b*e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(c^2*d
^2+e^2)/(c*e*x+c*d)*arctanh(1/(c^2*x^2+1)^(1/2))-1/2*b*e^2*(c^2*x^2+1)^(1/2)
)/((c^2*x^2+1)/c^2/x^2)^(1/2)/d^2/((c^2*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)/(
c*e*x+c*d)*ln(2*((c^2*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-c^2*d*x+e)/(
c*e*x+c*d))-1/2*b*e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/((c^2
```

$\frac{d^2+e^2}{e^2}^{1/2}/(c^2d^2+e^2)/(cex+cd) \ln(2((c^2x^2+1)^{1/2}((c^2d^2+e^2)/e^2)^{1/2}e^{-c^2dx+e})/(cex+cd))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} \left( \frac{2ic^3d(\log(icx+1) - \log(-icx+1))}{c^4d^4 + 2c^2d^2e^2 + e^4} + 4c^2 \int \frac{1}{2(c^2e^3x^4 + 2c^2de^2x^3 + 2de^2x + d^2e + (c^2d^2e + e^3)x^2 + (c^2d^2e + e^3)x^2 + (c^2d^2e + e^3)x^2)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^3,x, algorithm="maxima")

[Out]  $-1/4*(2*I*c^3*d*(\log(I*c*x + 1) - \log(-I*c*x + 1))/(c^4*d^4 + 2*c^2*d^2*e^2 + e^4) + 4*c^2*\text{integrate}(1/2*x/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2)*\text{sqrt}(c^2*x^2 + 1)), x) - 2*(3*c^2*d^2*e + e^3)*\log(e*x + d)/(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4) - (2*c^4*d^6*\log(c) + 2*d^2*e^4*\log(c) - 2*d^2*e^4 + 2*(2*d^4*e^2*\log(c) - d^4*e^2)*c^2 - 2*(c^2*d^3*e^3 + d*e^5)*x + (c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*d^2*e^4)*x^2 + 2*(c^4*d^5*e - c^2*d^3*e^3)*x)*\log(c^2*x^2 + 1) - 2*((c^4*d^4*e^2 + 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e + 2*c^2*d^3*e^3 + d*e^5)*x)*\log(x) - 2*(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4)*\log(\text{sqrt}(c^2*x^2 + 1) + 1))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*x)*b - 1/2*a/(e^3*x^2 + 2*d*e^2*x + d^2*e)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(d + e\*x)^3,x)

[Out] int((a + b\*asinh(1/(c\*x)))/(d + e\*x)^3, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/(e\*x+d)\*\*3,x)

[Out] Integral((a + b\*acsch(c\*x))/(d + e\*x)\*\*3, x)

### 3.51 $\int x^2 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=918

$$\frac{32b \sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}} \sqrt{c^2x^2+1} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right) \middle| \frac{2e}{\sqrt{-c^2}d+e}\right) d^4}{105ce^3 \sqrt{1 + \frac{1}{c^2x^2} x \sqrt{d+ex}}} + \frac{32bc \sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}} \sqrt{c^2x^2+1} F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\right)}{105(-c^2)^{3/2} e^2 \sqrt{1 + \frac{1}{c^2x^2} x \sqrt{d+ex}}}$$

[Out]  $2/3*d^2*(e*x+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^3-4/5*d*(e*x+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^3+2/7*(e*x+d)^{(7/2)}*(a+b*\operatorname{arccsch}(c*x))/e^3+4/35*b*(c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/(1+1/c^2/x^2)^{(1/2)}+8/105*b*d*(c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/e/x/(1+1/c^2/x^2)^{(1/2)}-32/105*b*d^4*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/35*b*c*d^2*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}+4/105*b*c*(2*c^2*d^2+9*e^2)*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(5/2)}/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}+32/105*b*c*d^3*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/105*b*c*d*(c^2*d^2+e^2)*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(5/2)}/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 3.25, antiderivative size = 918, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 16, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$ , Rules used = {43, 6310, 12, 6721, 6742, 743, 844, 719, 424, 419, 958, 932, 168, 538, 537, 833}

$$\frac{32b \sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}} \sqrt{c^2x^2+1} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right) \middle| \frac{2e}{\sqrt{-c^2}d+e}\right) d^4}{105ce^3 \sqrt{1 + \frac{1}{c^2x^2} x \sqrt{d+ex}}} + \frac{32bc \sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}} \sqrt{c^2x^2+1} F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\right)}{105(-c^2)^{3/2} e^2 \sqrt{1 + \frac{1}{c^2x^2} x \sqrt{d+ex}}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*\operatorname{Sqrt}[d+e*x]*(a+b*\operatorname{ArcCsch}[c*x]),x]$

[Out]  $(-4*b*d*\operatorname{Sqrt}[d+e*x]*(1+c^2*x^2))/(105*c^3*e*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x) + (4*b*(d+e*x)^{(3/2)}*(1+c^2*x^2))/(35*c^3*e*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x) + (2*d^2*(d+e*x)^{(3/2)}*(a+b*\operatorname{ArcCsch}[c*x]))/(3*e^3) - (4*d*(d+e*x)^{(5/2)}*(a+b*\operatorname{ArcCsch}[c*x]))/(5*e^3) + (2*(d+e*x)^{(7/2)}*(a+b*\operatorname{ArcCsch}[c*x]))/(7*e^3) - (32*b*c*d^2*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)))/(105*(-c^2)^{(3/2)}*e^2*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d+e*x))/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]) - (4*b*c*(c^2*d^2-3*e^2)*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)))/(35*(-c^2)^{(5/2)}*e^2*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d+e*x))/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]) + (32*b*c*d^3*\operatorname{Sqrt}[(c^2*(d+e*x))/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)))/(105*(-c^2)^{(3/2)}*e^2*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x]) - (4*b*c*d*(c^2*d^2+e^2)*\operatorname{Sqrt}[(c^2*(d+e*x))/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-$

$\text{Sqrt}[-c^2*x]/\text{Sqrt}[2]], (-2*\text{Sqrt}[-c^2]*e)/(c^2*d - \text{Sqrt}[-c^2]*e)]/(105*(-c^2)^{(5/2)}*e^2*\text{Sqrt}[1 + 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x]) - (32*b*d^4*\text{Sqrt}[(\text{Sqrt}[-c^2]*(d + e*x))/(\text{Sqrt}[-c^2]*d + e)]*\text{Sqrt}[1 + c^2*x^2]*\text{EllipticPi}[2, \text{ArcSin}[\text{Sqrt}[1 - \text{Sqrt}[-c^2]*x]/\text{Sqrt}[2]], (2*e)/(\text{Sqrt}[-c^2]*d + e))]/(105*c*e^3*\text{Sqrt}[1 + 1/(c^2*x^2)]*x*\text{Sqrt}[d + e*x])$

### Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

### Rule 43

$\text{Int}[(a_*) + (b_*)(x_)]^{(m_*)}((c_*) + (d_*)(x_)]^{(n_*)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (\text{!IntegerQ}[n] \|\| (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \|\| \text{LtQ}[9*m + 5*(n + 1), 0] \|\| \text{GtQ}[m + n + 2, 0])$

### Rule 168

$\text{Int}[1/(((a_*) + (b_*)(x_))*\text{Sqrt}[(c_*) + (d_*)(x_)]*\text{Sqrt}[(e_*) + (f_*)(x_)]*\text{Sqrt}[(g_*) + (h_*)(x_)]), x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(\text{Simp}[b*c - a*d - b*x^2, x]*\text{Sqrt}[\text{Simp}[(d*e - c*f)/d + (f*x^2)/d, x]]*\text{Sqrt}[\text{Simp}[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x\} \&\& \text{GtQ}[(d*e - c*f)/d, 0]$

### Rule 419

$\text{Int}[1/(\text{Sqrt}[(a_*) + (b_*)(x_)]^2*\text{Sqrt}[(c_*) + (d_*)(x_)]^2), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0] \&\& \text{!(NegQ}[b/a] \&\& \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

### Rule 424

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)(x_)]^2/\text{Sqrt}[(c_*) + (d_*)(x_)]^2, x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$

### Rule 537

$\text{Int}[1/(((a_*) + (b_*)(x_)]^2*\text{Sqrt}[(c_*) + (d_*)(x_)]^2*\text{Sqrt}[(e_*) + (f_*)(x_)]^2), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticPi}[(b*c)/(a*d), \text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (c*f)/(d*e)])/(\text{Sqrt}[c]*\text{Sqrt}[e]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{!GtQ}[d/c, 0] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[e, 0] \&\& \text{!(GtQ}[f/e, 0] \&\& \text{SimplerSqrtQ}[-(f/e), -(d/c)])$

### Rule 538

$\text{Int}[1/(((a_*) + (b_*)(x_)]^2*\text{Sqrt}[(c_*) + (d_*)(x_)]^2*\text{Sqrt}[(e_*) + (f_*)(x_)]^2), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/((a + b*x^2)*\text{Sqrt}[1 + (d*x^2)/c]*\text{Sqrt}[e + f*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{!GtQ}[c, 0]$

### Rule 719

$\text{Int}[(d_*) + (e_*)(x_)]^{(m_*)}/\text{Sqrt}[(a_*) + (c_*)(x_)]^2, x\_Symbol] \rightarrow \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + e*x)^m*\text{Sqrt}[1 + (c*x^2)/a])/(\text{Sqrt}[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/$

a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

### Rule 743

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m - 1)\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 1)), x] + Dist[1/(c\*(m + 2\*p + 1)), Int[(d + e\*x)^(m - 2)\*Simp[c\*d^2\*(m + 2\*p + 1) - a\*e^2\*(m - 1) + 2\*c\*d\*e\*(m + p)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2\*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

### Rule 833

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(g\*(d + e\*x)^(m)\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^p\*Simp[c\*d\*f\*(m + 2\*p + 2) - a\*e\*g\*m + c\*(e\*f\*(m + 2\*p + 2) + d\*g\*m)\*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[m, 0] && NeQ[m + 2\*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

### Rule 844

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^(m)\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 932

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(f\_) + (g\_)\*(x\_)]\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

### Rule 958

Int[((f\_) + (g\_)\*(x\_))^(n\_)/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), (f + g\*x)^(n + 1/2)/(d + e\*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[n + 1/2]

### Rule 6310

Int[((a\_) + ArcCsch[(c\_)\*(x\_)]\*(b\_))\*(u\_), x\_Symbol] := With[{v = IntHide[u, x]}, Dist[a + b\*ArcCsch[c\*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2\*Sqrt[1 + 1/(c^2\*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]

### Rule 6721

Int[(u\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(x^(n\*FracPart[p])\*(1 + a/(x^n\*b))^FracPart[p]), Int[u\*x^(n\*p)\*(1 + a/(x^n\*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

### Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
]
```

Rubi steps





**Mathematica [C]** time = 14.40, size = 1094, normalized size = 1.19

$$b \frac{c \left( \frac{d}{x} + e \right) x \left( -\frac{16c^3 \operatorname{csch}^{-1}(cx) d^3}{105e^3} - \frac{2}{7} c^3 x^3 \operatorname{csch}^{-1}(cx) - \frac{2c^2 x^2 \left( 2 \sqrt{1 + \frac{1}{c^2 x^2}} e + c d \operatorname{csch}^{-1}(cx) \right)}{35e} - \frac{8cx \left( cde \sqrt{1 + \frac{1}{c^2 x^2}} - c^2 d^2 \operatorname{csch}^{-1}(cx) \right)}{105e^2} + \frac{4(5c^2 d^2 + 9e^2) \sqrt{1 + \frac{1}{c^2 x^2}}}{105e^2} \right)}{\sqrt{d+ex}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2\*Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]),x]

[Out]  $-\left( (a d^3 \sqrt{d+e x}) \operatorname{Beta}\left[-\left(\frac{e x}{d}\right), 3, \frac{3}{2}\right] / \left(e^3 \sqrt{1+\frac{e x}{d}}\right) + (b \left( -\left( c \left( \frac{d}{x} + e \right) x \left( \frac{4 \left( 5 c^2 d^2 + 9 e^2 \right) \sqrt{1+\frac{1}{c^2 x^2}}}{105 e^2} - \frac{16 c^3 d^3 \operatorname{ArcCsch}[c x]}{105 e^3} - \frac{2 c^3 x^3 \operatorname{ArcCsch}[c x]}{7} - \left( 2 c^2 x^2 \left( 2 e \sqrt{1+\frac{1}{c^2 x^2}} + c d \operatorname{ArcCsch}[c x] \right) / (35 e) - \left( 8 c x \left( c d e \sqrt{1+\frac{1}{c^2 x^2}} - c^2 d^2 \operatorname{ArcCsch}[c x] \right) / (105 e^2) \right) \right) / \sqrt{d+e x} \right) - \left( 2 \sqrt{e+\frac{d}{x}} \sqrt{c x} \left( -\left( \sqrt{2} \left( 9 c^3 d^3 e + c d e^3 \right) \sqrt{1+I c x} \right) \left( I + c x \right) \sqrt{\frac{c d + c e x}{c d - I e}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\left(\frac{e(I+c x)}{c d - I e}\right)}\right], \frac{I c d + e}{2 e}\right] / \left( \sqrt{1+\frac{1}{c^2 x^2}} \right) \sqrt{e+\frac{d}{x}} \left( c x \right)^{3/2} \sqrt{\frac{e(1-I c x)}{I c d + e}} \right) + \left( I \sqrt{2} \left( c d - I e \right) \left( 8 c^4 d^4 - 5 c^2 d^2 e^2 - 9 e^4 \right) \sqrt{1+I c x} \sqrt{\frac{e(I+c x)(c d + c e x)}{I c d + e}} \operatorname{EllipticPi}\left[1+\frac{I c d}{e}, \operatorname{ArcSin}\left[\sqrt{-\left(\frac{e(I+c x)}{c d - I e}\right)}\right], \frac{I c d + e}{2 e}\right] / \left( e \sqrt{1+\frac{1}{c^2 x^2}} \right) \sqrt{e+\frac{d}{x}} \left( c x \right)^{3/2} - \left( 2 \left( -5 c^3 d^3 e - 9 c d e^3 \right) \operatorname{Cosh}\left[2 \operatorname{ArcCsch}[c x]\right] \left( -\left( c d + c e x \right) \left( 1 + c^2 x^2 \right) + \left( c x \left( c d \sqrt{2 + (2 I) c x} \right) \left( I + c x \right) \sqrt{\frac{c d + c e x}{c d - I e}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{-\left(\frac{e(I+c x)}{c d - I e}\right)}\right], \frac{I c d + e}{2 e}\right] + 2 \sqrt{-\left(\frac{e(-I+c x)}{c d + I e}\right)} \right) \left( I + c x \right) \sqrt{\frac{c d + c e x}{c d - I e}} \left( c d + I e \right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{\frac{c d + c e x}{c d - I e}}\right], \frac{c d - I e}{c d + I e} \right] - I e \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{c d + c e x}{c d - I e}}\right], \frac{c d - I e}{c d + I e} \right) + \left( I c d + e \right) \sqrt{2 + (2 I) c x} \sqrt{-\left(\frac{e(I+c x)}{c d - I e}\right)} \sqrt{\frac{e(I+c x)(c d + c e x)}{I c d + e}} \operatorname{EllipticPi}\left[1+\frac{I c d}{e}, \operatorname{ArcSin}\left[\sqrt{-\left(\frac{e(I+c x)}{c d - I e}\right)}\right], \frac{I c d + e}{2 e}\right] \right) / \left( 2 \sqrt{-\left(\frac{e(I+c x)}{c d - I e}\right)} \right) \right) / \left( c d \sqrt{1+\frac{1}{c^2 x^2}} \sqrt{e+\frac{d}{x}} \sqrt{c x} \left( 2 + c^2 x^2 \right) \right) / \left( 105 e^3 \sqrt{d+e x} \right) \right) / c^4$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d} (b \operatorname{arcsch}(cx) + a) x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)\*x^2, x)

**maple [C]** time = 0.36, size = 2515, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x)

[Out] 
$$\frac{2}{e^3} \left( a \left( \frac{1}{7} (e*x+d)^{7/2} - \frac{2}{5} d (e*x+d)^{5/2} + \frac{1}{3} d^2 (e*x+d)^{3/2} \right) + b \left( \frac{1}{7} \operatorname{arccsch}(c*x) (e*x+d)^{7/2} - \frac{2}{5} \operatorname{arccsch}(c*x) d (e*x+d)^{5/2} + \frac{1}{3} \operatorname{arccsch}(c*x) d^2 (e*x+d)^{3/2} + \frac{2}{105} c^4 (-I * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{1/2} * c*d*e^3 - 3 * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{7/2} * c^4*d - I * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{1/2} * c^3*d^3*e + 7 * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{5/2} * c^4*d^2 - 7 * I * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{5/2} * c^3*d*e + I * (-I * (e*x+d) * c*e + (e*x+d) * c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2) \right)^{1/2} * ((I * (e*x+d) * c*e - (e*x+d) * c^2*d + c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2}, (-2 * I * c*d*e - c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * c*d*e^3 + 3 * I * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{3/2} * c*e^3 - 4 * (-I * (e*x+d) * c*e + (e*x+d) * c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2) \right)^{1/2} * ((I * (e*x+d) * c*e - (e*x+d) * c^2*d + c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2}, (-2 * I * c*d*e - c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * c^4*d^4 - 5 * (-I * (e*x+d) * c*e + (e*x+d) * c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2) \right)^{1/2} * ((I * (e*x+d) * c*e - (e*x+d) * c^2*d + c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * \operatorname{EllipticE}((e*x+d)^{1/2} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2}, (-2 * I * c*d*e - c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * c^4*d^4 + 8 * (-I * (e*x+d) * c*e + (e*x+d) * c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2) \right)^{1/2} * ((I * (e*x+d) * c*e - (e*x+d) * c^2*d + c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * \operatorname{EllipticPi}((e*x+d)^{1/2} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{1/2}, 1 / (I*e+c*d) / c * (c^2*d^2+e^2) / d, (-I * e - c*d) * c / (c^2*d^2+e^2) \right)^{1/2} / ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2} * c^4*d^4 - 5 * ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{3/2} * c^4*d^3 + 3 * I * ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{7/2} * c^3*e + ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{1/2} * c^4*d^4 + 9 * I * (-I * (e*x+d) * c*e + (e*x+d) * c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2) \right)^{1/2} * ((I * (e*x+d) * c*e - (e*x+d) * c^2*d + c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2}, (-2 * I * c*d*e - c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * c^2*d^2*e^2 - 14 * (-I * (e*x+d) * c*e + (e*x+d) * c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2) \right)^{1/2} * ((I * (e*x+d) * c*e - (e*x+d) * c^2*d + c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * \operatorname{EllipticE}((e*x+d)^{1/2} * ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2}, (-2 * I * c*d*e - c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * c^2*d^2*e^2 - 3 * ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{3/2} * c^2*d*e^2 + 5 * I * ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{3/2} * c^3*d^2*e + ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2} (e*x+d)^{1/2} * c^2*d^2*e^2 + 9 * (-I * (e*x+d) * c*e + (e*x+d) * c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2) \right)^{1/2} * ((I * (e*x+d) * c*e - (e*x+d) * c^2*d + c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2}, (-2 * I * c*d*e - c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * e^4 - 9 * (-I * (e*x+d) * c*e + (e*x+d) * c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2) \right)^{1/2} * ((I * (e*x+d) * c*e - (e*x+d) * c^2*d + c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * \operatorname{EllipticE}((e*x+d)^{1/2} * ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2}, (-2 * I * c*d*e - c^2*d^2 + e^2) / (c^2*d^2+e^2))^{1/2} * e^4) / (((e*x+d)^2 * c^2 - 2 * (e*x+d) * c^2*d + c^2*d^2 + e^2) / c^2 / x^2 / e^2)^{1/2} / x / ((I * e + c*d) * c / (c^2*d^2+e^2))^{1/2} / (I * e - c*d))$$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) \sqrt{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a + b\*asinh(1/(c\*x)))\*(d + e\*x)^(1/2),x)

[Out] int(x^2\*(a + b\*asinh(1/(c\*x)))\*(d + e\*x)^(1/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))\*(e\*x+d)\*\*(1/2),x)

[Out] Timed out

### 3.52 $\int x\sqrt{d+ex} \left(a + b\operatorname{csch}^{-1}(cx)\right) dx$

**Optimal.** Leaf size=679

$$-\frac{2d(d+ex)^{3/2} \left(a + b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{2(d+ex)^{5/2} \left(a + b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{8bd^3\sqrt{c^2x^2+1} \sqrt{\frac{\sqrt{-c^2(d+ex)}}{\sqrt{-c^2d+e}}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-v}}{\sqrt{1-v}}\right)\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

[Out]  $-2/3*d*(e*x+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^2+2/5*(e*x+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^2+4/15*b*(c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/x/(1+1/c^2/x^2)^{(1/2)}+8/15*b*d^3*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+8/15*b*c*d*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/e/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}-8/15*b*c*d^2*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/15*b*c*(c^2*d^2+e^2)*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(5/2)}/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 2.52, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 15, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.790$ , Rules used = {43, 6310, 12, 6721, 6742, 743, 844, 719, 424, 419, 958, 932, 168, 538, 537}

$$-\frac{2d(d+ex)^{3/2} \left(a + b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{2(d+ex)^{5/2} \left(a + b\operatorname{csch}^{-1}(cx)\right)}{5e^2} + \frac{4bc\sqrt{c^2x^2+1} (c^2d^2 + e^2) \sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-v}}{\sqrt{1-v}}\right)\right)}{15(-c^2)^{5/2} ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $(4*b*\operatorname{Sqrt}[d + e*x]*(1 + c^2*x^2))/(15*c^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x) - (2*d*(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(3*e^2) + (2*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(5*e^2) + (8*b*c*d*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(3/2)}*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]) - (8*b*c*d^2*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(3/2)}*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) + (4*b*c*(c^2*d^2 + e^2)*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(5/2)}*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) + (8*b*d^3*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e)))/(15*c*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{MatchQ}[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(!GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a]]/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 743

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[
(e*(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
```

$[m, -2]] \&\& \text{NeQ}[m + 2*p + 1, 0] \&\& \text{IntQuadraticQ}[a, 0, c, d, e, m, p, x]$

#### Rule 844

$\text{Int}[\frac{(d_.) + (e_.)x^{m_}}{(f_.) + (g_.)x^{n_}}(a_.) + (c_.)x^{2p_}], x\_Symbol] \rightarrow \text{Dist}[g/e, \text{Int}[(d + ex)^{m+1}(a + cx^2)^p, x], x] + \text{Dist}[(ef - d^2g)/e, \text{Int}[(d + ex)^m(a + cx^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[c^2d + ae^2, 0] \&\& \text{!IGtQ}[m, 0]$

#### Rule 932

$\text{Int}[1/((d_.) + (e_.)x^{m_})\sqrt{(f_.) + (g_.)x^{n_}}\sqrt{(a_.) + (c_.)x^{2p_}}], x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 2]\}, \text{Dist}[1/\sqrt{a}, \text{Int}[1/((d + ex)\sqrt{f + gx}\sqrt{1 - qx}\sqrt{1 + qx})], x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x] \&\& \text{NeQ}[ef - d^2g, 0] \&\& \text{NeQ}[c^2d + ae^2, 0] \&\& \text{GtQ}[a, 0]$

#### Rule 958

$\text{Int}[\frac{(f_.) + (g_.)x^{n_}}{(d_.) + (e_.)x^{m_}}\sqrt{(a_.) + (c_.)x^{2p_}}], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[1/(\sqrt{f + gx}\sqrt{a + cx^2}), (f + gx)^{n+1/2}/(d + ex), x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x] \&\& \text{NeQ}[ef - d^2g, 0] \&\& \text{NeQ}[c^2d + ae^2, 0] \&\& \text{IntegerQ}[n + 1/2]$

#### Rule 6310

$\text{Int}[\frac{(a_.) + \text{ArcCsch}[c_.]x^{m_}}{(b_.)}(u_.)], x\_Symbol] \rightarrow \text{With}\{v = \text{IntHide}[u, x]\}, \text{Dist}[a + b\text{ArcCsch}[cx], v, x] + \text{Dist}[b/c, \text{Int}[\text{SimplifyIntegrand}[v/(x^2\sqrt{1 + 1/(c^2x^2)})], x], x], x] /; \text{InverseFunctionFreeQ}[v, x] /; \text{FreeQ}\{a, b, c\}, x]$

#### Rule 6721

$\text{Int}[(u_.)((a_.) + (b_.)x^{n_})^p], x\_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[p]}(a + bx^n)^{\text{FracPart}[p]})/(x^{n\text{FracPart}[p]}(1 + a/(x^n b))^{\text{FracPart}[p]}), \text{Int}[u x^{n p} (1 + a/(x^n b))^p, x], x] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{!IntegerQ}[p] \&\& \text{ILtQ}[n, 0] \&\& \text{!RationalFunctionQ}[u, x] \&\& \text{IntegerQ}[p + 1/2]$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

#### Rubi steps



**Mathematica [C]** time = 1.68, size = 418, normalized size = 0.62

$$\frac{1}{15} \left( \frac{2a\sqrt{d+ex}(-2d^2+dex+3e^2x^2)}{e^2} + \frac{4bx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}{c} + \frac{4ib\sqrt{-\frac{e(cx-i)}{cd+ie}}\sqrt{-\frac{e(cx+i)}{cd-ie}}}{\left((c^2d^2-2icde+e^2)F\left(\right)\right)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]),x]

[Out] ((4\*b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x])/c + (2\*a\*Sqrt[d + e\*x]\*(-2\*d^2 + d\*e\*x + 3\*e^2\*x^2))/e^2 + (2\*b\*Sqrt[d + e\*x]\*(-2\*d^2 + d\*e\*x + 3\*e^2\*x^2)\*ArcCsch[c\*x])/e^2 + ((4\*I)\*b\*Sqrt[-((e\*(-I + c\*x))/(c\*d + I\*e))]\*Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]\*(2\*c\*d\*(c\*d + I\*e)\*EllipticE[I\*ArcSinh[Sqrt[-(c/(c\*d - I\*e))]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e)] + (c^2\*d^2 - (2\*I)\*c\*d\*e + e^2)\*EllipticF[I\*ArcSinh[Sqrt[-(c/(c\*d - I\*e))]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e)] - 2\*c^2\*d^2\*EllipticPi[1 - (I\*e)/(c\*d), I\*ArcSinh[Sqrt[-(c/(c\*d - I\*e))]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e))]/(c^3\*Sqrt[-(c/(c\*d - I\*e))]\*e^2\*Sqrt[1 + 1/(c^2\*x^2)]\*x))/15

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex+d}(b \operatorname{arcsch}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)\*x, x)

**maple [C]** time = 0.08, size = 1964, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x)

[Out] 2/e^2\*(a\*(1/5\*(e\*x+d)^(5/2)-1/3\*d\*(e\*x+d)^(3/2))+b\*(1/5\*arccsch(c\*x)\*(e\*x+d)^(5/2)-1/3\*arccsch(c\*x)\*d\*(e\*x+d)^(3/2)+2/15/c^3\*(-3\*I\*(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticF((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), (-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*c^2\*d^2\*e-((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^(5/2)\*c^3\*d-2\*I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^(3/2)\*c^2\*d\*e+2\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^(3/2)\*c^3\*d^2+I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^(5/2)\*c^2\*e+(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticF((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), (-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*c^3\*d^3+2\*(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)



$2) * \text{EllipticE}((e*x+d)^{(1/2)} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e - c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)} * c^3*d^3 + I * ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(1/2)} * e^3 - 2 * (-I*(e*x+d)*c*e + (e*x+d)*c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I*(e*x+d)*c*e - (e*x+d)*c^2*d + c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)}$   
 $) * \text{EllipticPi}((e*x+d)^{(1/2)} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)}, 1 / (I*e+c*d) / c * (c^2*d^2+e^2) / d, (-I*e-c*d)*c / (c^2*d^2+e^2))^{(1/2)} / ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)} * c^3*d^3 + I * ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(1/2)} * c^2*d^2 * e - ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(1/2)} * c^3*d^3 + 2*I * (-I*(e*x+d)*c*e + (e*x+d)*c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I*(e*x+d)*c*e - (e*x+d)*c^2*d + c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticPi}((e*x+d)^{(1/2)} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)}, 1 / (I*e+c*d) / c * (c^2*d^2+e^2) / d, (-I*e-c*d)*c / (c^2*d^2+e^2))^{(1/2)} / ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)} * c^2*d^2 * e - (-I*(e*x+d)*c*e + (e*x+d)*c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I*(e*x+d)*c*e - (e*x+d)*c^2*d + c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticF}((e*x+d)^{(1/2)} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e - c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)} * c*d * e^2 + 2 * (-I*(e*x+d)*c*e + (e*x+d)*c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I*(e*x+d)*c*e - (e*x+d)*c^2*d + c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticE}((e*x+d)^{(1/2)} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e - c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)} * c*d * e^2 - I * (-I*(e*x+d)*c*e + (e*x+d)*c^2*d - c^2*d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I*(e*x+d)*c*e - (e*x+d)*c^2*d + c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticF}((e*x+d)^{(1/2)} * ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e - c^2*d^2+e^2) / (c^2*d^2+e^2))^{(1/2)} * e^3 - ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(1/2)} * c*d * e^2 / (((e*x+d)^2 * c^2 - 2 * (e*x+d)*c^2*d + c^2*d^2+e^2) / c^2 / x^2 / e^2)^{(1/2)} / x / ((I*e+c*d)*c / (c^2*d^2+e^2))^{(1/2)} / (I*e-c*d))$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) \sqrt{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a + b\*asinh(1/(c\*x)))\*(d + e\*x)^(1/2),x)

[Out] int(x\*(a + b\*asinh(1/(c\*x)))\*(d + e\*x)^(1/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))\*(e\*x+d)\*\*(1/2),x)

[Out] Timed out

### 3.53 $\int \sqrt{d+ex} \left( a + b \operatorname{csch}^{-1}(cx) \right) dx$

**Optimal.** Leaf size=429

$$\frac{2(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e} - \frac{4bd^2 \sqrt{c^2x^2+1} \sqrt{\frac{\sqrt{-c^2(d+ex)}}{\sqrt{-c^2d+e}}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right) \middle| \frac{2e}{\sqrt{-c^2d+e}}\right)}{3cex \sqrt{\frac{1}{c^2x^2}+1} \sqrt{d+ex}} + \frac{4bcd \sqrt{c^2x^2+1} \sqrt{d+ex}}{3e}$$

[Out]  $\frac{2}{3}*(e*x+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e+4/3*b*c*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)})*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^((1/2))*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/x/(1+1/c^2/x^2)^{(1/2)}/((e*x+d)/(d+e/(-c^2)^{(1/2)}))^((1/2))+4/3*b*c*d*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)})*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^((1/2))*(c^2*x^2+1)^{(1/2)}*((e*x+d)/(d+e/(-c^2)^{(1/2)}))^((1/2)}/(-c^2)^{(3/2)}/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/3*b*d^2*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)})*x)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^((1/2))*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^((1/2)}/c/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 0.71, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {6290, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424}

$$\frac{2(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e} - \frac{4bd^2 \sqrt{c^2x^2+1} \sqrt{\frac{\sqrt{-c^2(d+ex)}}{\sqrt{-c^2d+e}}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right) \middle| \frac{2e}{\sqrt{-c^2d+e}}\right)}{3cex \sqrt{\frac{1}{c^2x^2}+1} \sqrt{d+ex}} + \frac{4bcd \sqrt{c^2x^2+1} \sqrt{d+ex}}{3e}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]), x]`

[Out]  $(2*(d+e*x)^{(3/2)}*(a+b*\operatorname{ArcCsch}[c*x]))/(3*e) + (4*b*c*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)))/(3*(-c^2)^{(3/2)}*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[(d+e*x)/(d+e/\operatorname{Sqrt}[-c^2])]) + (4*b*c*d*\operatorname{Sqrt}[(d+e*x)/(d+e/\operatorname{Sqrt}[-c^2])])* \operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)))/(3*(-c^2)^{(3/2)}*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x]) - (4*b*d^2*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d+e*x))/(\operatorname{Sqrt}[-c^2]*d+e)]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d+e)))/(3*c*e*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x])$

#### Rule 168

`Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]`

#### Rule 419

`Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])`

#### Rule 424

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/Sqrt[(c\_) + (d\_.)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

#### Rule 537

Int[1/(((a\_) + (b\_.)\*(x\_)^2)\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]\*Sqrt[(e\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticPi[(b\*c)/(a\*d), ArcSin[Rt[-(d/c), 2]\*x], (c\*f)/(d\*e)]/(a\*Sqrt[c]\*Sqrt[e]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

#### Rule 538

Int[1/(((a\_) + (b\_.)\*(x\_)^2)\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]\*Sqrt[(e\_) + (f\_.)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/((a + b\*x^2)\*Sqrt[1 + (d\*x^2)/c]\*Sqrt[e + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

#### Rule 719

Int[((d\_) + (e\_.)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_.)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

#### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

#### Rule 933

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(f\_.) + (g\_.)\*(x\_)]\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c\*x^2)/a]/Sqrt[a + c\*x^2], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && !GtQ[a, 0]

#### Rule 958

Int[((f\_.) + (g\_.)\*(x\_))^(n\_)/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), (f + g\*x)^(n + 1/2)/(d + e\*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[n + 1/2]

#### Rule 1574

Int[(x\_)^(m\_.)\*((a\_.) + (c\_.)\*(x\_)^(mn2\_.))^p\_)\*((d\_) + (e\_.)\*(x\_)^(n\_.))^q\_.), x\_Symbol] := Dist[(x^(2\*n\*FracPart[p])\*(a + c/x^(2\*n))^FracPart[p])/(c + a\*x^(2\*n))^FracPart[p], Int[x^(m - 2\*n\*p)\*(d + e\*x^n)^q\*(c + a\*x^(2\*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2\*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]

#### Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:= Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{d+ex} (a + bcsch^{-1}(cx)) dx &= \frac{2(d+ex)^{3/2} (a + bcsch^{-1}(cx))}{3e} + \frac{(2b) \int \frac{(d+ex)^{3/2}}{\sqrt{1+\frac{1}{c^2x^2}}} dx}{3ce} \\
 &= \frac{2(d+ex)^{3/2} (a + bcsch^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{(d+ex)^{3/2}}{x\sqrt{\frac{1}{c^2} + x^2}} dx}{3ce\sqrt{1 + \frac{1}{c^2x^2}} x} \\
 &= \frac{2(d+ex)^{3/2} (a + bcsch^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2} + x^2}\right) \int \left(\frac{2de}{\sqrt{d+ex}\sqrt{\frac{1}{c^2} + x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2} + x^2}}\right) dx}{3ce\sqrt{1 + \frac{1}{c^2x^2}} x} \\
 &= \frac{2(d+ex)^{3/2} (a + bcsch^{-1}(cx))}{3e} + \frac{\left(4bd\sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2} + x^2}} dx}{3c\sqrt{1 + \frac{1}{c^2x^2}} x} + \frac{(2bd^2)}{3c} \\
 &= \frac{2(d+ex)^{3/2} (a + bcsch^{-1}(cx))}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2} + x^2}} dx}{3c\sqrt{1 + \frac{1}{c^2x^2}} x} - \frac{(2bd\sqrt{\frac{1}{c^2} + x^2})}{3c} \\
 &= \frac{2(d+ex)^{3/2} (a + bcsch^{-1}(cx))}{3e} + \frac{8b\sqrt{-c^2} d \sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}} \sqrt{1+c^2x^2} E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}}}{\sqrt{2}}\right)\right)}{3c^3\sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{d+ex}} \\
 &= \frac{2(d+ex)^{3/2} (a + bcsch^{-1}(cx))}{3e} + \frac{4b\sqrt{-c^2} \sqrt{d+ex} \sqrt{1+c^2x^2} E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}}}{\sqrt{2}}\right)\right)}{3c^3\sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
 &= \frac{2(d+ex)^{3/2} (a + bcsch^{-1}(cx))}{3e} + \frac{4b\sqrt{-c^2} \sqrt{d+ex} \sqrt{1+c^2x^2} E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}}}{\sqrt{2}}\right)\right)}{3c^3\sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}
 \end{aligned}$$

**Mathematica [C]** time = 14.08, size = 926, normalized size = 2.16

$$\frac{2a(d+ex)^{3/2}}{3e} + \frac{b}{\sqrt{d+ex}} \left( -\frac{(cd+ce)x \left( -\frac{2}{3}cx \operatorname{arcsch}(cx) - \frac{2cd \operatorname{arcsch}^{-1}(cx)}{3e} - \frac{4}{3} \sqrt{1 + \frac{1}{c^2x^2}} \right)}{\sqrt{d+ex}} - \frac{2(cd+ce)x}{\sqrt{1 + \frac{1}{c^2x^2}} \sqrt{\frac{d}{x} + e} (cx)^{3/2} \sqrt{\frac{e(1-icx)}{icd+e}}} \right) \frac{\sqrt{2}cde \sqrt{icx+1} (cx+i) \sqrt{\frac{cd+ce}{cd-ie}} F\left(\sin^{-1}\left(\sqrt{\frac{e(cx+i)}{cd-ie}}\right)\right) \frac{icd-2e}{2}}{\sqrt{1 + \frac{1}{c^2x^2}} \sqrt{\frac{d}{x} + e} (cx)^{3/2} \sqrt{\frac{e(1-icx)}{icd+e}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]),x]

[Out]  $(2*a*(d + e*x)^{(3/2)})/(3*e) + (b*(-(((c*d + c*e*x)*((-4*\sqrt{1 + 1/(c^2*x^2)})))/3 - (2*c*d*ArcCsch[c*x])/(3*e) - (2*c*x*ArcCsch[c*x])/3))/\sqrt{d + e*x}) - (2*(c*d + c*e*x)*(-((\sqrt{2}*c*d*e*\sqrt{1 + I*c*x}*(I + c*x)*\sqrt{(c*d + c*e*x)/(c*d - I*e)})*EllipticF[ArcSin[\sqrt{-((e*(I + c*x))/(c*d - I*e))}], (I*c*d + e)/(2*e)])/(\sqrt{1 + 1/(c^2*x^2)}*\sqrt{e + d/x}*(c*x)^{(3/2)}*\sqrt{(e*(1 - I*c*x))/(I*c*d + e)})) + (I*\sqrt{2}*(c*d - I*e)*(c^2*d^2 + e^2)*\sqrt{1 + I*c*x}*\sqrt{(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2}*EllipticPi[1 + (I*c*d)/e, ArcSin[\sqrt{-((e*(I + c*x))/(c*d - I*e))}], (I*c*d + e)/(2*e)])/(e*\sqrt{1 + 1/(c^2*x^2)}*\sqrt{e + d/x}*(c*x)^{(3/2)} - (2*e*Cosh[2*ArcCsch[c*x]])*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*\sqrt{2 + (2*I)*c*x}*(I + c*x)*\sqrt{(c*d + c*e*x)/(c*d - I*e)})*EllipticF[ArcSin[\sqrt{-((e*(I + c*x))/(c*d - I*e))}], (I*c*d + e)/(2*e)] + 2*\sqrt{-((e*(-I + c*x))/(c*d + I*e))}*(I + c*x)*\sqrt{(c*d + c*e*x)/(c*d - I*e)}*((c*d + I*e)*EllipticE[ArcSin[\sqrt{(c*d + c*e*x)/(c*d - I*e)}], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[\sqrt{(c*d + c*e*x)/(c*d - I*e)}], (c*d - I*e)/(c*d + I*e)] + (I*c*d + e)*\sqrt{2 + (2*I)*c*x}*\sqrt{-((e*(I + c*x))/(c*d - I*e))}*\sqrt{(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2}*EllipticPi[1 + (I*c*d)/e, ArcSin[\sqrt{-((e*(I + c*x))/(c*d - I*e))}], (I*c*d + e)/(2*e)]))/(2*\sqrt{-((e*(I + c*x))/(c*d - I*e))}))/(\sqrt{1 + 1/(c^2*x^2)}*\sqrt{e + d/x}*\sqrt{c*x}*(2 + c^2*x^2)))/(3*e*\sqrt{e + d/x}*\sqrt{c*x}*\sqrt{d + e*x}))/c^2$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d} (b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a), x)

**maple** [C] time = 0.08, size = 840, normalized size = 1.96

$$\frac{2a(ex+d)^{\frac{3}{2}}}{3} + 2b \left( \frac{(ex+d)^{\frac{3}{2}} \operatorname{arccsch}(cx)}{3} + \frac{2\sqrt{-\frac{i(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}} \sqrt{\frac{i(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}}{\sqrt{-\frac{i(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}} \sqrt{\frac{i(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}} \left( i \operatorname{EllipticF} \left( \sqrt{ex+d} \sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}}, \sqrt{-\frac{-c^2d^2+2cd+e^2}{c^2d^2+e^2}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsch(c*x))*(e*x+d)^(1/2),x)`

[Out]  $2/e*(1/3*a*(e*x+d)^{(3/2)}+b*(1/3*(e*x+d)^{(3/2)}*\operatorname{arccsch}(c*x)+2/3/c^2*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^{(1/2)}*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*(I*\operatorname{EllipticF}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)},(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)})*c*d*e-2*\operatorname{EllipticF}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)},(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)})*c^2*d^2+\operatorname{EllipticE}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)},(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)})*c^2*d^2-I*\operatorname{EllipticPi}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)},1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-I*e-c*d)*c/(c^2*d^2+e^2))^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)})*c*d*e+\operatorname{EllipticPi}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)},1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-I*e-c*d)*c/(c^2*d^2+e^2))^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)})*c^2*d^2-\operatorname{EllipticF}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)},(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)})*e^2+\operatorname{EllipticE}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)},(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)})*e^2)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^{(1/2)}/x/((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}/(I*e-c*d))$

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")`

[Out] Timed out

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) \sqrt{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)`

[Out] `int((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) \sqrt{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))*(e*x+d)**(1/2),x)`

[Out] `Integral((a + b*acsch(c*x))*sqrt(d + e*x), x)`

$$3.54 \quad \int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x} dx$$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x, x)

**Rubi** [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/x, x]

[Out] Defer[Int][(Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/x, x]

Rubi steps

$$\int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x} dx = \int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x} dx$$

**Mathematica** [A] time = 19.51, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/x, x]

[Out] Integrate[(Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/x, x]

**fricas** [A] time = 0.57, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d} (b \operatorname{arcsch}(cx) + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x, x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d} (b \operatorname{arcsch}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x, x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/x, x)

**maple** [A] time = 6.87, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx)) \sqrt{ex + d}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x,x)

[Out] int((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\left( \sqrt{d} \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right) + 2\sqrt{ex+d} \right) a - \left( \left( \sqrt{d} \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right) + 2\sqrt{ex+d} \right) \log(c) + \int \frac{\sqrt{ex+d} \log(x)}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x,x, algorithm="maxima")

[Out] (sqrt(d)\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d))) + 2\*sqrt(e\*x + d))\*a - ((sqrt(d)\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d))) + 2\*sqrt(e\*x + d))\*log(c) + integrate(sqrt(e\*x + d)\*log(x)/x, x) - integrate(sqrt(e\*x + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/x, x))\*b

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right) \sqrt{d + ex}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*asinh(1/(c\*x)))\*(d + e\*x)^(1/2))/x,x)

[Out] int(((a + b\*asinh(1/(c\*x)))\*(d + e\*x)^(1/2))/x, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*(e\*x+d)\*\*(1/2)/x,x)

[Out] Timed out



$$3.55 \quad \int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x^2}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x^2, x)

**Rubi** [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/x^2, x]

[Out] Defer[Int][(Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/x^2, x]

Rubi steps

$$\int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x^2} dx = \int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x^2} dx$$

**Mathematica** [A] time = 7.97, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/x^2, x]

[Out] Integrate[(Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/x^2, x]

**fricas** [A] time = 0.92, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d} (b \operatorname{arcsch}(cx) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x^2, x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex+d} (b \operatorname{arcsch}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x^2, x, algorithm="giac")

[Out] integrate(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/x^2, x)

**maple** [A] time = 5.92, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx)) \sqrt{ex + d}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x^2,x)

[Out] int((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( \frac{e \log \left( \frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}} \right)}{\sqrt{d}} - \frac{2\sqrt{ex+d}}{x} \right) a - \frac{1}{2} \left( \left( \frac{e \log \left( \frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}} \right)}{\sqrt{d}} - \frac{2\sqrt{ex+d}}{x} \right) \log(c) + 2 \int \frac{\sqrt{ex+d} \log(x)}{x^2} dx - 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/x^2,x, algorithm="maxima")

[Out] 1/2\*(e\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/sqrt(d) - 2\*sqrt(e\*x + d)/x)\*a - 1/2\*((e\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/sqrt(d) - 2\*sqrt(e\*x + d)/x)\*log(c) + 2\*integrate(sqrt(e\*x + d)\*log(x)/x^2, x) - 2\*integrate(sqrt(e\*x + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/x^2, x))\*b

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right) \sqrt{d + ex}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*asinh(1/(c\*x)))\*(d + e\*x)^(1/2))/x^2,x)

[Out] int(((a + b\*asinh(1/(c\*x)))\*(d + e\*x)^(1/2))/x^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*(e\*x+d)\*\*(1/2)/x\*\*2,x)

[Out] Timed out

### 3.56 $\int (d + ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=486

$$\frac{2(d + ex)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e} - \frac{4bd^3 \sqrt{c^2 x^2 + 1} \sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right) \middle| \frac{2e}{\sqrt{-c^2}d+e}\right)}{5cex \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{d + ex}} - \frac{4bc \sqrt{c^2 x^2 + 1}}{5e}$$

[Out]  $2/5*(e*x+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e+4/15*b*e*(c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/x/(1+1/c^2/x^2)^{(1/2)}+28/15*b*c*d*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/x/(1+1/c^2/x^2)^{(1/2)}/((e*x+d)/(d+e/(-c^2)^{(1/2)}))^{(1/2)}-4/15*b*c*(2*c^2*d^2-e^2)*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*((e*x+d)/(d+e/(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(5/2)}/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/5*b*d^3*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 1.02, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$ , Rules used = {6290, 1574, 958, 719, 419, 933, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2(d + ex)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e} - \frac{4bc \sqrt{c^2 x^2 + 1} (2c^2 d^2 - e^2) \sqrt{\frac{d+ex}{\sqrt{-c^2}+d}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{-c^2}e}{c^2 d - \sqrt{-c^2}e}\right)}{15(-c^2)^{5/2} x \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{d + ex}} - \frac{4bd^3}{5e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $(4*b*e*\operatorname{Sqrt}[d + e*x]*(1 + c^2*x^2))/(15*c^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x) + (2*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(5*e) + (28*b*c*d*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(3/2)}*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(d + e*x)/(d + e/\operatorname{Sqrt}[-c^2])]) - (4*b*c*(2*c^2*d^2 - e^2)*\operatorname{Sqrt}[(d + e*x)/(d + e/\operatorname{Sqrt}[-c^2])]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(5/2)}*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (4*b*d^3*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e)))/(5*c*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

#### Rule 168

$\operatorname{Int}[1/(((a_.) + (b_.)*(x_.))*\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)]*\operatorname{Sqrt}[(e_.) + (f_.)*(x_.)]*\operatorname{Sqrt}[(g_.) + (h_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/(\operatorname{Simp}[b*c - a*d - b*x^2, x]*\operatorname{Sqrt}[\operatorname{Simp}[(d*e - c*f)/d + (f*x^2)/d, x]]*\operatorname{Sqrt}[\operatorname{Simp}[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, \operatorname{Sqrt}[c + d*x]], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, h\}, x \&\& \operatorname{GtQ}[(d*e - c*f)/d, 0]$

#### Rule 419

$\operatorname{Int}[1/(\operatorname{Sqrt}[(a_.) + (b_.)*(x_.)^2]*\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)^2]), x\_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Rt}[-(d/c), 2]*x], (b*c)/(a*d)]/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c]*\operatorname{Rt}[-(d/c), 2]), x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NegQ}[d/c] \&\& \operatorname{GtQ}[c, 0] \&\& \operatorname{GtQ}$

[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

#### Rule 424

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Simp[(Sqrt[a]\*EllipticE[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

#### Rule 537

Int[1/(((a\_) + (b\_)\*(x\_)^2)\*Sqrt[(c\_) + (d\_)\*(x\_)^2]\*Sqrt[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticPi[(b\*c)/(a\*d), ArcSin[Rt[-(d/c), 2]\*x], (c\*f)/(d\*e)]/(a\*Sqrt[c]\*Sqrt[e]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

#### Rule 538

Int[1/(((a\_) + (b\_)\*(x\_)^2)\*Sqrt[(c\_) + (d\_)\*(x\_)^2]\*Sqrt[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/((a + b\*x^2)\*Sqrt[1 + (d\*x^2)/c]\*Sqrt[e + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

#### Rule 719

Int[((d\_) + (e\_)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

#### Rule 844

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

#### Rule 931

Int[((d\_) + (e\_)\*(x\_))^(m\_)/(Sqrt[(f\_) + (g\_)\*(x\_)]\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Simp[(2\*e^2\*(d + e\*x)^(m - 2)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(c\*g\*(2\*m - 1)), x] - Dist[1/(c\*g\*(2\*m - 1)), Int[((d + e\*x)^(m - 3))\*Simp[a\*e^2\*(d\*g + 2\*e\*f\*(m - 2)) - c\*d^3\*g\*(2\*m - 1) + e\*(e\*(a\*e\*g\*(2\*m - 3)) + c\*d\*(2\*e\*f - 3\*d\*g\*(2\*m - 1)))\*x + 2\*e^2\*(c\*e\*f - 3\*c\*d\*g)\*(m - 1)\*x^2, x]/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[2\*m] && GeQ[m, 2]

#### Rule 933

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(f\_) + (g\_)\*(x\_)]\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c\*x^2)/a]/Sqrt[a + c\*x^2], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && !GtQ[a, 0]

#### Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (c_.)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

#### Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_.) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

#### Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

#### Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}
\int (d+ex)^{3/2} (a+b\operatorname{csch}^{-1}(cx)) dx &= \frac{2(d+ex)^{5/2} (a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{(2b) \int \frac{(d+ex)^{5/2}}{\sqrt{1+\frac{1}{c^2x^2}}} dx}{5ce} \\
&= \frac{2(d+ex)^{5/2} (a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right) \int \frac{(d+ex)^{5/2}}{x\sqrt{\frac{1}{c^2}+x^2}} dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2(d+ex)^{5/2} (a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right) \int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} + \frac{d^3}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}}\right) dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2(d+ex)^{5/2} (a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{\left(6bd^2\sqrt{\frac{1}{c^2}+x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{\left(2bd^3\sqrt{\frac{1}{c^2}+x^2}\right) \int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{\left(6bd\sqrt{\frac{1}{c^2}+x^2}\right) \int \frac{1}{x\sqrt{d+ex}\sqrt{\frac{1}{c^2}+x^2}} dx}{5c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{12b\sqrt{-c^2}d^2\sqrt{\frac{1}{c^2}+x^2}}{5c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{12b\sqrt{-c^2}d\sqrt{d+ex}}{5c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{12b\sqrt{-c^2}d\sqrt{d+ex}}{5c\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2(d+ex)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{28b\sqrt{-c^2}d\sqrt{d+ex}}{5c\sqrt{1+\frac{1}{c^2x^2}}x}
\end{aligned}$$

**Mathematica [C]** time = 1.61, size = 380, normalized size = 0.78

$$2 \left( 3a(d+ex)^{5/2} + \frac{2be^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}{c} + \frac{2ib\sqrt{-\frac{e(cx-i)}{cd+ie}}\sqrt{-\frac{e(cx+i)}{cd-ie}}\left((-9c^2d^2-7icde+e^2)F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)\frac{cd-ie}{cd+ie}\right)+3c^2d^2\Pi\left(1-\frac{cd-ie}{cd+ie}\right)\right)}{c^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} \right)$$

15e

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] (2\*((2\*b\*e^2\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x])/c + 3\*a\*(d + e\*x)^(5/2) + 3\*b\*(d + e\*x)^(5/2)\*ArcCsch[c\*x] + ((2\*I)\*b\*Sqrt[-((e\*(-I + c\*x))/(c\*d + I\*e))]\*Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]\*(7\*c\*d\*(c\*d + I\*e)\*EllipticE[I\*ArcSinh[Sqrt[-(c/(c\*d - I\*e))]]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e)] + (-9\*c^2\*d^2 - (7\*I)\*c\*d\*e + e^2)\*EllipticF[I\*ArcSinh[Sqrt[-(c/(c\*d - I\*e))]]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e)] + 3\*c^2\*d^2\*EllipticPi[1 - (I\*e)/(c\*d), I\*ArcSinh[Sqrt[-(c/(c\*d - I\*e))]]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e)))/(c^3\*Sqrt[-(c/(c\*d - I\*e))]\*Sqrt[1 + 1/(c^2\*x^2)]\*x))/(15\*e)

**fricas [F]** time = 11.81, size = 0, normalized size = 0.00

$$\text{integral}\left((aex + ad + (bex + bd) \operatorname{arcsch}(cx))\sqrt{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^(3/2)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral((a\*e\*x + a\*d + (b\*e\*x + b\*d)\*arccsch(c\*x))\*sqrt(e\*x + d), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex + d)^{\frac{3}{2}}(b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^(3/2)\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x + d)^(3/2)\*(b\*arccsch(c\*x) + a), x)

**maple [C]** time = 0.07, size = 1939, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)\*(a+b\*arccsch(c\*x)), x)

[Out] 2/e\*(1/5\*(e\*x+d)^(5/2)\*a+b\*(1/5\*arccsch(c\*x)\*(e\*x+d)^(5/2)+2/15/c^3\*(-I\*(-(I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticF((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), (-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2))\*e^3-((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^(5/2)\*c^3\*d+I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^(1/2)\*e^3+2\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^(3/2)\*c^3\*d^2+I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^(5/2)\*c^2\*e-9\*(-(I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticF((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), (-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2))\*c^3\*d^3+7\*(-(I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticE((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), (-2\*I\*c

```

*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3-3*I*(-(I*(e*x+d)*c*e+(e*x+d)
)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2))*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2))*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(
1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+3*(-(I*(e*x+d)*c*e+(e*x+d)
)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2))*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2))*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(
1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3+2*I*(-(I*(e*x+d)*c*e+(e*x+d)
)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2))*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2))*((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^3*d^3+I*((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2)*(e*x+d)^(1/2)*c^2*d^2*e-6*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*
d^2-e^2)/(c^2*d^2+e^2))^(1/2))*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c
^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2))*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2+7*(-(I*(e*x+d)*c*
e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2))*((I*(e*x+d)*c*e-(e*x+d)*c
^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2))*((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e
^2-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e-((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c*d*e^2)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*
d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-
c*d)))

```

**maxima** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] Timed out
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) (d + ex)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^(3/2), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```



$$3.57 \quad \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=939

$$\frac{64b \sqrt{\frac{\sqrt{-c^2(d+ex)}}{\sqrt{-c^2 d+e}}} \sqrt{c^2 x^2 + 1} \Pi \left( 2; \sin^{-1} \left( \frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}} \right) \middle| \frac{2e}{\sqrt{-c^2 d+e}} \right) d^4}{35ce^4 \sqrt{1 + \frac{1}{c^2 x^2} x \sqrt{d+ex}}} - \frac{2\sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx)) d^3}{e^4} - \frac{64bc \sqrt{\frac{c^2(d+ex)}{c^2 d+e}}}{e^4}$$

[Out]  $2*d^2*(e*x+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^4 - 6/5*d*(e*x+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^4 + 2/7*(e*x+d)^{(7/2)}*(a+b*\operatorname{arccsch}(c*x))/e^4 - 2*d^3*(a+b*\operatorname{arccsch}(c*x))*(e*x+d)^{(1/2)}/e^4 + 4/35*b*(c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/e/(1+1/c^2/x^2)^{(1/2)} - 4/21*b*d*(c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/e^2/x/(1+1/c^2/x^2)^{(1/2)} + 64/35*b*d^4*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}, 2, 2^{(1/2)}*(e/(d*(-c^2)^{(1/2)+e}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)+e}))^{(1/2)}/c/e^4/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)} + 24/35*b*c*d^2*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)} + 4/105*b*c*(2*c^2*d^2+9*e^2)*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(5/2)}/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)} - 64/35*b*c*d^3*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)} - 32/105*b*c*d*(c^2*d^2+e^2)*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(5/2)}/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 2.87, antiderivative size = 939, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 17, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$ , Rules used = {43, 6310, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537, 833, 844, 942, 1654}

$$\frac{64b \sqrt{\frac{\sqrt{-c^2(d+ex)}}{\sqrt{-c^2 d+e}}} \sqrt{c^2 x^2 + 1} \Pi \left( 2; \sin^{-1} \left( \frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}} \right) \middle| \frac{2e}{\sqrt{-c^2 d+e}} \right) d^4}{35ce^4 \sqrt{1 + \frac{1}{c^2 x^2} x \sqrt{d+ex}}} - \frac{2\sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx)) d^3}{e^4} - \frac{64bc \sqrt{\frac{c^2(d+ex)}{c^2 d+e}}}{e^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^3*(a + b*\operatorname{ArcSch}[c*x]))/\operatorname{Sqrt}[d + e*x], x]$

[Out]  $(4*b*\operatorname{Sqrt}[d + e*x]*(1 + c^2*x^2))/(35*c^3*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]) - (4*b*d*\operatorname{Sqrt}[d + e*x]*(1 + c^2*x^2))/(21*c^3*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x) - (2*d^3*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcSch}[c*x]))/e^4 + (2*d^2*(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcSch}[c*x]))/e^4 - (6*d*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcSch}[c*x]))/(5*e^4) + (2*(d + e*x)^{(7/2)}*(a + b*\operatorname{ArcSch}[c*x]))/(7*e^4) + (24*b*c*d^2*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(35*(-c^2)^{(3/2)}*e^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]) + (4*b*c*(2*c^2*d^2 + 9*e^2)*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(105*(-c^2)^{(5/2)}*e^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]) - (64*b*c*d^3*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d -$

```

Sqrt[-c^2*e]))/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
) - (32*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)))/(105*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x
^2)]*x*Sqrt[d + e*x]) + (64*b*d^4*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
+ e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2
]], (2*e)/(Sqrt[-c^2]*d + e)))/(35*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x])

```

### Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

### Rule 43

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

```

### Rule 168

```

Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]

```

### Rule 419

```

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

```

### Rule 424

```

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

```

### Rule 537

```

Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_
)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])

```

### Rule 538

```

Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_
)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]

```

### Rule 719

```

Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(

```

$2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a]/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m)$ , Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

### Rule 833

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[(g\*(d + e\*x)^m\*(a + c\*x^2)^(p + 1))/(c\*(m + 2\*p + 2)), x] + Dist[1/(c\*(m + 2\*p + 2)), Int[(d + e\*x)^(m - 1)\*(a + c\*x^2)^p\*Simp[c\*d\*f\*(m + 2\*p + 2) - a\*e\*g\*m + c\*(e\*f\*(m + 2\*p + 2) + d\*g\*m)\*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[m, 0] && NeQ[m + 2\*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2\*m, 2\*p]) && !(IGtQ[m, 0] && EqQ[f, 0])

### Rule 844

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Dist[g/e, Int[(d + e\*x)^(m + 1)\*(a + c\*x^2)^p, x], x] + Dist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

### Rule 932

Int[1/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(f\_.) + (g\_.)\*(x\_)])\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

### Rule 942

Int((((d\_.) + (e\_.)\*(x\_))^(m\_)\*Sqrt[(f\_.) + (g\_.)\*(x\_)])/Sqrt[(a\_) + (c\_.)\*(x\_)^2], x\_Symbol] :> Simp[(2\*e\*(d + e\*x)^(m - 1)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(c\*(2\*m + 1)), x] - Dist[1/(c\*(2\*m + 1)), Int[((d + e\*x)^(m - 2)\*Simp[a\*e\*(d\*g + 2\*e\*f\*(m - 1)) - c\*d^2\*f\*(2\*m + 1) + (a\*e^2\*g\*(2\*m - 1) - c\*d\*(4\*e\*f\*m + d\*g\*(2\*m + 1)))\*x - c\*e\*(e\*f + d\*g\*(4\*m - 1))\*x^2, x])/Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[2\*m] && GtQ[m, 1]

### Rule 944

Int[Sqrt[(f\_.) + (g\_.)\*(x\_)]/(((d\_.) + (e\_.)\*(x\_))\*Sqrt[(a\_) + (c\_.)\*(x\_)^2]), x\_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x], x] + Dist[(e\*f - d\*g)/e, Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0]

### Rule 1654

Int[(Pq\_)\*((d\_) + (e\_.)\*(x\_))^(m\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(d + e\*x)^(m + q - 1)\*(a + c\*x^2)^(p + 1))/(c\*e^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(c\*e^q\*(m + q + 2\*p + 1)), Int[(d + e\*x)^m\*(a + c\*x^2)^p\*ExpandToSum[c\*e^q\*(m + q + 2\*p + 1)\*Pq - c\*f\*(m + q + 2\*p + 1)\*(d + e\*x)^q - f\*(d + e\*x)^(q - 2)\*(a\*e^2\*(m + q - 1) - c\*d^2\*(m + q + 2\*p + 1) - 2\*c\*d\*e\*(m + q + p)\*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[c\*d^2 + a\*e^2, 0] && !(EqQ[d, 0] && True) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +

1/2, 0]))

### Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*(u_), x_Symbol] := With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /;
FreeQ[{a, b, c}, x]
```

### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d+ex}} dx &= -\frac{2d^3 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \frac{2d^2 (d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{e^4} - \frac{6d(d+ex)}{e^4} \\
&= -\frac{2d^3 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \frac{2d^2 (d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{e^4} - \frac{6d(d+ex)}{e^4} \\
&= -\frac{2d^3 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \frac{2d^2 (d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{e^4} - \frac{6d(d+ex)}{e^4} \\
&= -\frac{2d^3 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \frac{2d^2 (d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{e^4} - \frac{6d(d+ex)}{e^4} \\
&= -\frac{2d^3 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \frac{2d^2 (d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{e^4} - \frac{6d(d+ex)}{e^4} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{8bd\sqrt{d+ex} (1+c^2x^2)}{35c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d^3\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{e^4} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex} (1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d^3\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{e^4} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex} (1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d^3\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{e^4} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex} (1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d^3\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{e^4} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex} (1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} - \frac{2d^3\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{e^4}
\end{aligned}$$

**Mathematica** [C] time = 14.30, size = 1098, normalized size = 1.17

$$\frac{a\sqrt{\frac{ex}{d}} + 1 B_{-\frac{ex}{d}}\left(4, \frac{1}{2}\right) d^4}{e^4 \sqrt{d+ex}} + \frac{b \left( 2\sqrt{\frac{d}{x}+e} \sqrt{cx} \left( \frac{\sqrt{2}(40c^3d^3e-8cde^3)\sqrt{icx+1}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}} F\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right)\right)\frac{icd+e}{2e}} \right) + i\sqrt{2}(cd-ie)(48c^4d^4-16c^2e^2d^2+9e^4)}{\sqrt{1+\frac{1}{c^2x^2}} \sqrt{\frac{d}{x}+e} (cx)^{3/2} \sqrt{\frac{e(1-icx)}{icd+e}}} \right)}{e^4 \sqrt{d+ex}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x], x]
[Out] (a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (b
*((-((c*(e + d/x)*x*((4*(-16*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)]))/(105*e^
3) + (32*c^3*d^3*ArcCsch[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsch[c*x])/(7*e) -
(4*c^2*x^2*(e*Sqrt[1 + 1/(c^2*x^2)] - 3*c*d*ArcCsch[c*x]))/(35*e^2) + (4*c*
x*(5*c*d*e*Sqrt[1 + 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsch[c*x]))/(105*e^3)))/Sq
rt[d + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*(-(Sqrt[2]*(40*c^3*d^3*e - 8*c*d
*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[A
rcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)))/(Sqrt[1 + 1/
(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e])) +
(I*Sqrt[2]*(c*d - I*e)*(48*c^4*d^4 - 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[1 + I*c*x
]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e,
ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)))/(e*Sqrt[1
+ 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-16*c^3*d^3*e + 9*c*d*e^3)*
Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (
2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))
/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellip
ticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*
e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e
)) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sq
rt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, Arc
Sin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)))/(2*Sqrt[-((e*
(I + c*x))/(c*d - I*e))]))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c
*x]*(2 + c^2*x^2)))/(105*e^4*Sqrt[d + e*x])))/c^4
```

**fricas** [F] time = 15.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3 \operatorname{arcsch}(cx) + ax^3}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2), x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)/sqrt(e*x + d), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x + d), x)
```

**maple [C]** time = 0.08, size = 2543, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e^4*(a*(1/7*(e*x+d)^(7/2)-3/5*d*(e*x+d)^(5/2)+d^2*(e*x+d)^(3/2)-d^3*(e*x+d)^(1/2))+b*(1/7*arccsch(c*x)*(e*x+d)^(7/2)-3/5*arccsch(c*x)*d*(e*x+d)^(5/2)+arccsch(c*x)*d^2*(e*x+d)^(3/2)-arccsch(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^4*(-40*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e-3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(7/2)*c^4*d-8*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^3*d^3*e+14*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^4*d^2+8*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^3-14*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(5/2)*c^3*d*e-8*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c*d*e^3-19*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^4*d^3+24*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4+16*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-48*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4+3*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c*e^3+8*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^4*d^4+19*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^3*d^2*e+48*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e-3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e^2-15*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2+7*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2+3*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(7/2)*c^3*e+8*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*c^2*d^2*e^2+9*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4-9*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4)/((e*x+d)^(1/2))
```

$2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^{(1/2)}/x/((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)/(I*e-c*d))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{35} a \left( \frac{5(ex+d)^{\frac{7}{2}}}{e^4} - \frac{21(ex+d)^{\frac{5}{2}}d}{e^4} + \frac{35(ex+d)^{\frac{3}{2}}d^2}{e^4} - \frac{35\sqrt{ex+d}d^3}{e^4} \right) + \frac{1}{35} b \left( \frac{2(5e^4x^4 - de^3x^3 + 2d^2e^2x^2 - 8d^3ex)}{\sqrt{ex+d}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x+d)^(1/2), x, algorithm="maxima")

[Out]  $\frac{2}{35} a (5(e^4 x^4 - d e^3 x^3 + 2 d^2 e^2 x^2 - 8 d^3 e x) \sqrt{e x + d} + 35 e^4 x^4 - 21 e^3 x^3 d + 35 e^2 x^2 d^2 - 35 e d^3 + 35 \sqrt{e x + d} d^3) + \frac{1}{35} b (2(5 e^4 x^4 - d e^3 x^3 + 2 d^2 e^2 x^2 - 8 d^3 e x) \sqrt{e x + d} + 35 e^4 x^4 - 21 e^3 x^3 d + 35 e^2 x^2 d^2 - 35 e d^3 + 35 \sqrt{e x + d} d^3) / ((c^2 e^4 x^2 + e^4) \sqrt{e x + d}) + 35 \int (2/35 (5 c^2 e^4 x^5 - c^2 d e^3 x^4 + 2 c^2 d^2 e^2 x^3 - 8 c^2 d^3 e x^2 - 16 c^2 d^4 x) / ((c^2 e^4 x^2 + e^4) \sqrt{e x + d}) + (c^2 e^4 x^2 + e^4) \sqrt{e x + d}), x) - 35 \int (-1/35 (2 c^2 d e^3 x^4 + 16 c^2 d^3 e x^2 - 5 (7 e^4 \log(c) + 2 e^4) c^2 x^5 + 32 c^2 d^4 x - (4 c^2 d^2 e^2 + 35 e^4 \log(c)) x^3 - 35 (c^2 e^4 x^5 + e^4 x^3) \log(x)) / ((c^2 e^4 x^2 + e^4) \sqrt{e x + d}), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{c x} \right) \right)}{\sqrt{d + e x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(1/2), x)

[Out] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acsch}(c x))}{\sqrt{d + e x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsch(c\*x))/(e\*x+d)\*\*(1/2), x)

[Out] Integral(x\*\*3\*(a + b\*acsch(c\*x))/sqrt(d + e\*x), x)



$$3.58 \quad \int \frac{x^2(a+b\operatorname{csch}^{-1}(cx))}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=707

$$\frac{2d^2\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e^3} - \frac{32bd^3\sqrt{c^2x^2}}{e^3}$$

[Out]  $-4/3*d*(e*x+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^{3+2/5}*(e*x+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^{3+2*d^2}*(a+b*\operatorname{arccsch}(c*x))*(e*x+d)^{(1/2)}/e^{3+4/15}*b*(c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/e/x/(1+1/c^2/x^2)^{(1/2)}-32/15*b*d^3*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},2,2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/5*b*c*d*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}+32/15*b*c*d^2*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/15*b*c*(c^2*d^2+e^2)*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(5/2)}/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 2.14, antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {43, 6310, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537, 833, 844}

$$\frac{2d^2\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e^3} + \frac{32bcd^2\sqrt{c^2x^2}}{e^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^2*(a + b*\operatorname{ArcCsCh}[c*x]))/\operatorname{Sqrt}[d + e*x], x]$

[Out]  $(4*b*\operatorname{Sqrt}[d + e*x]*(1 + c^2*x^2))/(15*c^3*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x) + (2*d^2*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcCsCh}[c*x]))/e^3 - (4*d*(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcCsCh}[c*x]))/(3*e^3) + (2*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcCsCh}[c*x]))/(5*e^3) - (4*b*c*d*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(5*(-c^2)^{(3/2)}*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]) + (32*b*c*d^2*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(3/2)}*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) + (4*b*c*(c^2*d^2 + e^2)*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(5/2)}*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (32*b*d^3*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e)))/(15*c*e^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

#### Rule 168

`Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]], x], x, Sqrt[c + d*x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]`

#### Rule 419

`Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])]`

#### Rule 424

`Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]`

#### Rule 537

`Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])]`

#### Rule 538

`Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]`

#### Rule 719

`Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]`

#### Rule 833

`Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)`

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x]
/; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

#### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

#### Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

#### Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] := Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
0]
```

#### Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] := With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /;
FreeQ[{a, b, c}, x]
```

#### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d+ex}} dx &= \frac{2d^2 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
&= \frac{2d^2 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
&= \frac{2d^2 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
&= \frac{2d^2 \sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{2(d+ex)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}x}} + \frac{2d^2\sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}x}} + \frac{2d^2\sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}x}} + \frac{2d^2\sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}x}} + \frac{2d^2\sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} \\
&= \frac{4b\sqrt{d+ex} (1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}x}} + \frac{2d^2\sqrt{d+ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4d(d+ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3}
\end{aligned}$$

**Mathematica [C]** time = 14.43, size = 1012, normalized size = 1.43

$$b \frac{c \left( \frac{d}{x} + e \right) x \left( -\frac{16c^2 \operatorname{csch}^{-1}(cx) d^2}{15e^3} + \frac{4c \sqrt{1 + \frac{1}{c^2 x^2}} d}{5e^2} - \frac{2c^2 x^2 \operatorname{csch}^{-1}(cx)}{5e} - \frac{4cx \left( e \sqrt{1 + \frac{1}{c^2 x^2}} - 2cd \operatorname{csch}^{-1}(cx) \right)}{15e^2} \right)}{\sqrt{d+ex}} - \frac{2\sqrt{\frac{d}{x}+e} \sqrt{cx} \left( -\frac{\sqrt{2}(7c^2 d^2 e - e^3) \sqrt{icx+1} (cx+i) \sqrt{\frac{cd+e}{cd-}}}{\sqrt{1+\frac{1}{c^2 x^2}} \sqrt{\frac{d}{x}+e}} \right)}{\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x],x]

[Out] -((a\*d^3\*Sqrt[1 + (e\*x)/d]\*Beta[-((e\*x)/d), 3, 1/2])/(e^3\*Sqrt[d + e\*x])) + (b\*(-((c\*(e + d/x)\*x\*((4\*c\*d\*Sqrt[1 + 1/(c^2\*x^2)])/(5\*e^2) - (16\*c^2\*d^2\*ArcCsch[c\*x])/(15\*e^3) - (2\*c^2\*x^2\*ArcCsch[c\*x])/(5\*e) - (4\*c\*x\*(e\*Sqrt[1 + 1/(c^2\*x^2)] - 2\*c\*d\*ArcCsch[c\*x]))/(15\*e^2)))/Sqrt[d + e\*x]) - (2\*Sqrt[e + d/x]\*Sqrt[c\*x]\*(-(Sqrt[2]\*(7\*c^2\*d^2\*e - e^3)\*Sqrt[1 + I\*c\*x]\*(I + c\*x)\*Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]\*EllipticF[ArcSin[Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]]], (I\*c\*d + e)/(2\*e)))/(Sqrt[1 + 1/(c^2\*x^2)]\*Sqrt[e + d/x]\*(c\*x)^(3/2)\*Sqrt[(e\*(1 - I\*c\*x))/(I\*c\*d + e)])) + (I\*Sqrt[2]\*(c\*d - I\*e)\*(8\*c^3\*d^3 - 3\*c\*d\*e^2)\*Sqrt[1 + I\*c\*x]\*Sqrt[(e\*(I + c\*x)\*(c\*d + c\*e\*x))/(I\*c\*d + e)^2]\*EllipticPi[1 + (I\*c\*d)/e, ArcSin[Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]]], (I\*c\*d + e)/(2\*e)))/(e\*Sqrt[1 + 1/(c^2\*x^2)]\*Sqrt[e + d/x]\*(c\*x)^(3/2)) + (6\*c\*d\*e\*Cosh[2\*ArcCsch[c\*x]]\*(-((c\*d + c\*e\*x)\*(1 + c^2\*x^2)) + (c\*x\*(c\*d\*Sqrt[2 + (2\*I)\*c\*x]\*(I + c\*x)\*Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]\*EllipticF[ArcSin[Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]]], (I\*c\*d + e)/(2\*e)] + 2\*Sqrt[-((e\*(-I + c\*x))/(c\*d + I\*e))]\*(I + c\*x)\*Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]\*((c\*d + I\*e)\*EllipticE[ArcSin[Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]], (c\*d - I\*e)/(c\*d + I\*e)] - I\*e\*EllipticF[ArcSin[Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]], (c\*d - I\*e)/(c\*d + I\*e)] + (I\*c\*d + e)\*Sqrt[2 + (2\*I)\*c\*x]\*Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]\*Sqrt[(e\*(I + c\*x)\*(c\*d + c\*e\*x))/(I\*c\*d + e)^2]\*EllipticPi[1 + (I\*c\*d)/e, ArcSin[Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]]], (I\*c\*d + e)/(2\*e)))/(2\*Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]))/(Sqrt[1 + 1/(c^2\*x^2)]\*Sqrt[e + d/x]\*Sqrt[c\*x]\*(2 + c^2\*x^2)))/(15\*e^3\*Sqrt[d + e\*x]))/c^3

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2/sqrt(e\*x + d), x)

**maple** [C] time = 0.07, size = 1991, normalized size = 2.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(1/2),x)

[Out] 
$$\frac{2}{e^3} \left( \frac{3(ex+d)^{\frac{5}{2}}}{e^3} - \frac{10(ex+d)^{\frac{3}{2}}d}{e^3} + \frac{15\sqrt{ex+d}d^2}{e^3} \right) + \frac{1}{15} b \left( \frac{2(3e^3x^3 - de^2x^2 + 4d^2ex + 8d^3) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex+d}e^3} \right)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

[Out] 
$$\frac{2}{15} a \left( \frac{3(ex+d)^{\frac{5}{2}}}{e^3} - \frac{10(ex+d)^{\frac{3}{2}}d}{e^3} + \frac{15\sqrt{ex+d}d^2}{e^3} \right) + \frac{1}{15} b \left( \frac{2(3e^3x^3 - de^2x^2 + 4d^2ex + 8d^3) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex+d}e^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(1/2),x, algorithm="maxima")

```
[Out] 2/15*a*(3*(e*x + d)^(5/2)/e^3 - 10*(e*x + d)^(3/2)*d/e^3 + 15*sqrt(e*x + d)
*d^2/e^3) + 1/15*b*(2*(3*e^3*x^3 - d*e^2*x^2 + 4*d^2*e*x + 8*d^3)*log(sqrt(
c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^3) + 15*integrate(2/15*(3*c^2*e^3*x^4 -
c^2*d*e^2*x^3 + 4*c^2*d^2*e*x^2 + 8*c^2*d^3*x)/((c^2*e^3*x^2 + e^3)*sqrt(c^
2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3*x^2 + e^3)*sqrt(e*x + d)), x) - 15*inte
grate(-1/15*(2*c^2*d*e^2*x^3 - 3*(5*e^3*log(c) + 2*e^3)*c^2*x^4 - 16*c^2*d^
3*x - (8*c^2*d^2*e + 15*e^3*log(c))*x^2 - 15*(c^2*e^3*x^4 + e^3*x^2)*log(x)
)/((c^2*e^3*x^2 + e^3)*sqrt(e*x + d)), x))
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

```
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acsch}(cx))}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(1/2), x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

$$3.59 \quad \int \frac{x(a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=474

$$-\frac{2d\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{8bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}}}{\sqrt{2}}\right)\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

[Out]  $2/3*(e*x+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^2-2*d*(a+b*\operatorname{arccsch}(c*x))*(e*x+d)^{(1/2)}/e^2+8/3*b*d^2*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/3*b*c*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/e/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}-8/3*b*c*d*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 1.75, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$ , Rules used = {43, 6310, 12, 6721, 6742, 719, 424, 944, 419, 932, 168, 538, 537}

$$-\frac{2d\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{8bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}}}{\sqrt{2}}\right)\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x], x]

[Out]  $(-2*d*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcCsch}[c*x]))/e^2 + (2*(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(3*e^2) + (4*b*c*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e))]/(3*(-c^2)^{(3/2)}*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]) - (8*b*c*d*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e))]/(3*(-c^2)^{(3/2)}*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) + (8*b*d^2*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e)]/(3*c*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 168



```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

#### Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

#### Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

#### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])
```

#### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

#### Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

#### Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

#### Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

#### Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] := With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /;
FreeQ[{a, b, c}, x]
```

#### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b\operatorname{csch}^{-1}(cx))}{\sqrt{d+ex}} dx &= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{b \int \frac{2(-2d+ex)\sqrt{d+ex}}{3e^2\sqrt{1+\frac{1}{c^2x}}} dx}{c} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(2b) \int \frac{(-2d+ex)\sqrt{d+ex}}{\sqrt{1+\frac{1}{c^2x}}} dx}{3ce^2} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(2b\sqrt{1+c^2x^2})}{3ce^2} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(2b\sqrt{1+c^2x^2})}{3ce^2} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} - \frac{(4bd\sqrt{1+c^2x^2})}{3ce^2\sqrt{1+c^2x^2}} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} - \frac{(4bd^2\sqrt{1+c^2x^2})}{3ce^2\sqrt{1+c^2x^2}} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{4b\sqrt{-c^2}\sqrt{d+ex}}{3ce^2} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{4b\sqrt{-c^2}\sqrt{d+ex}}{3ce^2} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{4b\sqrt{-c^2}\sqrt{d+ex}}{3ce^2} \\
&= -\frac{2d\sqrt{d+ex}(a + b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2(d+ex)^{3/2}(a + b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{4b\sqrt{-c^2}\sqrt{d+ex}}{3ce^2}
\end{aligned}$$

**Mathematica [C]** time = 1.34, size = 343, normalized size = 0.72

$$2 \left( a \sqrt{d+ex} (ex-2d) + \frac{2b \sqrt{-\frac{e(cx-i)}{cd+ie}} \sqrt{-\frac{e(cx+i)}{cd-ie}} \left( (e+icd) F \left( i \sinh^{-1} \left( \sqrt{-\frac{c}{cd-ie}} \sqrt{d+ex} \right) \middle| \frac{cd-ie}{cd+ie} \right) + (-e+icd) E \left( i \sinh^{-1} \left( \sqrt{-\frac{c}{cd-ie}} \sqrt{d+ex} \right) \middle| \frac{cd-ie}{cd+ie} \right) - 2ic \right)}{c^2 x \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{-\frac{c}{cd-ie}}} \right) \frac{1}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x], x]
```

```
[Out] (2*(a*(-2*d + e*x)*Sqrt[d + e*x] + b*(-2*d + e*x)*Sqrt[d + e*x]*ArcCsch[c*x] + (2*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(I*c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + (I*c*d + e)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] - (2*I)*c*d*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^2*Sqrt[-(c/(c*d - I*e))]*Sqrt[1 + 1/(c^2*x^2)]*x))/(3*e^2)
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2), x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{\sqrt{ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2), x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/sqrt(e*x + d), x)
```

**maple [C]** time = 0.07, size = 868, normalized size = 1.83

$$2a \left( \frac{(ex+d)^{\frac{3}{2}}}{3} - d\sqrt{ex+d} \right) + 2b \left( \frac{(ex+d)^{\frac{3}{2}} \operatorname{arccsch}(cx)}{3} - \operatorname{arccsch}(cx) d\sqrt{ex+d} - \frac{2\sqrt{-\frac{i(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}} \sqrt{\frac{i(ex+d)ce-(ex+d)c^2}{c^2d^2+e^2}}}{c^2d^2+e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2), x)
```

```
[Out] 2/e^2*(a*(1/3*(e*x+d)^(3/2)-d*(e*x+d)^(1/2))+b*(1/3*(e*x+d)^(3/2)*arccsch(c*x)-arccsch(c*x)*d*(e*x+d)^(1/2)-2/3/c^2*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(2*I*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2), (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2), (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2), (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2-2*I*Ellipt
```

icPi((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), 1/(I\*e+c\*d)/c\*(c^2\*d^2+e^2)/d, (-I\*e-c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2))\*c\*d\*e+2\*EllipticPi((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), 1/(I\*e+c\*d)/c\*(c^2\*d^2+e^2)/d, (-I\*e-c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2))\*c^2\*d^2+EllipticF((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), (-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2))\*e^2-EllipticE((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), (-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2))\*e^2)/(((e\*x+d)^2\*c^2-2\*(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/(I\*e-c\*d)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3} a \left( \frac{(ex+d)^{\frac{3}{2}}}{e^2} - \frac{3\sqrt{ex+d}d}{e^2} \right) + \frac{1}{3} b \left( \frac{2(e^2x^2 - dex - 2d^2) \log(\sqrt{c^2x^2+1} + 1)}{\sqrt{ex+d}e^2} + 3 \int \frac{2(c^2e^2x^3 + e^2)\sqrt{c^2x^2+1}}{(c^2e^2x^2 + e^2)\sqrt{c^2x^2+1}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(1/2), x, algorithm="maxima")

[Out] 2/3\*a\*((e\*x + d)^(3/2)/e^2 - 3\*sqrt(e\*x + d)\*d/e^2) + 1/3\*b\*(2\*(e^2\*x^2 - d\*e\*x - 2\*d^2)\*log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x + d)\*e^2) + 3\*integrate(2/3\*(c^2\*e^2\*x^3 - c^2\*d\*e\*x^2 - 2\*c^2\*d^2\*x)/((c^2\*e^2\*x^2 + e^2)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x + d) + (c^2\*e^2\*x^2 + e^2)\*sqrt(e\*x + d)), x) - 3\*integrate(-1/3\*(2\*c^2\*d\*e\*x^2 - (3\*e^2\*log(c) + 2\*e^2)\*c^2\*x^3 + (4\*c^2\*d^2 - 3\*e^2\*log(c))\*x - 3\*(c^2\*e^2\*x^3 + e^2\*x)\*log(x))/((c^2\*e^2\*x^2 + e^2)\*sqrt(e\*x + d)), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(1/2), x)

[Out] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{acsch}(cx))}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))/(e\*x+d)\*\*(1/2), x)

[Out] Integral(x\*(a + b\*acsch(c\*x))/sqrt(d + e\*x), x)

$$3.60 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=284

$$\frac{2\sqrt{d+ex} (a + b\operatorname{csch}^{-1}(cx))}{e} + \frac{4bc\sqrt{c^2x^2+1} \sqrt{\frac{d+ex}{\sqrt{-c^2}+d}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2} x \sqrt{\frac{1}{c^2x^2}+1} \sqrt{d+ex}} - \frac{4bd\sqrt{c^2x^2+1} \sqrt{\frac{\sqrt{-c^2}}{\sqrt{-c^2}}}}{cex}$$

[Out] 2\*(a+b\*arccsch(c\*x))\*(e\*x+d)^(1/2)/e+4\*b\*c\*EllipticF(1/2\*(1-(-c^2)^(1/2)\*x)^(1/2)\*2^(1/2),(-2\*e\*(-c^2)^(1/2)/(c^2\*d-e\*(-c^2)^(1/2)))^(1/2))\*(c^2\*x^2+1)^(1/2)\*((e\*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/(e\*x+d)^(1/2)-4\*b\*d\*EllipticPi(1/2\*(1-(-c^2)^(1/2)\*x)^(1/2)\*2^(1/2),2,2^(1/2)\*(e/(d\*(-c^2)^(1/2)+e))^(1/2))\*(c^2\*x^2+1)^(1/2)\*((e\*x+d)\*(-c^2)^(1/2)/(d\*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e\*x+d)^(1/2)

**Rubi [A]** time = 0.41, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$ , Rules used = {6290, 1574, 944, 719, 419, 933, 168, 538, 537}

$$\frac{2\sqrt{d+ex} (a + b\operatorname{csch}^{-1}(cx))}{e} + \frac{4bc\sqrt{c^2x^2+1} \sqrt{\frac{d+ex}{\sqrt{-c^2}+d}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2} x \sqrt{\frac{1}{c^2x^2}+1} \sqrt{d+ex}} - \frac{4bd\sqrt{c^2x^2+1} \sqrt{\frac{\sqrt{-c^2}}{\sqrt{-c^2}}}}{cex}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/Sqrt[d + e\*x],x]

[Out] (2\*Sqrt[d + e\*x]\*(a + b\*ArcCsch[c\*x]))/e + (4\*b\*c\*Sqrt[(d + e\*x)/(d + e/Sqrt[-c^2]])\*Sqrt[1 + c^2\*x^2]\*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]\*x]/Sqrt[2]], (-2\*Sqrt[-c^2]\*e)/(c^2\*d - Sqrt[-c^2]\*e)]/((-c^2)^(3/2)\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x]) - (4\*b\*d\*Sqrt[(Sqrt[-c^2]\*(d + e\*x))/(Sqrt[-c^2]\*d + e)]\*Sqrt[1 + c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]\*x]/Sqrt[2]], (2\*e)/(Sqrt[-c^2]\*d + e)]/(c\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x])

#### Rule 168

Int[1/(((a\_) + (b\_)\*(x\_))\*Sqrt[(c\_) + (d\_)\*(x\_)]\*Sqrt[(e\_) + (f\_)\*(x\_)]\*Sqrt[(g\_) + (h\_)\*(x\_)]), x\_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b\*c - a\*d - b\*x^2, x]\*Sqrt[Simp[(d\*e - c\*f)/d + (f\*x^2)/d, x]]\*Sqrt[Simp[(d\*g - c\*h)/d + (h\*x^2)/d, x]]), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d\*e - c\*f)/d, 0]

#### Rule 419

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)^2]\*Sqrt[(c\_) + (d\_)\*(x\_)^2]), x\_Symbol] :> Simp[(1\*EllipticF[ArcSin[Rt[-(d/c), 2]\*x], (b\*c)/(a\*d)]/(Sqrt[a]\*Sqrt[c]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

#### Rule 537

Int[1/(((a\_) + (b\_)\*(x\_)^2)\*Sqrt[(c\_) + (d\_)\*(x\_)^2]\*Sqrt[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] :> Simp[(1\*EllipticPi[(b\*c)/(a\*d), ArcSin[Rt[-(d/c), 2]\*x], (c\*f)/(d\*e)]/(a\*Sqrt[c]\*Sqrt[e]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_)*(x_)^m)/Sqrt[(a_) + (c_)*(x_)^2], x_Symbol] := Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 933

```
Int[1/(((d_) + (e_)*(x_)^m)*Sqrt[(f_) + (g_)*(x_)^n]*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]
```

Rule 944

```
Int[Sqrt[(f_) + (g_)*(x_)^m]/(((d_) + (e_)*(x_)^n)*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1574

```
Int[(x_)^m*((a_) + (c_)*(x_)^mn)^p*((d_) + (e_)*(x_)^n)^q, x_Symbol] := Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

Rule 6290

```
Int[((a_) + ArcCsch[(c_)*(x_)^m]*(b_))*((d_) + (e_)*(x_)^m), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x])/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{d + ex}} dx &= \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{(2b) \int \frac{\sqrt{d+ex}}{\sqrt{1 + \frac{1}{c^2 x^2}} x^2} dx}{ce} \\
 &= \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{\left(2b\sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{\sqrt{d+ex}}{x\sqrt{\frac{1}{c^2} + x^2}} dx}{ce\sqrt{1 + \frac{1}{c^2 x^2}} x} \\
 &= \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{\left(2b\sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2} + x^2}} dx}{c\sqrt{1 + \frac{1}{c^2 x^2}} x} + \frac{\left(2bd\sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{x\sqrt{1 - \sqrt{-c^2} x}\sqrt{1 + \sqrt{-c^2} x}\sqrt{d+ex}} dx}{ce\sqrt{1 + \frac{1}{c^2 x^2}} x} \\
 &= \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{\left(2bd\sqrt{1 + c^2 x^2}\right) \int \frac{1}{x\sqrt{1 - \sqrt{-c^2} x}\sqrt{1 + \sqrt{-c^2} x}\sqrt{d+ex}} dx}{ce\sqrt{1 + \frac{1}{c^2 x^2}} x} + \frac{4b\sqrt{-c^2}}{c^3\sqrt{1 + \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \left(\sin^{-1}\left(\frac{\sqrt{1 - \sqrt{-c^2} x}}{\sqrt{2}}\right) - \frac{2}{c^2 d}\right) \\
 &= \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{4b\sqrt{-c^2}}{c^3\sqrt{1 + \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \left(\sin^{-1}\left(\frac{\sqrt{1 - \sqrt{-c^2} x}}{\sqrt{2}}\right) - \frac{2}{c^2 d}\right) \\
 &= \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{4b\sqrt{-c^2}}{c^3\sqrt{1 + \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \left(\sin^{-1}\left(\frac{\sqrt{1 - \sqrt{-c^2} x}}{\sqrt{2}}\right) - \frac{2}{c^2 d}\right) \\
 &= \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{4b\sqrt{-c^2}}{c^3\sqrt{1 + \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \left(\sin^{-1}\left(\frac{\sqrt{1 - \sqrt{-c^2} x}}{\sqrt{2}}\right) - \frac{2}{c^2 d}\right)
 \end{aligned}$$

**Mathematica [C]** time = 5.22, size = 307, normalized size = 1.08

$$\frac{2 \left( a e (d + e x) - \frac{b \left( \frac{d}{x} + e \right) \left( -c e x \operatorname{csch}^{-1}(c x) + \frac{\sqrt{2} \sqrt{1 + i c x} \left( c d (e + i c d) \sqrt{-\frac{e (c x + i)}{c d - i e}} \sqrt{\frac{c e (c x + i) (d + e x)}{(e + i c d)^2}} \Pi \left( \frac{i c d}{e} + 1; \sin^{-1} \left( \sqrt{-\frac{e (c x + i)}{c d - i e}} \right) \middle| \frac{i c d + e}{2 e} \right) - e^{2 (c x + i)} \sqrt{\frac{c (d + e x)}{c d - i e}} F \left( \sin^{-1} \left( \sqrt{\frac{1 - \sqrt{-c^2} x}}{\sqrt{2}} \right) \middle| -\frac{2}{c^2 d} \right) \right)}{\sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{-\frac{e (c x + i)}{c d - i e}} (c d + c e x)} \right)}{e^2 \sqrt{d + e x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x], x]
```

```
[Out] (2*(a*e*(d + e*x) - (b*(e + d/x)*(-(c*e*x*ArcCsch[c*x]) + (Sqrt[2]*Sqrt[1 + I*c*x]*(-(e^2*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e))) + c*d*(I*c*d + e)*S
```



```

qrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e
)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)]/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[-((e*(I + c*x))/(c*d - I*e
))]*(c*d + c*e*x))/c)/(e^2*Sqrt[d + e*x])

```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

[Out] integrate((b\*arccsch(c\*x) + a)/sqrt(e\*x + d), x)

**maple** [C] time = 0.07, size = 395, normalized size = 1.39

$$2a\sqrt{ex + d} + 2b \left( \sqrt{ex + d} \operatorname{arcsch}(cx) + \frac{2\sqrt{-\frac{i(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}} \sqrt{\frac{i(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}} \left( \operatorname{EllipticF}\left(\sqrt{ex+d}, \sqrt{\frac{(cd+ie)}{c^2d^2+e^2}}\right) \right)}{c\sqrt{\frac{(ex+d)^2c^2-2(ex+d)c^2d+c^2d^2}{c^2x^2e^2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(1/2),x)
```

```
[Out] 2/e*(a*(e*x+d)^(1/2)+b*((e*x+d)^(1/2)*arccsch(c*x)+2/c*(-(I*(e*x+d)*c*e+(e*
x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+
c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2), (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-EllipticP
i((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2), 1/(I*e+c*d)/c*(c^2*d^2+e^
2)/d, (-I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))
/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \left( \frac{2\sqrt{ex + d} \log\left(\sqrt{c^2x^2 + 1} + 1\right)}{e} + \int \frac{2(c^2ex^2 + c^2dx)}{(c^2ex^2 + e)\sqrt{c^2x^2 + 1}\sqrt{ex + d} + (c^2ex^2 + e)\sqrt{ex + d}} dx - \int \frac{e \log(c)}{e} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*(2*sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/e + integrate(2*(c^2*e*x^2 +
c^2*d*x)/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e*x^2 + e)
*sqrt(e*x + d)), x) - integrate(((e*log(c) + 2*e)*c^2*x^2 + 2*c^2*d*x + e*1
```

$\log(c) + (c^2 e x^2 + e) \log(x) / ((c^2 e x^2 + e) \sqrt{e x + d}), x) + 2 \sqrt{e x + d} a / e$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{c x}\right)}{\sqrt{d + e x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))/(d + e*x)^(1/2), x)`

[Out] `int((a + b*asinh(1/(c*x)))/(d + e*x)^(1/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(c x)}{\sqrt{d + e x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))/(e*x+d)**(1/2), x)`

[Out] `Integral((a + b*acsch(c*x))/sqrt(d + e*x), x)`

$$3.61 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x \sqrt{d+ex}} dx$$

**Optimal.** Leaf size=24

$$\operatorname{Int}\left(\frac{a + b \operatorname{csch}^{-1}(cx)}{x \sqrt{d+ex}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x/(e\*x+d)^(1/2), x)

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x \sqrt{d+ex}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[d + e\*x]), x]

[Out] Defer[Int][(a + b\*ArcCsch[c\*x])/(x\*Sqrt[d + e\*x]), x]

Rubi steps

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x \sqrt{d+ex}} dx = \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x \sqrt{d+ex}} dx$$

**Mathematica [A]** time = 6.55, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x \sqrt{d+ex}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[d + e\*x]), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[d + e\*x]), x]

**fricas [A]** time = 1.12, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arcsch}(cx) + a)}{ex^2 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/(e\*x^2 + d\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex+d} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(sqrt(e\*x + d)\*x), x)

**maple** [A] time = 1.53, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x\sqrt{ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x+d)^(1/2),x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x+d)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\left( \frac{\log(c) \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{\sqrt{d}} + \int \frac{\log(x)}{\sqrt{ex+d}x} dx - \int \frac{\log\left(\sqrt{c^2x^2+1}+1\right)}{\sqrt{ex+d}x} dx \right) b + \frac{a \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] -(log(c)\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/sqrt(d) + integrate(log(x)/(sqrt(e\*x + d)\*x), x) - integrate(log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x + d)\*x), x))\*b + a\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/sqrt(d)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x\sqrt{d+ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x)^(1/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x\sqrt{d+ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(e\*x+d)\*\*(1/2),x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*sqrt(d + e\*x)), x)

$$3.62 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 \sqrt{d+ex}} dx$$

**Optimal.** Leaf size=24

$$\operatorname{Int}\left(\frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 \sqrt{d+ex}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(1/2), x)

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 \sqrt{d+ex}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

[Out] Defer[Int][(a + b\*ArcCsch[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

Rubi steps

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 \sqrt{d+ex}} dx = \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 \sqrt{d+ex}} dx$$

**Mathematica [A]** time = 9.21, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 \sqrt{d+ex}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*Sqrt[d + e\*x]), x]

**fricas [A]** time = 0.83, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arcsch}(cx) + a)}{ex^3 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/(e\*x^3 + d\*x^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex+d} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(sqrt(e\*x + d)\*x^2), x)

**maple** [A] time = 5.26, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 \sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(1/2),x)

[Out] int((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( \left( \frac{2\sqrt{ex+d}e}{(ex+d)d-d^2} + \frac{e \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right) \log(c) - 2 \int \frac{\log(x)}{\sqrt{ex+d}x^2} dx + 2 \int \frac{\log\left(\sqrt{c^2x^2+1}+1\right)}{\sqrt{ex+d}x^2} dx \right) b - \frac{1}{2} a \left( \frac{2\sqrt{ex+d}}{(ex+d)d-d^2} + \frac{e \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(1/2),x, algorithm="maxima")

[Out] 1/2\*((2\*sqrt(e\*x + d)\*e/((e\*x + d)\*d - d^2) + e\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(3/2))\*log(c) - 2\*integrate(log(x)/(sqrt(e\*x + d)\*x^2), x) + 2\*integrate(log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x + d)\*x^2), x))\*b - 1/2\*a\*(2\*sqrt(e\*x + d)\*e/((e\*x + d)\*d - d^2) + e\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(3/2))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2 \sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x)^(1/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^2 \sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*2/(e\*x+d)\*\*(1/2),x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*\*2\*sqrt(d + e\*x)), x)

$$3.63 \quad \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex)^{3/2}} dx$$

Optimal. Leaf size=731

$$\frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \frac{2(d + ex)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^4}$$

[Out]  $-2*d*(e*x+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^4+2/5*(e*x+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^4+2*d^3*(a+b*\operatorname{arccsch}(c*x))/e^4/(e*x+d)^{(1/2)}+6*d^2*(a+b*\operatorname{arccsch}(c*x))*(e*x+d)^{(1/2)}/e^4+4/15*b*(c^2*x^2+1)*(e*x+d)^{(1/2)}/c^3/e^2/x/(1+1/c^2/x^2)^{(1/2)}-64/5*b*d^3*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},2,2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^4/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-32/15*b*c*d*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}+8*b*c*d^2*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/15*b*c*(2*c^2*d^2-e^2)*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(5/2)}/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

Rubi [A] time = 2.64, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {43, 6310, 12, 6721, 6742, 719, 419, 932, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} + \frac{6d^2 \sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} - \frac{2d(d + ex)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \frac{2(d + ex)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^3*(a + b*\operatorname{ArcCsch}[c*x]))/(d + e*x)^{(3/2)}, x]$

[Out]  $(4*b*\operatorname{Sqrt}[d + e*x]*(1 + c^2*x^2))/(15*c^3*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x) + (2*d^3*(a + b*\operatorname{ArcCsch}[c*x]))/(e^4*\operatorname{Sqrt}[d + e*x]) + (6*d^2*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcCsch}[c*x]))/e^4 - (2*d*(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/e^4 + (2*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(5*e^4) - (32*b*c*d*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(3/2)}*e^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]) + (8*b*c*d^2*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/((-c^2)^{(3/2)}*e^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (4*b*c*(2*c^2*d^2 - e^2)*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(15*(-c^2)^{(5/2)}*e^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (64*b*d^3*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e)))/(5*c*e^4*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

#### Rule 168

`Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]], x], x, Sqrt[c + d*x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]`

#### Rule 419

`Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])`

#### Rule 424

`Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]`

#### Rule 537

`Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])`

#### Rule 538

`Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]`

#### Rule 719

`Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]`

#### Rule 844

`Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D`



ist[(e\*f - d\*g)/e, Int[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && !IGtQ[m, 0]

#### Rule 931

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)/(Sqrt[(f\_.) + (g\_.)\*(x\_.)]\*Sqrt[(a\_.) + (c\_.)\*(x\_.)^2]), x\_Symbol] := Simp[(2\*e^2\*(d + e\*x)^(m - 2)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]/(c\*g\*(2\*m - 1)), x] - Dist[1/(c\*g\*(2\*m - 1)), Int[((d + e\*x)^(m - 3))\*Simp[a\*e^2\*(d\*g + 2\*e\*f\*(m - 2)) - c\*d^3\*g\*(2\*m - 1) + e\*(e\*(a\*e\*g\*(2\*m - 3)) + c\*d\*(2\*e\*f - 3\*d\*g\*(2\*m - 1)))\*x + 2\*e^2\*(c\*e\*f - 3\*c\*d\*g)\*(m - 1)\*x^2, x]]/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[2\*m] && GeQ[m, 2]

#### Rule 932

Int[1/(((d\_.) + (e\_.)\*(x\_.))\*Sqrt[(f\_.) + (g\_.)\*(x\_.)]\*Sqrt[(a\_.) + (c\_.)\*(x\_.)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

#### Rule 1584

Int[(u\_.)\*(x\_.)^(m\_.)\*((a\_.)\*(x\_.)^(p\_.) + (b\_.)\*(x\_.)^(q\_.))^(n\_.), x\_Symbol] := Int[u\*x^(m + n\*p)\*(a + b\*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

#### Rule 6310

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))\*(u\_), x\_Symbol] := With[{v = IntHid e[u, x]}, Dist[a + b\*ArcCsch[c\*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2\*Sqrt[1 + 1/(c^2\*x^2)])], x], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]

#### Rule 6721

Int[(u\_.)\*((a\_.) + (b\_.)\*(x\_.)^(n\_.))^(p\_.), x\_Symbol] := Dist[(b^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(x^(n\*FracPart[p])\*(1 + a/(x^n\*b))^FracPart[p]), Int[u\*x^(n\*p)\*(1 + a/(x^n\*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rubi steps



**Mathematica [C]** time = 14.70, size = 1042, normalized size = 1.43

$$\frac{a \left(\frac{ex}{d} + 1\right)^{3/2} B_{-\frac{ex}{d}}\left(4, -\frac{1}{2}\right) d^4}{e^4 (d + ex)^{3/2}} + \frac{b \left( \frac{c^2 \left(\frac{d}{x} + e\right)^2 \left( \frac{2c^2 \operatorname{csch}^{-1}(cx)d^2}{e^3 \left(\frac{d}{x} + e\right)} - \frac{32c^2 \operatorname{csch}^{-1}(cx)d^2}{5e^4} + \frac{32c \sqrt{1 + \frac{1}{c^2 x^2}} d}{15e^3} - \frac{2c^2 x^2 \operatorname{csch}^{-1}(cx)}{5e^2} - \frac{2cx \left(2e \sqrt{1 + \frac{1}{c^2 x^2}} - 9cd \operatorname{csch}^{-1}(cx)\right)}{15e^3} \right)}{(d+ex)^{3/2}} \right)}{e^4 (d + ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x)^(3/2), x]

[Out] (a\*d^4\*(1 + (e\*x)/d)^(3/2)\*Beta[-((e\*x)/d), 4, -1/2])/(e^4\*(d + e\*x)^(3/2)) + (b\*(-((c^2\*(e + d/x)^2\*x^2\*((32\*c\*d\*Sqrt[1 + 1/(c^2\*x^2)]))/(15\*e^3) - (32\*c^2\*d^2\*ArcCsch[c\*x])/(5\*e^4) + (2\*c^2\*d^2\*ArcCsch[c\*x])/(e^3\*(e + d/x)) - (2\*c^2\*x^2\*ArcCsch[c\*x])/(5\*e^2) - (2\*c\*x\*(2\*e\*Sqrt[1 + 1/(c^2\*x^2)] - 9\*c\*d\*ArcCsch[c\*x]))/(15\*e^3)))/(d + e\*x)^(3/2)) - (2\*(e + d/x)^(3/2)\*(c\*x)^(3/2)\*(-(Sqrt[2]\*(32\*c^2\*d^2\*e - e^3)\*Sqrt[1 + I\*c\*x]\*(I + c\*x)\*Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]\*EllipticF[ArcSin[Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]], (I\*c\*d + e)/(2\*e)]/(Sqrt[1 + 1/(c^2\*x^2)]\*Sqrt[e + d/x]\*(c\*x)^(3/2)\*Sqrt[(e\*(1 - I\*c\*x))/(I\*c\*d + e)])) + (I\*Sqrt[2]\*(c\*d - I\*e)\*(48\*c^3\*d^3 - 8\*c\*d\*e^2)\*Sqrt[1 + I\*c\*x]\*Sqrt[(e\*(I + c\*x)\*(c\*d + c\*e\*x))/(I\*c\*d + e]^2)\*EllipticPi[1 + (I\*c\*d)/e, ArcSin[Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]], (I\*c\*d + e)/(2\*e)]/(e\*Sqrt[1 + 1/(c^2\*x^2)]\*Sqrt[e + d/x]\*(c\*x)^(3/2)) + (16\*c\*d\*e\*Cosh[2\*ArcCsch[c\*x]]\*(-((c\*d + c\*e\*x)\*(1 + c^2\*x^2)) + (c\*x\*(c\*d\*Sqrt[2 + (2\*I)\*c\*x]\*(I + c\*x)\*Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]\*EllipticF[ArcSin[Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]], (I\*c\*d + e)/(2\*e)] + 2\*Sqrt[-((e\*(-I + c\*x))/(c\*d + I\*e))]\*(I + c\*x)\*Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]\*((c\*d + I\*e)\*EllipticE[ArcSin[Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]], (c\*d - I\*e)/(c\*d + I\*e)] - I\*e\*EllipticF[ArcSin[Sqrt[(c\*d + c\*e\*x)/(c\*d - I\*e)]], (c\*d - I\*e)/(c\*d + I\*e)])) + (I\*c\*d + e)\*Sqrt[2 + (2\*I)\*c\*x]\*Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]\*Sqrt[(e\*(I + c\*x)\*(c\*d + c\*e\*x))/(I\*c\*d + e]^2)\*EllipticPi[1 + (I\*c\*d)/e, ArcSin[Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]], (I\*c\*d + e)/(2\*e)])))/(2\*Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]))/(Sqrt[1 + 1/(c^2\*x^2)]\*Sqrt[e + d/x]\*Sqrt[c\*x]\*(2 + c^2\*x^2)))/(15\*e^4\*(d + e\*x)^(3/2)))/c^4

**fricas [F]** time = 16.60, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(bx^3 \operatorname{arcsch}(cx) + ax^3)\sqrt{ex + d}}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2), x, algorithm="fricas")

[Out] integral((b\*x^3\*arccsch(c\*x) + a\*x^3)\*sqrt(e\*x + d)/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^3/(e\*x + d)^(3/2), x)

**maple** [C] time = 0.08, size = 2019, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2), x)

[Out] 
$$\frac{2}{e^4} \left( a \left( \frac{1}{5} (e*x+d)^{5/2} - d (e*x+d)^{3/2} + 3d^2 (e*x+d)^{1/2} + d^3 (e*x+d)^{-1/2} \right) + b \left( \frac{1}{5} \operatorname{arccsch}(c*x) (e*x+d)^{5/2} - \operatorname{arccsch}(c*x) d (e*x+d)^{3/2} + 3 \operatorname{arccsch}(c*x) d^2 (e*x+d)^{1/2} + \operatorname{arccsch}(c*x) d^3 (e*x+d)^{-1/2} + \frac{2}{15} c^3 (-48 I * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticPi}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, 1 / (I * e + c * d) / c * (c^2 * d^2 + e^2) / d, (-I * e - c * d) * c / (c^2 * d^2 + e^2))^{1/2} / ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * c^2 * d^2 * e - (I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{5/2} * c^3 * d - 2 * I * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{3/2} * c^2 * d * e + I * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{5/2} * c^2 * e - I * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * e^3 + 2 * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{3/2} * c^3 * d^2 - 24 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * c^3 * d^3 - 8 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticE}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * c^3 * d^3 + 48 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticPi}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, 1 / (I * e + c * d) / c * (c^2 * d^2 + e^2) / d, (-I * e - c * d) * c / (c^2 * d^2 + e^2))^{1/2} / (((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}) * c^3 * d^3 + I * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^2 * d^2 * e - ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^3 * d^3 + I * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * e^3 + 9 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * c * d * e^2 - 8 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticE}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * c * d * e^2 + 32 * I * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * c^2 * d^2 * e - ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c * d * e^2 / (((e*x+d)^2 * c^2 - 2 * (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / c^2 / x^2 / e^2)^{1/2} / x / ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} / (I * e - c * d))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{5} a \left( \frac{(ex+d)^5}{e^4} - \frac{5(ex+d)^3 d}{e^4} + \frac{15 \sqrt{ex+d} d^2}{e^4} + \frac{5d^3}{\sqrt{ex+d} e^4} \right) + \frac{1}{5} b \left( \frac{2(e^3 x^3 - 2de^2 x^2 + 8d^2 ex + 16d^3) \log(\sqrt{c^2 x^2 + e^2})}{\sqrt{ex+d} e^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out]  $\frac{2}{5}a \frac{(ex+d)^{5/2}}{e^4} - 5 \frac{(ex+d)^{3/2}d}{e^4} + 15 \frac{\sqrt{ex+d}d^2}{e^4} + 5 \frac{d^3}{\sqrt{ex+d}e^4} + \frac{1}{5}b \frac{(e^3x^3 - 2de^2x^2 + 8d^2ex + 16d^3) \log(\sqrt{c^2x^2+1} + 1)}{\sqrt{ex+d}e^4} + 5 \int \frac{(2/5(c^2e^3x^4 - 2c^2de^2x^3 + 8c^2d^2ex^2 + 16c^2d^3x) / ((c^2e^4x^2 + e^4)\sqrt{c^2x^2+1})\sqrt{ex+d} + (c^2e^4x^2 + e^4)\sqrt{ex+d})}{(c^2e^5x^3 + c^2de^4x^2 + e^5x + de^4)\sqrt{ex+d}}, x) - 5 \int \frac{(-1/5(2c^2de^3x^4 - 48c^2d^3ex^2 - (5e^4 \log(c) + 2e^4)c^2x^5 - 32c^2d^4x - (12c^2d^2e^2 + 5e^4 \log(c))x^3 - 5(c^2e^4x^5 + e^4x^3)\log(x)) / ((c^2e^5x^3 + c^2de^4x^2 + e^5x + de^4)\sqrt{ex+d}))}{(c^2e^5x^3 + c^2de^4x^2 + e^5x + de^4)\sqrt{ex+d}}, x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(d + ex)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(3/2),x)

[Out] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acsch}(cx))}{(d + ex)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsch(c\*x))/(e\*x+d)\*\*(3/2),x)

[Out] Integral(x\*\*3\*(a + b\*acsch(c\*x))/(d + e\*x)\*\*(3/2), x)

$$3.64 \quad \int \frac{x^2(a+b\operatorname{csch}^{-1}(cx))}{(d+ex)^{3/2}} dx$$

Optimal. Leaf size=499

$$\frac{2d^2(a+b\operatorname{csch}^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^3} + \frac{32bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}}{\sqrt{-c}}}}{3ce^3}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3-2*d^2*(a+b*arccsch(c*x))/e^3/(e*x+d)^(1/2)-4*d*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3+32/3*b*d^2*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*c*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-20/3*b*c*d*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

**Rubi [A]** time = 2.01, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {43, 6310, 12, 6721, 6742, 719, 419, 932, 168, 538, 537, 844, 424}

$$\frac{2d^2(a+b\operatorname{csch}^{-1}(cx))}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^3} + \frac{2(d+ex)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^3} + \frac{32bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}}{\sqrt{-c}}}}{3ce^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2), x]
```

```
[Out] (-2*d^2*(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) + (4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)]/(3*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (20*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)]/(3*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (32*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)]/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
```

$Q[7*m + 4*n + 4, 0] \parallel LtQ[9*m + 5*(n + 1), 0] \parallel GtQ[m + n + 2, 0]$

### Rule 168

$Int[1/(((a_.) + (b_.)*(x_.))*Sqrt[(c_.) + (d_.)*(x_.)]*Sqrt[(e_.) + (f_.)*(x_.)]*Sqrt[(g_.) + (h_.)*(x_.)]), x\_Symbol] \rightarrow Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] \&\& GtQ[(d*e - c*f)/d, 0]$

### Rule 419

$Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x\_Symbol] \rightarrow Simp[1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] \&\& NegQ[d/c] \&\& GtQ[c, 0] \&\& GtQ[a, 0] \&\& !(NegQ[b/a] \&\& SimplerSqrtQ[-(b/a), -(d/c)])$

### Rule 424

$Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x\_Symbol] \rightarrow Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] \&\& NegQ[d/c] \&\& GtQ[c, 0] \&\& GtQ[a, 0]$

### Rule 537

$Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x\_Symbol] \rightarrow Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] \&\& !GtQ[d/c, 0] \&\& GtQ[c, 0] \&\& GtQ[e, 0] \&\& !( !GtQ[f/e, 0] \&\& SimplerSqrtQ[-(f/e), -(d/c)])$

### Rule 538

$Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2]), x\_Symbol] \rightarrow Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] \&\& !GtQ[c, 0]$

### Rule 719

$Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x\_Symbol] \rightarrow Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& EqQ[m^2, 1/4]$

### Rule 844

$Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x\_Symbol] \rightarrow Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& !IGtQ[m, 0]$

### Rule 932

$Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x\_Symbol] \rightarrow With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e}$

, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && GtQ[a, 0]

#### Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*(u_), x_Symbol] := With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /;
FreeQ[{a, b, c}, x]
```

#### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rubi steps





**Mathematica [C]** time = 14.49, size = 979, normalized size = 1.96

$$b \left( 2 \left( \frac{d}{x} + e \right)^{3/2} (cx)^{3/2} \frac{5\sqrt{2}cde\sqrt{icx+1}(cx+i)\sqrt{\frac{cd+cex}{cd-ie}} F\left(\sin^{-1}\left(\sqrt{\frac{-e(cx+i)}{cd-ie}}\right)\right) \frac{icd+e}{2e} + i\sqrt{2}(cd-ie)(8c^2d^2-e^2)\sqrt{icx+1}\sqrt{\frac{e(cx+i)(cd+cex)}{(icd+e)^2}} \Pi\left(\frac{icd}{e}+1, \sin^{-1}\left(\sqrt{\frac{-e(cx+i)}{cd-ie}}\right)\right) \frac{icd+e}{2e}}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}\sqrt{\frac{e(1-icx)}{icd+e}}} + \frac{e\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}}{e\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{d}{x}+e}(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2), x]
[Out] -((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/2))) + (b*(-((c^2*(e + d/x)^2*x^2*(-4*sqrt[1 + 1/(c^2*x^2)]))/(3*e^2) + (16*c*d*ArcCsch[c*x])/(3*e^3) - (2*c*d*ArcCsch[c*x])/(e^2*(e + d/x)) - (2*c*x*ArcCsch[c*x])/(3*e^2)))/(d + e*x)^(3/2)) + (2*(e + d/x)^(3/2)*(c*x)^(3/2)*((-5*sqrt[2]*c*d*e*sqrt[1 + I*c*x]*(I + c*x)*sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(sqrt[1 + 1/(c^2*x^2)]*sqrt[e + d/x]*(c*x)^(3/2)*sqrt[(e*(1 - I*c*x))/(I*c*d + e)]) + (I*sqrt[2]*(c*d - I*e)*(8*c^2*d^2 - e^2)*sqrt[1 + I*c*x]*sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e]^2)*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*sqrt[1 + 1/(c^2*x^2)]*sqrt[e + d/x]*(c*x)^(3/2)) + (2*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*sqrt[2 + (2*I)*c*x]*(I + c*x)*sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*sqrt[-((e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)])) + (I*c*d + e)*sqrt[2 + (2*I)*c*x]*sqrt[-((e*(I + c*x))/(c*d - I*e))]*sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e]^2)*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*sqrt[-((e*(I + c*x))/(c*d - I*e))]))/(sqrt[1 + 1/(c^2*x^2)]*sqrt[e + d/x]*sqrt[c*x]*(2 + c^2*x^2)))/(3*e^3*(d + e*x)^(3/2)))/c^3
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2), x, algorithm="fricas")
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2/(e\*x + d)^(3/2), x)

**maple** [C] time = 0.07, size = 896, normalized size = 1.80

$$2a \left( \frac{(ex+d)^{\frac{3}{2}}}{3} - 2d\sqrt{ex+d} - \frac{d^2}{\sqrt{ex+d}} \right) + 2b \left( \frac{(ex+d)^{\frac{3}{2}} \operatorname{arccsch}(cx)}{3} - 2 \operatorname{arccsch}(cx) d\sqrt{ex+d} - \frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}} - \frac{2\sqrt{-\frac{i(ex+d)ce}{e^2}}}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2),x)

[Out]  $\frac{2}{e^3} \left( a \left( \frac{1}{3} (ex+d)^{3/2} - 2d \sqrt{ex+d} - \frac{d^2}{\sqrt{ex+d}} \right) + b \left( \frac{1}{3} (ex+d)^{3/2} \operatorname{arccsch}(cx) - 2 \operatorname{arccsch}(cx) d \sqrt{ex+d} - \frac{\operatorname{arccsch}(cx) d^2}{\sqrt{ex+d}} - \frac{2 \sqrt{-\frac{i(ex+d)ce}{e^2}}}{\dots} \right) \right)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2}{3} a \left( \frac{(ex+d)^{\frac{3}{2}}}{e^3} - \frac{6\sqrt{ex+d}d}{e^3} - \frac{3d^2}{\sqrt{ex+d}e^3} \right) + \frac{1}{3} b \left( \frac{2(e^2x^2 - 4dex - 8d^2) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex+d}e^3} + 3 \int \frac{1}{3((c^2e^3x^2 + e^3) \sqrt{c^2x^2 + 1} \sqrt{ex+d} + (c^2e^3x^2 + e^3) \sqrt{ex+d})} dx - 3 \int \frac{-1/3(6c^2de^2x^3 - (3e^3 \log(c) + 2e^3)c^2x^4 + 16c^2d^3x + 3(8c^2d^2e - e^3 \log(c))x^2 - 3(c^2e^3x^4 + e^3x^2) \log(x))}{(c^2e^4x^3 + c^2de^3x^2 + e^4x + de^3) \sqrt{ex+d}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out]  $\frac{2}{3} a \left( \frac{(ex+d)^{3/2}}{e^3} - \frac{6\sqrt{ex+d}d}{e^3} - \frac{3d^2}{\sqrt{ex+d}e^3} \right) + \frac{1}{3} b \left( \frac{2(e^2x^2 - 4dex - 8d^2) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex+d}e^3} + 3 \int \frac{1}{3((c^2e^3x^2 + e^3) \sqrt{c^2x^2 + 1} \sqrt{ex+d} + (c^2e^3x^2 + e^3) \sqrt{ex+d})} dx - 3 \int \frac{-1/3(6c^2de^2x^3 - (3e^3 \log(c) + 2e^3)c^2x^4 + 16c^2d^3x + 3(8c^2d^2e - e^3 \log(c))x^2 - 3(c^2e^3x^4 + e^3x^2) \log(x))}{(c^2e^4x^3 + c^2de^3x^2 + e^4x + de^3) \sqrt{ex+d}} dx \right)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(d + ex)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)`

[Out] `int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acsch}(cx))}{(d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(3/2), x)`

[Out] `Integral(x**2*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)`

$$3.65 \quad \int \frac{x(a+b\operatorname{csch}^{-1}(cx))}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=318

$$\frac{2\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2d(a+b\operatorname{csch}^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{8bd\sqrt{c^2x^2+1} \sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\right) \frac{2e}{\sqrt{-c^2}}}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

[Out]  $2*d*(a+b*\operatorname{arccsch}(c*x))/e^2/(e*x+d)^{(1/2)}+2*(a+b*\operatorname{arccsch}(c*x))*(e*x+d)^{(1/2)}/e^2-8*b*d*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}},2,2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4*b*c*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 1.69, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$ , Rules used = {43, 6310, 12, 6721, 6742, 719, 419, 932, 168, 538, 537}

$$\frac{2\sqrt{d+ex} (a+b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{2d(a+b\operatorname{csch}^{-1}(cx))}{e^2\sqrt{d+ex}} - \frac{8bd\sqrt{c^2x^2+1} \sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\right) \frac{2e}{\sqrt{-c^2}}}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x*(a + b*\operatorname{ArcCsch}[c*x]))/(d + e*x)^{(3/2)}, x]$

[Out]  $(2*d*(a + b*\operatorname{ArcCsch}[c*x]))/(e^2*\operatorname{Sqrt}[d + e*x]) + (2*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcCsch}[c*x]))/e^2 + (4*b*c*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]/((-c^2)^{(3/2)}*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (8*b*d*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e)]/(c*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

**Rule 12**

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

**Rule 43**

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (\operatorname{!IntegerQ}[n] \operatorname{||} (\operatorname{EqQ}[c, 0] \&\& \operatorname{LeQ}[7*m + 4*n + 4, 0]) \operatorname{||} \operatorname{LtQ}[9*m + 5*(n + 1), 0] \operatorname{||} \operatorname{GtQ}[m + n + 2, 0])$

**Rule 168**

$\operatorname{Int}[1/(((a_*) + (b_*)*(x_*)*\operatorname{Sqrt}[(c_*) + (d_*)*(x_*)]*\operatorname{Sqrt}[(e_*) + (f_*)*(x_*)]*\operatorname{Sqrt}[(g_*) + (h_*)*(x_*)]), x\_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/(\operatorname{Simp}[b*c - a*d - b*x^2, x]*\operatorname{Sqrt}[\operatorname{Simp}[(d*e - c*f)/d + (f*x^2)/d, x]]*\operatorname{Sqrt}[\operatorname{Simp}[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, \operatorname{Sqrt}[c + d*x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \&\& \operatorname{GtQ}[(d*e - c*f)/d, 0]$

**Rule 419**

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

### Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

### Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

### Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] := Dist[(
2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

### Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_
)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

### Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] := With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /;
FreeQ[{a, b, c}, x]
```

### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \operatorname{csch}^{-1}(cx))}{(d + ex)^{3/2}} dx &= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{b \int \frac{2(2d+ex)}{e^2 \sqrt{1 + \frac{1}{c^2 x^2}} x^2 \sqrt{d+ex}} dx}{c} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(2b) \int \frac{2d+ex}{\sqrt{1 + \frac{1}{c^2 x^2}} x^2 \sqrt{d+ex}} dx}{ce^2} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(2b\sqrt{1 + c^2 x^2}) \int \frac{2d+ex}{x\sqrt{d+ex}} dx}{ce^2 \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(2b\sqrt{1 + c^2 x^2}) \int \left( \frac{2d+ex}{\sqrt{d+ex}} \right) dx}{ce^2 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(4bd\sqrt{1 + c^2 x^2}) \int \frac{2d+ex}{x\sqrt{d+ex}} dx}{ce^2 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(4bd\sqrt{1 + c^2 x^2}) \int \frac{2d+ex}{x\sqrt{1-\sqrt{d+ex}}} dx}{ce^2 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{4b\sqrt{-c^2} \sqrt{\frac{c^2(d+ex)}{c^2 d - \sqrt{-c^2} e}} \sqrt{1 + \frac{1}{c^2 x^2}}}{c^3 e} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{4b\sqrt{-c^2} \sqrt{\frac{c^2(d+ex)}{c^2 d - \sqrt{-c^2} e}} \sqrt{1 + \frac{1}{c^2 x^2}}}{c^3 e} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{4b\sqrt{-c^2} \sqrt{\frac{c^2(d+ex)}{c^2 d - \sqrt{-c^2} e}} \sqrt{1 + \frac{1}{c^2 x^2}}}{c^3 e}
\end{aligned}$$

**Mathematica [C]** time = 1.58, size = 264, normalized size = 0.83

$$\frac{2 \left( \frac{a(2d+ex)}{\sqrt{d+ex}} - \frac{2ib \sqrt{-\frac{e(cx-i)}{cd+ie}} \sqrt{-\frac{e(cx+i)}{cd-ie}} \left( F \left( i \sinh^{-1} \left( \sqrt{-\frac{c}{cd-ie}} \sqrt{d+ex} \right) \Big|_{\frac{cd-ie}{cd+ie}} \right) - 2\pi \left( 1 - \frac{ie}{cd} ; i \sinh^{-1} \left( \sqrt{-\frac{c}{cd-ie}} \sqrt{d+ex} \right) \Big|_{\frac{cd-ie}{cd+ie}} \right) \right)}{cx \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{-\frac{c}{cd-ie}}} + \frac{b \operatorname{csch}^{-1}(cx)(2d+ex)}{\sqrt{d+ex}} \right)}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x)^(3/2), x]

[Out] (2\*((a\*(2\*d + e\*x))/Sqrt[d + e\*x] + (b\*(2\*d + e\*x)\*ArcCsch[c\*x])/Sqrt[d + e\*x] - ((2\*I)\*b\*Sqrt[-((e\*(-I + c\*x))/(c\*d + I\*e))]\*Sqrt[-((e\*(I + c\*x))/(c\*

$d - Ie)))(\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-(c/(c*d - I*e))]*\text{Sqrt}[d + e*x]], (c*d - I*e)/(c*d + I*e)] - 2*\text{EllipticPi}[1 - (I*e)/(c*d), I*\text{ArcSinh}[\text{Sqrt}[-(c/(c*d - I*e))]*\text{Sqrt}[d + e*x]], (c*d - I*e)/(c*d + I*e))])/(c*\text{Sqrt}[-(c/(c*d - I*e))]*\text{Sqrt}[1 + 1/(c^2*x^2)]*x)))/e^2$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x/(e\*x + d)^(3/2), x)

**maple** [C] time = 0.07, size = 418, normalized size = 1.31

$$2a \left( \sqrt{ex + d} + \frac{d}{\sqrt{ex + d}} \right) + 2b \left( \sqrt{ex + d} \operatorname{arcsch}(cx) + \frac{\operatorname{arcsch}(cx)d}{\sqrt{ex + d}} + \frac{2\sqrt{-\frac{i(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}} \sqrt{\frac{i(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}}{e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2),x)

[Out]  $2/e^2*(a*((e*x+d)^{(1/2)}+d/(e*x+d)^{(1/2)})+b*((e*x+d)^{(1/2)}*\operatorname{arccsch}(c*x)+\operatorname{arccsch}(c*x)*d/(e*x+d)^{(1/2)}+2/c*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^{(1/2)}*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*(\text{EllipticF}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)})-2*\text{EllipticPi}((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}, 1/(I*e+c*d)/c*(c^2*d^2+e^2)/d, (-I*e-c*d)*c/(c^2*d^2+e^2))^{(1/2)})/((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)})/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^{(1/2)}/x/((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)})$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \left( \frac{2(ex + 2d) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex + d} e^2} + \int \frac{2(c^2ex^2 + 2c^2dx)}{(c^2e^2x^2 + e^2)\sqrt{c^2x^2 + 1}\sqrt{ex + d} + (c^2e^2x^2 + e^2)\sqrt{ex + d}} dx - \int \frac{6c^2dex}{(c^2e^2x^2 + e^2)\sqrt{c^2x^2 + 1}\sqrt{ex + d}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(3/2),x, algorithm="maxima")

[Out]  $b*(2*(e*x + 2*d)*\log(\text{sqrt}(c^2*x^2 + 1) + 1)/(\text{sqrt}(e*x + d)*e^2) + \text{integrate}(2*(c^2*e*x^2 + 2*c^2*d*x)/((c^2*e^2*x^2 + e^2)*\text{sqrt}(c^2*x^2 + 1)*\text{sqrt}(e*x + d) + (c^2*e^2*x^2 + e^2)*\text{sqrt}(e*x + d)), x) - \text{integrate}((6*c^2*d*e*x^2 +$



$(e^{2\log(c)} + 2e^2)c^2x^3 + (4c^2d^2 + e^{2\log(c)})x + (c^2e^{2x^3} + e^{2x})\log(x) / ((c^2e^{3x^3} + c^2de^{2x^2} + e^{3x} + d)e^2)\sqrt{ex + d}$ ,  $x$ ) +  $2a(\sqrt{ex + d}/e^2 + d/(\sqrt{ex + d}e^2))$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(d + ex)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(3/2), x)

[Out] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{acsch}(cx))}{(d + ex)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))/(e\*x+d)\*\*(3/2), x)

[Out] Integral(x\*(a + b\*acsch(c\*x))/(d + e\*x)\*\*(3/2), x)

$$3.66 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=149

$$\frac{4b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2(d+ex)}}{\sqrt{-c^2d+e}}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|\frac{2e}{\sqrt{-c^2d+e}}\right)}{cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} - \frac{2(a+b\operatorname{csch}^{-1}(cx))}{e\sqrt{d+ex}}$$

[Out]  $-2*(a+b*\operatorname{arccsch}(c*x))/e/(e*x+d)^{(1/2)}+4*b*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}},2,2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)})*(c^2*x^2+1)^{(1/2)*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 0.28, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6290, 1574, 933, 168, 538, 537}

$$\frac{4b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2(d+ex)}}{\sqrt{-c^2d+e}}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|\frac{2e}{\sqrt{-c^2d+e}}\right)}{cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}} - \frac{2(a+b\operatorname{csch}^{-1}(cx))}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/(d + e\*x)^(3/2),x]

[Out]  $(-2*(a + b*\operatorname{ArcCsch}[c*x]))/(e*\operatorname{Sqrt}[d + e*x]) + (4*b*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e))]/(c*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

#### Rule 168

Int[1/(((a\_) + (b\_)\*(x\_))\*Sqrt[(c\_) + (d\_)\*(x\_)]\*Sqrt[(e\_) + (f\_)\*(x\_)]\*Sqrt[(g\_) + (h\_)\*(x\_)]), x\_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b\*c - a\*d - b\*x^2, x]\*Sqrt[Simp[(d\*e - c\*f)/d + (f\*x^2)/d, x]]\*Sqrt[Simp[(d\*g - c\*h)/d + (h\*x^2)/d, x]]), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d\*e - c\*f)/d, 0]

#### Rule 537

Int[1/(((a\_) + (b\_)\*(x\_)^2)\*Sqrt[(c\_) + (d\_)\*(x\_)^2]\*Sqrt[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] :> Simp[(1\*EllipticPi[(b\*c)/(a\*d), ArcSin[Rt[-(d/c), 2]\*x], (c\*f)/(d\*e))]/(a\*Sqrt[c]\*Sqrt[e]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

#### Rule 538

Int[1/(((a\_) + (b\_)\*(x\_)^2)\*Sqrt[(c\_) + (d\_)\*(x\_)^2]\*Sqrt[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] :> Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/((a + b\*x^2)\*Sqrt[1 + (d\*x^2)/c]\*Sqrt[e + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

#### Rule 933

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(f\_) + (g\_)\*(x\_)]\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] :> With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c\*x^2)/a]/Sqrt[

$a + c*x^2$ ], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && !GtQ[a, 0]

### Rule 1574

Int[(x\_)^(m\_)\*((a\_) + (c\_)\*(x\_)^(mn2\_))^(p\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[(x^(2\*n\*FracPart[p])\*(a + c/x^(2\*n))^(FracPart[p]))/(c + a\*x^(2\*n))^(FracPart[p]), Int[x^(m - 2\*n\*p)\*(d + e\*x^n)^q\*(c + a\*x^(2\*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2\*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]

### Rule 6290

Int[((a\_) + ArcCsch[(c\_)\*(x\_)])\*(b\_))\*((d\_) + (e\_)\*(x\_))^(m\_), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcCsch[c\*x]))/(e\*(m + 1)), x] + Dist[b/(c\*e\*(m + 1)), Int[(d + e\*x)^(m + 1)/(x^2\*Sqrt[1 + 1/(c^2\*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex)^{3/2}} dx &= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{e\sqrt{d + ex}} - \frac{(2b) \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^2 \sqrt{d + ex}} dx}{ce} \\ &= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{e\sqrt{d + ex}} - \frac{(2b\sqrt{\frac{1}{c^2} + x^2}) \int \frac{1}{x\sqrt{d + ex} \sqrt{\frac{1}{c^2} + x^2}} dx}{ce\sqrt{1 + \frac{1}{c^2 x^2}} x} \\ &= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{e\sqrt{d + ex}} - \frac{(2b\sqrt{1 + c^2 x^2}) \int \frac{1}{x\sqrt{1 - \sqrt{-c^2} x} \sqrt{1 + \sqrt{-c^2} x} \sqrt{d + ex}} dx}{ce\sqrt{1 + \frac{1}{c^2 x^2}} x} \\ &= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{e\sqrt{d + ex}} + \frac{(4b\sqrt{1 + c^2 x^2}) \operatorname{Subst}\left(\int \frac{1}{(1 - x^2)\sqrt{2 - x^2} \sqrt{d + \frac{e}{\sqrt{-c^2}} - \frac{ex^2}{\sqrt{-c^2}}}} dx, x, \sqrt{1 + \frac{1}{c^2 x^2}}\right)}{ce\sqrt{1 + \frac{1}{c^2 x^2}} x} \\ &= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{e\sqrt{d + ex}} + \frac{\left(4b\sqrt{1 + c^2 x^2} \sqrt{1 + \frac{e(-1 + \sqrt{-c^2} x)}{\sqrt{-c^2} d + e}}\right) \operatorname{Subst}\left(\int \frac{1}{(1 - x^2)\sqrt{2 - x^2} \sqrt{1 - \frac{e}{\sqrt{-c^2}} - \frac{ex^2}{\sqrt{-c^2}}}} dx, x, \sqrt{1 + \frac{1}{c^2 x^2}}\right)}{ce\sqrt{1 + \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \\ &= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{e\sqrt{d + ex}} + \frac{4b\sqrt{1 + c^2 x^2} \sqrt{1 - \frac{e(1 - \sqrt{-c^2} x)}{\sqrt{-c^2} d + e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1 - \sqrt{-c^2} x}}{\sqrt{2}}\right) \middle| \frac{2e}{\sqrt{-c^2} d}\right)}{ce\sqrt{1 + \frac{1}{c^2 x^2}} x\sqrt{d + ex}} \end{aligned}$$

**Mathematica [C]** time = 0.68, size = 166, normalized size = 1.11

$$\frac{-2e(c^2 x^2 + 1)(a + b \operatorname{csch}^{-1}(cx)) + 2bcx\sqrt{\frac{2}{c^2 x^2}} + 2\sqrt{1 + icx}(e + icd)\sqrt{\frac{ce(cx+i)(d+ex)}{(e+icd)^2}} \Pi\left(\frac{icd}{e} + 1; \sin^{-1}\left(\sqrt{\frac{e(cx+i)}{cd-ie}}\right)\right)}{e^2(c^2 x^2 + 1)\sqrt{d + ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2), x]
```

```
[Out] (-2*e*(1 + c^2*x^2)*(a + b*ArcCsch[c*x]) + 2*b*c*(I*c*d + e)*Sqrt[2 + 2/(c^2*x^2)]*x*Sqrt[1 + I*c*x]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]/(e^2*Sqrt[d + e*x]*(1 + c^2*x^2))
```

```
fricas [F] time = 9.00, size = 0, normalized size = 0.00
```

$$\text{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{arcsch}(cx) + a)}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2), x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2), x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(3/2), x)
```

```
maple [C] time = 0.07, size = 328, normalized size = 2.20
```

$$-\frac{2a}{\sqrt{ex+d}} + 2b \left( -\frac{\operatorname{arccsch}(cx)}{\sqrt{ex+d}} + \frac{2\sqrt{\frac{i(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}} \sqrt{\frac{i(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}} \operatorname{EllipticPi}\left(\sqrt{ex+d} \sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}}, \frac{c^2d^2+e^2}{(cd+ie)cd}, \frac{\sqrt{\frac{-(cd+ie)c}{c^2d^2+e^2}}}{\sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}}}\right)}{c\sqrt{\frac{(ex+d)^2c^2-2(ex+d)c^2d+c^2d^2+e^2}{c^2x^2e^2}} xd \sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(3/2), x)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arccsch(c*x)+2/c/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/d/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2), 1/(I*e+c*d)/c*(c^2*d^2+e^2)/d, (-I*e-c*d)*c/(c^2*d^2+e^2)^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)))
```

```
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$-\left(2c^2 \int \frac{x}{(c^2ex^2 + e)\sqrt{c^2x^2 + 1} \sqrt{ex + d} + (c^2ex^2 + e)\sqrt{ex + d}} dx + \frac{2 \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex + d} e} + \int \frac{(e \log(c) - 2e)c}{(c^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2), x, algorithm="maxima")
```

```
[Out] -(2*c^2*integrate(x/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2
*e*x^2 + e)*sqrt(e*x + d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d
)*e) + integrate(((e*log(c) - 2*e)*c^2*x^2 - 2*c^2*d*x + e*log(c) + (c^2*e*
x^2 + e)*log(x))/((c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqrt(e*x + d)),
x))*b - 2*a/(sqrt(e*x + d)*e)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(d + ex)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(3/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{(d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

$$3.67 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{3/2}} dx$$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{3/2}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x/(e\*x+d)^(3/2), x)

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x)^(3/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x)^(3/2)), x]

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{3/2}} dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{3/2}} dx$$

**Mathematica [A]** time = 12.17, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x)^(3/2)), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x)^(3/2)), x]

**fricas [A]** time = 0.81, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{e^2x^3+2dex^2+d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/(e^2\*x^3 + 2\*d\*e\*x^2 + d^2\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(3/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x + d)^(3/2)\*x), x)

**maple** [A] time = 6.99, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x+d)^(3/2), x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x+d)^(3/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-b \left( \frac{\left( \frac{e \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{\frac{3}{d^2}} + \frac{2e}{\sqrt{ex+d}} \right) \log(c)}{e} + \int \frac{\log(x)}{\sqrt{ex+d}ex^2 + \sqrt{ex+d}} dx - \int \frac{\log\left(\sqrt{c^2x^2+1}+1\right)}{\sqrt{ex+d}ex^2 + \sqrt{ex+d}} dx \right) + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(3/2), x, algorithm="maxima")

[Out] -b\*((e\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(3/2) + 2\*e/(sqrt(e\*x + d)\*d))\*log(c)/e + integrate(log(x)/(sqrt(e\*x + d)\*e\*x^2 + sqrt(e\*x + d)\*d\*x), x) - integrate(log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x + d)\*e\*x^2 + sqrt(e\*x + d)\*d\*x), x)) + a\*(log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(3/2) + 2/(sqrt(e\*x + d)\*d))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x(d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x)^(3/2)), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x)^(3/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x(d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(e\*x+d)\*\*(3/2), x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*(d + e\*x)\*\*(3/2)), x)

$$3.68 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}} dx$$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(3/2), x)

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}} dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}} dx$$

**Mathematica [A]** time = 15.68, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x)^(3/2)), x]

**fricas [A]** time = 1.35, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{e^2x^4+2dex^3+d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/(e^2\*x^4 + 2\*d\*e\*x^3 + d^2\*x^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(3/2), x, algorithm="giac")



[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x + d)^(3/2)\*x^2), x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 (ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(3/2), x)

[Out] int((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(3/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \left( \frac{\left( \frac{3e^2 \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{2(3(ex+d)e^2-2de^2)}{(ex+d)^{\frac{3}{2}}d^2-\sqrt{ex+d}d^3} \right) \log(c)}{e} - 2 \int \frac{\log(x)}{\sqrt{ex+d}ex^3 + \sqrt{ex+d}dx^2} dx + 2 \int \frac{\log\left(\sqrt{c^2x^2}\right)}{\sqrt{ex+d}ex^3 + \sqrt{ex+d}dx^2} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(3/2), x, algorithm="maxima")

[Out] 1/2\*b\*((3\*e^2\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(5/2) + 2\*(3\*(e\*x + d)\*e^2 - 2\*d\*e^2)/((e\*x + d)^(3/2)\*d^2 - sqrt(e\*x + d)\*d^3))\*log(c)/e - 2\*integrate(log(x)/(sqrt(e\*x + d)\*e\*x^3 + sqrt(e\*x + d)\*d\*x^2), x) + 2\*integrate(log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x + d)\*e\*x^3 + sqrt(e\*x + d)\*d\*x^2), x) - 1/2\*a\*(2\*(3\*(e\*x + d)\*e - 2\*d\*e)/((e\*x + d)^(3/2)\*d^2 - sqrt(e\*x + d)\*d^3) + 3\*e\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(5/2))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2 (d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x)^(3/2)), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x)^(3/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^2 (d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*2/(e\*x+d)\*\*(3/2), x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*\*2\*(d + e\*x)\*\*(3/2)), x)

$$3.69 \quad \int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=777

$$\frac{2d^3(a+b\operatorname{csch}^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\operatorname{csch}^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^4} + \dots$$

[Out]  $\frac{2}{3}d^3(a+b\operatorname{arccsch}(cx))/e^4/(e*x+d)^{(3/2)} + \frac{2}{3}(e*x+d)^{(3/2)}(a+b\operatorname{arccsch}(cx))/e^4 - 6d^2(a+b\operatorname{arccsch}(cx))/e^4/(e*x+d)^{(1/2)} + \frac{4}{3}b*d^2(c^2*x^2+1)/c/e^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)} - 6*d*(a+b\operatorname{arccsch}(cx))*(e*x+d)^{(1/2)}/e^4 + \frac{64}{3}b*d^2*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^4/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)} + \frac{4}{3}b*c*(2*c^2*d^2+e^2)*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/(-c^2)^{(3/2)}/e^3/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)} - \frac{8}{3}b*d^2*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(-c^2)^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/c/e^3/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)} - \frac{32}{3}b*c*d*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 3.19, antiderivative size = 777, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 18, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$ , Rules used = {43, 6310, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537, 835, 844, 419, 1651}

$$\frac{2d^3(a+b\operatorname{csch}^{-1}(cx))}{3e^4(d+ex)^{3/2}} - \frac{6d^2(a+b\operatorname{csch}^{-1}(cx))}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^4} + \frac{2(d+ex)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^4} + \dots$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x)^(5/2), x]

[Out]  $(4*b*d^2*(1+c^2*x^2))/(3*c*e^2*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x] + (2*d^3*(a+b\operatorname{ArcCsch}[c*x]))/(3*e^4*(d+e*x)^{(3/2)}) - (6*d^2*(a+b\operatorname{ArcCsch}[c*x]))/(e^4*\operatorname{Sqrt}[d+e*x]) - (6*d*\operatorname{Sqrt}[d+e*x]*(a+b\operatorname{ArcCsch}[c*x]))/e^4 + (2*(d+e*x)^{(3/2)}*(a+b\operatorname{ArcCsch}[c*x]))/(3*e^4) - (8*b*\operatorname{Sqrt}[-c^2]*d^2*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]/(3*c*e^3*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d+e*x))/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]) + (4*b*c*(2*c^2*d^2+e^2)*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]/(3*(-c^2)^{(3/2)}*e^3*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d+e*x))/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]) - (32*b*c*d*\operatorname{Sqrt}[(c^2*(d+e*x))/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)]/(3*(-c^2)^{(3/2)}*e^3*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x]) + (64*b*d^2*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d+e*x))/(\operatorname{Sqrt}[-c^2]*d+e)]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d+e)]/(3*c*e^4*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x])$

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 43

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 168

```
Int[1/(((a_) + (b_)*(x_))*Sqrt[(c_) + (d_)*(x_)]*Sqrt[(e_) + (f_)*(x_
)]*Sqrt[(g_) + (h_)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 719

```
Int[((d_) + (e_)*(x_))^(m_)/Sqrt[(a_) + (c_)*(x_)^2], x_Symbol] := Dist[
(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
```

$]x^2)/(c*d - a*e*Rt[-(c/a), 2])^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& EqQ[m^2, 1/4]$

#### Rule 745

$Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x\_Symbol] \rightarrow Simp[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& NeQ[m, -1] \&\& ((LtQ[m, -1] \&\& IntQuadraticQ[a, 0, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] \&\& IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])$

#### Rule 835

$Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x\_Symbol] \rightarrow Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& LtQ[m, -1] \&\& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])$

#### Rule 844

$Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x\_Symbol] \rightarrow Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& !IGtQ[m, 0]$

#### Rule 932

$Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)^2]), x\_Symbol] \rightarrow With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] \&\& NeQ[e*f - d*g, 0] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& GtQ[a, 0]$

#### Rule 958

$Int[((f_) + (g_)*(x_))^(n_)/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x\_Symbol] \rightarrow Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] \&\& NeQ[e*f - d*g, 0] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& IntegerQ[n + 1/2]$

#### Rule 1651

$Int[(Pq)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x\_Symbol] \rightarrow With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] \&\& PolyQ[Pq, x] \&\& NeQ[c*d^2 + a*e^2, 0] \&\& LtQ[m, -1]$

#### Rule 6310

$Int[((a_) + ArcCsch[(c_)*(x_)]*(b_))*(u_), x\_Symbol] \rightarrow With[{v = IntHide[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[$

```
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
FreeQ[{a, b, c}, x]
```

#### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex)^{5/2}} dx &= \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \dots \\
&= \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \dots \\
&= \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \dots \\
&= \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \dots \\
&= \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} - \frac{6d \sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^4} + \dots \\
&= \frac{68bd^2 (1 + c^2x^2)}{3ce^2 (c^2d^2 + e^2) \sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{d + ex}} + \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} + \dots \\
&= \frac{68bd^2 (1 + c^2x^2)}{3ce^2 (c^2d^2 + e^2) \sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{d + ex}} + \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} + \dots \\
&= \frac{4bd^2 (1 + c^2x^2)}{3ce^2 (c^2d^2 + e^2) \sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{d + ex}} + \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} + \dots \\
&= \frac{4bd^2 (1 + c^2x^2)}{3ce^2 (c^2d^2 + e^2) \sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{d + ex}} + \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} + \dots \\
&= \frac{4bd^2 (1 + c^2x^2)}{3ce^2 (c^2d^2 + e^2) \sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{d + ex}} + \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} + \dots \\
&= \frac{4bd^2 (1 + c^2x^2)}{3ce^2 (c^2d^2 + e^2) \sqrt{1 + \frac{1}{c^2x^2}} x \sqrt{d + ex}} + \frac{2d^3 (a + b \operatorname{csch}^{-1}(cx))}{3e^4(d + ex)^{3/2}} - \frac{6d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^4 \sqrt{d + ex}} + \dots
\end{aligned}$$

**Mathematica [C]** time = 14.49, size = 1108, normalized size = 1.43

$$\frac{a \left(\frac{ex}{d} + 1\right)^{5/2} B_{-\frac{ex}{d}}\left(4, -\frac{3}{2}\right) d^4}{e^4(d + ex)^{5/2}} + \frac{2 \left(\frac{d}{x} + e\right)^{5/2} (cx)^{5/2} \left( 2 \cosh\left(2 \operatorname{csch}^{-1}(cx)\right) \left( cx \left( cd \sqrt{2icx+2} (cx+i) \sqrt{\frac{cd+cx}{cd-ie}} F\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right)\right) \frac{icd+e}{2e}\right) + 2 \sqrt{-\frac{e(cx-i)}{cd+ie}} (cx) \right) \right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2), x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2))
+ (b*(-((c^3*(e + d/x)^3*x^3*(-4*sqrt[1 + 1/(c^2*x^2)]))/(3*e*(c^2*d^2 + e^2))
+ (32*c*d*ArcCsch[c*x])/(3*e^4) - (2*c*d*ArcCsch[c*x])/(3*e^2*(e + d/x)^2)
- (2*c*x*ArcCsch[c*x])/(3*e^3) - (2*(2*c^2*d^2*e*sqrt[1 + 1/(c^2*x^2)]
+ 7*c^3*d^3*ArcCsch[c*x] + 7*c*d*e^2*ArcCsch[c*x]))/(3*e^3*(c^2*d^2 + e^2)
*(e + d/x)))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*(-((sqrt[2]
*(8*c^3*d^3*e + 8*c*d*e^3)*sqrt[1 + I*c*x]*(I + c*x)*sqrt[(c*d + c*e*x)/(c*d - I*e)]
*EllipticF[ArcSin[sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/sqrt[1 + 1/(c^2*x^2)]
*sqrt[e + d/x]*(c*x)^(3/2)*sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*sqrt[2]*(c*d - I*e)
*(16*c^4*d^4 + 16*c^2*d^2*e^2 - e^4)*sqrt[1 + I*c*x]*sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]
*EllipticPi[1 + (I*c*d)/e, ArcSin[sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/
(e*sqrt[1 + 1/(c^2*x^2)]*sqrt[e + d/x]*(c*x)^(3/2)) + (2*e^3*Cosh[2*ArcCsch[c*x]]
*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*sqrt[2 + (2*I)*c*x]*(I + c*x)*sqrt[(c*d + c*e*x)/(c*d - I*e)]
*EllipticF[ArcSin[sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*sqrt[-((e*(-I + c*x))/(c*d + I*e))]
*(I + c*x)*sqrt[(c*d + c*e*x)/(c*d - I*e)]*(c*d + I*e)*EllipticE[ArcSin[sqrt[(c*d + c*e*x)/(c*d - I*e)]],
(c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]))
+ (I*c*d + e)*sqrt[2 + (2*I)*c*x]*sqrt[-((e*(I + c*x))/(c*d - I*e))]*sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]
*EllipticPi[1 + (I*c*d)/e, ArcSin[sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])))/(2*sqrt[-((e*(I + c*x))/(c*d - I*e))])
)/sqrt[1 + 1/(c^2*x^2)]*sqrt[e + d/x]*sqrt[c*x]*(2 + c^2*x^2)))/(3*e^4*(c^2*d^2 + e^2)*(d + e*x)^(5/2)))/c^4
```

**fricas [F]** time = 16.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(bx^3 \operatorname{arcsch}(cx) + ax^3)\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2), x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^3/(e\*x + d)^(5/2), x)

maple [C] time = 0.09, size = 2726, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x)

[Out] 
$$\frac{2}{e^4} \left( a \left( \frac{1}{3} (e*x+d)^{3/2} - 3*d*(e*x+d)^{1/2} - 3*d^2/(e*x+d)^{1/2} + \frac{1}{3} d^3/(e*x+d)^{3/2} \right) + b \left( \frac{1}{3} (e*x+d)^{3/2} * \operatorname{arccsch}(c*x) - 3 * \operatorname{arccsch}(c*x) * d * (e*x+d)^{1/2} - 3 * \operatorname{arccsch}(c*x) * d^2 / (e*x+d)^{1/2} + \frac{1}{3} * \operatorname{arccsch}(c*x) * d^3 / (e*x+d)^{3/2} + \frac{2}{3} / c^2 * (-2 * I * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d) * c^3 * d^3 * e - ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^2 * c^4 * d^3 + I * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * c * d^2 * e^3 + 8 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^4 * d^4 + 16 * I * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticPi}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, 1 / (I * e + c * d) / c * (c^2 * d^2 + e^2) / d, (-I * e - c * d) * c / (c^2 * d^2 + e^2))^{1/2} / ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c * d * e^3 - 16 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticPi}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, 1 / (I * e + c * d) / c * (c^2 * d^2 + e^2) / d, (-I * e - c * d) * c / (c^2 * d^2 + e^2))^{1/2} / ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^4 * d^4 - 8 * I * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^3 * d^3 * e + 2 * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d) * c^4 * d^4 + I * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^2 * c^3 * d^2 * e - ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * c^4 * d^5 + 16 * I * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticPi}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, 1 / (I * e + c * d) / c * (c^2 * d^2 + e^2) / d, (-I * e - c * d) * c / (c^2 * d^2 + e^2))^{1/2} / ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^3 * d^3 * e + 7 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^2 * d^2 * e^2 + (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticE}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^2 * d^2 * e^2 - 8 * I * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c * d * e^3 - 16 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticPi}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, 1 / (I * e + c * d) / c * (c^2 * d^2 + e^2) / d, (-I * e - c * d) * c / (c^2 * d^2 + e^2))^{1/2} / ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * c^2 * d^2 * e^2 + I * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * c^3 * d^4 * e - ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2} * c^2 * d^3 * e^2 - (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * \operatorname{EllipticF}((e*x+d)^{1/2} * ((I * e + c * d) * c / (c^2 * d^2 + e^2))^{1/2}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2 * d^2 + e^2))^{1/2} * (e*x+d)^{1/2} * e^4 + (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2 * d^2 + e^2))^{1/2} * ((I * (e*x+d)$$



) \* c \* e - (e \* x + d) \* c^2 \* d + c^2 \* d^2 + e^2) / (c^2 \* d^2 + e^2)^(1/2) \* EllipticE((e \* x + d)^(1/2) \* ((I \* e + c \* d) \* c / (c^2 \* d^2 + e^2))^(1/2), (-2 \* I \* c \* d \* e - c^2 \* d^2 + e^2) / (c^2 \* d^2 + e^2)^(1/2)) \* (e \* x + d)^(1/2) \* e^4 / (((e \* x + d)^2 \* c^2 - 2 \* (e \* x + d) \* c^2 \* d + c^2 \* d^2 + e^2) / c^2 / x^2 / e^2)^(1/2) / x / (c^2 \* d^2 + e^2) / (e \* x + d)^(1/2) / ((I \* e + c \* d) \* c / (c^2 \* d^2 + e^2))^(1/2) / (I \* e - c \* d))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} b \left( \frac{2(e^3 x^3 - 6 d e^2 x^2 - 24 d^2 e x - 16 d^3) \log(\sqrt{c^2 x^2 + 1} + 1)}{(e^5 x + d e^4) \sqrt{e x + d}} + 3 \int \frac{2(c^2 e^3 x^4 - 6 c^2 d e^2 x^3 + 24 c^2 d^2 e x^2 - 16 c^2 d^3 x)}{3((c^2 e^5 x^3 + c^2 d e^4 x^2 + e^5 x + d e^4) \sqrt{c^2 x^2 + 1})} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] 1/3\*b\*(2\*(e^3\*x^3 - 6\*d\*e^2\*x^2 - 24\*d^2\*e\*x - 16\*d^3)\*log(sqrt(c^2\*x^2 + 1) + 1)/((e^5\*x + d\*e^4)\*sqrt(e\*x + d)) + 3\*integrate(2/3\*(c^2\*e^3\*x^4 - 6\*c^2\*d\*e^2\*x^3 - 24\*c^2\*d^2\*e\*x^2 - 16\*c^2\*d^3\*x)/(c^2\*e^5\*x^3 + c^2\*d\*e^4\*x^2 + e^5\*x + d\*e^4)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x + d) + (c^2\*e^5\*x^3 + c^2\*d\*e^4\*x^2 + e^5\*x + d\*e^4)\*sqrt(e\*x + d)), x) - 3\*integrate(-1/3\*(10\*c^2\*d\*e^3\*x^4 + 80\*c^2\*d^3\*e\*x^2 - (3\*e^4\*log(c) + 2\*e^4)\*c^2\*x^5 + 32\*c^2\*d^4\*x + 3\*(20\*c^2\*d^2\*e^2 - e^4\*log(c))\*x^3 - 3\*(c^2\*e^4\*x^5 + e^4\*x^3)\*log(x))/(c^2\*e^6\*x^4 + 2\*c^2\*d\*e^5\*x^3 + 2\*d\*e^5\*x + d^2\*e^4 + (c^2\*d^2\*e^4 + e^6)\*x^2)\*sqrt(e\*x + d)), x) + 2/3\*a\*((e\*x + d)^(3/2)/e^4 - 9\*sqrt(e\*x + d)\*d/e^4 - 9\*d^2/(sqrt(e\*x + d)\*e^4) + d^3/((e\*x + d)^(3/2)\*e^4))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left( a + b \operatorname{asinh}\left(\frac{1}{c x}\right) \right)}{(d + e x)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(5/2),x)

[Out] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acsch}(c x))}{(d + e x)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsch(c\*x))/(e\*x+d)\*\*(5/2),x)

[Out] Integral(x\*\*3\*(a + b\*acsch(c\*x))/(d + e\*x)\*\*(5/2), x)

$$3.70 \quad \int \frac{x^2(a + b \operatorname{csch}^{-1}(cx))}{(d + ex)^{5/2}} dx$$

Optimal. Leaf size=569

$$-\frac{2d^2(a + b \operatorname{csch}^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d(a + b \operatorname{csch}^{-1}(cx))}{e^3\sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4bd(c^2x^2 + 1)}{3cex\sqrt{\frac{1}{c^2x^2} + 1}(c^2d^2 + e^2)\sqrt{d + ex}}$$

[Out]  $-2/3*d^2*(a+b*\operatorname{arccsch}(c*x))/e^3/(e*x+d)^{(3/2)}+4*d*(a+b*\operatorname{arccsch}(c*x))/e^3/(e*x+d)^{(1/2)}-4/3*b*d*(c^2*x^2+1)/c/e/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+2*(a+b*\operatorname{arccsch}(c*x))*(e*x+d)^{(1/2)}/e^3-32/3*b*d*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, 2, 2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)})*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^3/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/3*b*d*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(-c^2)^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/c/e^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}+4*b*c*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)}, (-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}/(-c^2)^{(3/2)}/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 2.50, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 17, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$ , Rules used = {43, 6310, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537, 835, 844, 419}

$$-\frac{2d^2(a + b \operatorname{csch}^{-1}(cx))}{3e^3(d + ex)^{3/2}} + \frac{4d(a + b \operatorname{csch}^{-1}(cx))}{e^3\sqrt{d + ex}} + \frac{2\sqrt{d + ex}(a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{4bd(c^2x^2 + 1)}{3cex\sqrt{\frac{1}{c^2x^2} + 1}(c^2d^2 + e^2)\sqrt{d + ex}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^2*(a + b*\operatorname{ArcSch}[c*x]))/(d + e*x)^{(5/2)}, x]$

[Out]  $(-4*b*d*(1 + c^2*x^2))/(3*c*e*(c^2*d^2 + e^2)*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (2*d^2*(a + b*\operatorname{ArcSch}[c*x]))/(3*e^3*(d + e*x)^{(3/2)}) + (4*d*(a + b*\operatorname{ArcSch}[c*x]))/(e^3*\operatorname{Sqrt}[d + e*x]) + (2*\operatorname{Sqrt}[d + e*x]*(a + b*\operatorname{ArcSch}[c*x]))/e^3 + (4*b*\operatorname{Sqrt}[-c^2]*d*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e))]/(3*c*e^2*(c^2*d^2 + e^2)*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]) + (4*b*c*\operatorname{Sqrt}[(c^2*(d + e*x))/(c^2*d - \operatorname{Sqrt}[-c^2]*e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e))]/((-c^2)^{(3/2)}*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (32*b*d*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e))]/(3*c*e^3*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 21

$\operatorname{Int}[(u_)*((a_*) + (b_*)(v_))^{(m_)}*((c_*) + (d_*)(v_))^{(n_)}, x\_Symbol] := \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m + n)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x]$

$\&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& ( !\text{IntegerQ}[n] \mid\mid \text{SimplerQ}[c + d*x, a + b*x])$

#### Rule 43

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& ( !\text{IntegerQ}[n] \mid\mid (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \mid\mid \text{LtQ}[9*m + 5*(n + 1), 0] \mid\mid \text{GtQ}[m + n + 2, 0])$

#### Rule 168

$\text{Int}[1/((a + b*x)*\text{Sqrt}[c + d*x]*\text{Sqrt}[e + f*x])*\text{Sqrt}[g + h*x], x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(\text{Simp}[b*c - a*d - b*x^2, x]*\text{Sqrt}[\text{Simp}[(d*e - c*f)/d + (f*x^2)/d, x]]*\text{Sqrt}[\text{Simp}[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{GtQ}[(d*e - c*f)/d, 0]$

#### Rule 419

$\text{Int}[1/(\text{Sqrt}[a + b*x]^2*\text{Sqrt}[c + d*x]^2), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)]/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0] \&\& !( \text{NegQ}[b/a] \&\& \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

#### Rule 424

$\text{Int}[\text{Sqrt}[a + b*x]^2/\text{Sqrt}[c + d*x]^2, x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)]/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$

#### Rule 537

$\text{Int}[1/((a + b*x)^2*\text{Sqrt}[c + d*x]^2*\text{Sqrt}[e + f*x]^2), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticPi}[(b*c)/(a*d), \text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (c*f)/(d*e)]/(\text{Sqrt}[c]*\text{Sqrt}[e]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& !\text{GtQ}[d/c, 0] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[e, 0] \&\& !( !\text{GtQ}[f/e, 0] \&\& \text{SimplerSqrtQ}[-(f/e), -(d/c)])$

#### Rule 538

$\text{Int}[1/((a + b*x)^2*\text{Sqrt}[c + d*x]^2*\text{Sqrt}[e + f*x]^2), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/((a + b*x^2)*\text{Sqrt}[1 + (d*x^2)/c]*\text{Sqrt}[e + f*x^2]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& !\text{GtQ}[c, 0]$

#### Rule 719

$\text{Int}[(d + e*x)^m/\text{Sqrt}[a + c*x^2], x\_Symbol] \rightarrow \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + e*x)^m*\text{Sqrt}[1 + (c*x^2)/a])/(\text{Sqrt}[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/a), 2]*x)/2]], x] /; \text{FreeQ}\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{EqQ}[m^2, 1/4]$

#### Rule 745

$\text{Int}[(d + e*x)^m*(a + c*x^2)^p, x\_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^{m+1}*(a + c*x^2)^{p+1})/((m+1)*(c*d^2 + a*e^2)), x] + D$

```

Int[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])

```

### Rule 835

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])

```

### Rule 844

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

```

### Rule 932

```

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

```

### Rule 958

```

Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
+ g*x)^(n + 1/2)/(d + e*x), x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]

```

### Rule 6310

```

Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] := With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x] /; InverseFunctionFreeQ[v, x] /;
FreeQ[{a, b, c}, x]

```

### Rule 6721

```

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(
a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), In
t[u*x^(n*p)*(1 + a/(x^n*b))^p, x] /; FreeQ[{a, b, p}, x] && !IntegerQ[
p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

```

### Rule 6742

```

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex)^{5/2}} dx &= -\frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= -\frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= -\frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= -\frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= -\frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} + \frac{2\sqrt{d + ex} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= -\frac{12bd (1 + c^2 x^2)}{ce (c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} \\
&= -\frac{12bd (1 + c^2 x^2)}{ce (c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} \\
&= -\frac{4bd (1 + c^2 x^2)}{3ce (c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} \\
&= -\frac{4bd (1 + c^2 x^2)}{3ce (c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} \\
&= -\frac{4bd (1 + c^2 x^2)}{3ce (c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}} \\
&= -\frac{4bd (1 + c^2 x^2)}{3ce (c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex)^{3/2}} + \frac{4d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex}}
\end{aligned}$$

**Mathematica [C]** time = 14.38, size = 1076, normalized size = 1.89

$$b \frac{c^3 \left( \frac{d}{x} + e \right)^3 \left( -\frac{4c \sqrt{1 + \frac{1}{c^2 x^2}} d}{3e^2 (c^2 d^2 + e^2)} + \frac{2 \operatorname{csch}^{-1}(cx)}{3e \left( \frac{d}{x} + e \right)^2} - \frac{16 \operatorname{csch}^{-1}(cx)}{3e^3} + \frac{4 \left( 2c^2 \operatorname{csch}^{-1}(cx) d^2 + ce \sqrt{1 + \frac{1}{c^2 x^2}} d + 2e^2 \operatorname{csch}^{-1}(cx) \right)}{3e^2 (c^2 d^2 + e^2) \left( \frac{d}{x} + e \right)} \right) x^3}{(d+ex)^{5/2}} - \frac{2 \left( \frac{d}{x} + e \right)^{5/2} (cx)^{5/2} \frac{\sqrt{2} (3e^3 + 3c^2 d^2 e) \sqrt{cx + \dots}}{\sqrt{1 + \dots}}}{(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x)^(5/2), x]

[Out]  $-\left( (a*d^3*(1 + (e*x)/d)^{(5/2)}*\operatorname{Beta}\left[-\left(\frac{e*x}{d}\right), 3, -3/2\right]/(e^3*(d + e*x)^{(5/2)})) + (b*(-((c^3*(e + d/x)^3*x^3*(-4*c*d*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]))/(3*e^2*(c^2*d^2 + e^2)) - (16*\operatorname{ArcCsch}[c*x])/(3*e^3) + (2*\operatorname{ArcCsch}[c*x])/(3*e*(e + d/x)^2) + (4*(c*d*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)] + 2*c^2*d^2*\operatorname{ArcCsch}[c*x] + 2*e^2*\operatorname{ArcCsch}[c*x]))/(3*e^2*(c^2*d^2 + e^2)*(e + d/x)))))/(d + e*x)^{(5/2)} - (2*(e + d/x)^{(5/2)}*(c*x)^{(5/2)}*(-((\operatorname{Sqrt}[2]*(3*c^2*d^2*e + 3*e^3)*\operatorname{Sqrt}[1 + I*c*x]*(I + c*x)*\operatorname{Sqrt}[(c*d + c*e*x)/(c*d - I*e)]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e)))/(\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*\operatorname{Sqrt}[e + d/x]*(c*x)^{(3/2)}*\operatorname{Sqrt}[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*\operatorname{Sqrt}[2]*(c*d - I*e)*(8*c^3*d^3 + 9*c*d*e^2)*\operatorname{Sqrt}[1 + I*c*x]*\operatorname{Sqrt}[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*\operatorname{EllipticPi}[1 + (I*c*d)/e, \operatorname{ArcSin}[\operatorname{Sqrt}[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e)))/(\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*\operatorname{Sqrt}[e + d/x]*(c*x)^{(3/2)}) - (2*c*d*e*\operatorname{Cosh}[2*\operatorname{ArcCsch}[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*\operatorname{Sqrt}[2 + (2*I)*c*x]*(I + c*x)*\operatorname{Sqrt}[(c*d + c*e*x)/(c*d - I*e)]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e)] + 2*\operatorname{Sqrt}[-(e*(-I + c*x))/(c*d + I*e)]*(I + c*x)*\operatorname{Sqrt}[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] + (I*c*d + e)*\operatorname{Sqrt}[2 + (2*I)*c*x]*\operatorname{Sqrt}[-((e*(I + c*x))/(c*d - I*e))]*\operatorname{Sqrt}[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*\operatorname{EllipticPi}[1 + (I*c*d)/e, \operatorname{ArcSin}[\operatorname{Sqrt}[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e)))/(\operatorname{Sqrt}[-((e*(I + c*x))/(c*d - I*e))])))/(\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*\operatorname{Sqrt}[e + d/x]*\operatorname{Sqrt}[c*x]*(2 + c^2*x^2)))/(3*e^3*(c^2*d^2 + e^2)*(d + e*x)^{(5/2)))/c^3$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

**maple [C]** time = 0.08, size = 2497, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x)
```

```
[Out] 2/e^3*(a*((e*x+d)^(1/2)+2*d/(e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2))+b*((e*x+d)^(1/2)*arccsch(c*x)+2*arccsch(c*x)*d/(e*x+d)^(1/2)-1/3*arccsch(c*x)*d^2/(e*x+d)^(3/2)-2/3*c*(8*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^2*d^2*e+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^2*d*e-3*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^2*d^2*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^2*c^3*d^2+4*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*d^3-8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*d^3-(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c^3*d^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e+2*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*c^3*d^3-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)*c^2*d^2*e+8*I*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*e^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*d*e^3-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^4+4*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^2-8*(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*c*d*e^2-(-(I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(e*x+d)^(1/2)*e^3-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d^2*e^2/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/(c^2*d^2+e^2)/(e*x+d)^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} b \left( \frac{2(3e^2x^2 + 12dex + 8d^2) \log(\sqrt{c^2x^2 + 1} + 1)}{(e^4x + de^3)\sqrt{ex + d}} + 3 \int \frac{2(3c^2e^2x^3 + 12c^2dex^2 + 8c^2d^2x)}{3((c^2e^4x^3 + c^2de^3x^2 + e^4x + de^3)\sqrt{c^2x^2 + 1}\sqrt{ex + d} + (c^2e^4x^3 + c^2d^2e^3x^2 + e^4x + de^3)\sqrt{ex + d})} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] 1/3\*b\*(2\*(3\*e^2\*x^2 + 12\*d\*e\*x + 8\*d^2)\*log(sqrt(c^2\*x^2 + 1) + 1)/((e^4\*x + d\*e^3)\*sqrt(e\*x + d)) + 3\*integrate(2/3\*(3\*c^2\*e^2\*x^3 + 12\*c^2\*d\*e\*x^2 + 8\*c^2\*d^2\*x)/((c^2\*e^4\*x^3 + c^2\*d\*e^3\*x^2 + e^4\*x + d\*e^3)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x + d) + (c^2\*e^4\*x^3 + c^2\*d\*e^3\*x^2 + e^4\*x + d\*e^3)\*sqrt(e\*x + d)), x) - 3\*integrate(1/3\*(30\*c^2\*d\*e^2\*x^3 + 3\*(e^3\*log(c) + 2\*e^3)\*c^2\*x^4 + 16\*c^2\*d^3\*x + (40\*c^2\*d^2\*e + 3\*e^3\*log(c))\*x^2 + 3\*(c^2\*e^3\*x^4 + e^3\*x^2)\*log(x))/((c^2\*e^5\*x^4 + 2\*c^2\*d\*e^4\*x^3 + 2\*d\*e^4\*x + d^2\*e^3 + (c^2\*d^2\*e^3 + e^5)\*x^2)\*sqrt(e\*x + d)), x) + 2/3\*a\*(3\*sqrt(e\*x + d)/e^3 + 6\*d/(sqrt(e\*x + d)\*e^3) - d^2/((e\*x + d)^(3/2)\*e^3))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(d + ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(5/2),x)

[Out] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(5/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acsch}(cx))}{(d + ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))/(e\*x+d)\*\*(5/2),x)

[Out] Integral(x\*\*2\*(a + b\*acsch(c\*x))/(d + e\*x)\*\*(5/2), x)



$$3.71 \quad \int \frac{x(a+b\operatorname{csch}^{-1}(cx))}{(d+ex)^{5/2}} dx$$

Optimal. Leaf size=393

$$\frac{2(a+b\operatorname{csch}^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2d(a+b\operatorname{csch}^{-1}(cx))}{3e^2(d+ex)^{3/2}} + \frac{4b(c^2x^2+1)}{3cx\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)\sqrt{d+ex}} - \frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}}{3cex\sqrt{\frac{1}{c^2x^2}+1}}$$

[Out]  $2/3*d*(a+b*\operatorname{arccsch}(c*x))/e^2/(e*x+d)^{(3/2)}-2*(a+b*\operatorname{arccsch}(c*x))/e^2/(e*x+d)^{(1/2)}+4/3*b*(c^2*x^2+1)/c/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+8/3*b*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},2,2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/e^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/3*b*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)}*x)^{(1/2)}*2^{(1/2)},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)})*(-c^2)^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/c/e/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}$

Rubi [A] time = 2.19, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$ , Rules used = {43, 6310, 12, 6721, 6742, 745, 21, 719, 424, 958, 932, 168, 538, 537}

$$\frac{2(a+b\operatorname{csch}^{-1}(cx))}{e^2\sqrt{d+ex}} + \frac{2d(a+b\operatorname{csch}^{-1}(cx))}{3e^2(d+ex)^{3/2}} + \frac{4b(c^2x^2+1)}{3cx\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)\sqrt{d+ex}} - \frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}}{3cex\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x)^(5/2), x]

[Out]  $(4*b*(1+c^2*x^2))/(3*c*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x])+(2*d*(a+b*\operatorname{ArcCsch}[c*x]))/(3*e^2*(d+e*x)^{(3/2)})-(2*(a+b*\operatorname{ArcCsch}[c*x]))/(e^2*\operatorname{Sqrt}[d+e*x])-(4*b*\operatorname{Sqrt}[-c^2]*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]],(-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)])/(3*c*e*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[(c^2*(d+e*x))/(c^2*d-\operatorname{Sqrt}[-c^2]*e)])+(8*b*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d+e*x))/(\operatorname{Sqrt}[-c^2]*d+e)]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticPi}[2,\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]],(2*e)/(\operatorname{Sqrt}[-c^2]*d+e)])/(3*c*e^2*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m+n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n},

$x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& ( !\text{IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \parallel \text{LtQ}[9*m + 5*(n + 1), 0] \parallel \text{GtQ}[m + n + 2, 0])$

### Rule 168

$\text{Int}[1/(((a_.) + (b_.)*(x_))\text{Sqrt}[(c_.) + (d_.)*(x_)]\text{Sqrt}[(e_.) + (f_.)*(x_)]\text{Sqrt}[(g_.) + (h_.)*(x_)]), x\_Symbol] \text{:>} \text{Dist}[-2, \text{Subst}[\text{Int}[1/(\text{Simp}[b*c - a*d - b*x^2, x]\text{Sqrt}[\text{Simp}[(d*e - c*f)/d + (f*x^2)/d, x]]\text{Sqrt}[\text{Simp}[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, \text{Sqrt}[c + d*x]], x] \text{/; FreeQ}\{a, b, c, d, e, f, g, h\}, x] \&\& \text{GtQ}[(d*e - c*f)/d, 0]$

### Rule 424

$\text{Int}[\text{Sqrt}[(a_) + (b_.)*(x_)^2]/\text{Sqrt}[(c_) + (d_.)*(x_)^2], x\_Symbol] \text{:>} \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] \text{/; FreeQ}\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$

### Rule 537

$\text{Int}[1/(((a_) + (b_.)*(x_)^2)\text{Sqrt}[(c_) + (d_.)*(x_)^2]\text{Sqrt}[(e_) + (f_.)*(x_)^2]), x\_Symbol] \text{:>} \text{Simp}[(1*\text{EllipticPi}[(b*c)/(a*d), \text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (c*f)/(d*e)])/(a*\text{Sqrt}[c]*\text{Sqrt}[e]*\text{Rt}[-(d/c), 2]), x] \text{/; FreeQ}\{a, b, c, d, e, f\}, x] \&\& !\text{GtQ}[d/c, 0] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[e, 0] \&\& !( !\text{GtQ}[f/e, 0] \&\& \text{SimplerSqrtQ}[-(f/e), -(d/c)])$

### Rule 538

$\text{Int}[1/(((a_) + (b_.)*(x_)^2)\text{Sqrt}[(c_) + (d_.)*(x_)^2]\text{Sqrt}[(e_) + (f_.)*(x_)^2]), x\_Symbol] \text{:>} \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/((a + b*x^2)*\text{Sqrt}[1 + (d*x^2)/c]*\text{Sqrt}[e + f*x^2]), x], x] \text{/; FreeQ}\{a, b, c, d, e, f\}, x] \&\& !\text{GtQ}[c, 0]$

### Rule 719

$\text{Int}(((d_) + (e_.)*(x_))^{(m)}/\text{Sqrt}[(a_) + (c_.)*(x_)^2], x\_Symbol] \text{:>} \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + e*x)^m*\text{Sqrt}[1 + (c*x^2)/a])/((c*\text{Sqrt}[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/a), 2]*x)/2]], x] \text{/; FreeQ}\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{EqQ}[m^2, 1/4]$

### Rule 745

$\text{Int}(((d_) + (e_.)*(x_))^{(m)}*((a_) + (c_.)*(x_)^2)^{(p)}, x\_Symbol] \text{:>} \text{Simp}[(e*(d + e*x)^{(m+1)}*(a + c*x^2)^{(p+1)})/((m+1)*(c*d^2 + a*e^2)), x] + \text{Dist}[c/((m+1)*(c*d^2 + a*e^2)), \text{Int}[(d + e*x)^{(m+1)}*\text{Simp}[d*(m+1) - e*(m+2*p+3)*x, x]*(a + c*x^2)^p, x], x] \text{/; FreeQ}\{a, c, d, e, m, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[m, -1] \&\& ((\text{LtQ}[m, -1] \&\& \text{IntQuadraticQ}[a, 0, c, d, e, m, p, x]) \parallel (\text{SumSimplerQ}[m, 1] \&\& \text{IntegerQ}[p]) \parallel \text{ILtQ}[\text{Simplify}[m + 2*p + 3], 0])$

### Rule 932

$\text{Int}[1/(((d_.) + (e_.)*(x_))\text{Sqrt}[(f_.) + (g_.)*(x_)]\text{Sqrt}[(a_) + (c_.)*(x_)^2]), x\_Symbol] \text{:>} \text{With}\{q = \text{Rt}[-(c/a), 2]\}, \text{Dist}[1/\text{Sqrt}[a], \text{Int}[1/((d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 - q*x]*\text{Sqrt}[1 + q*x]), x], x]] \text{/; FreeQ}\{a, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{GtQ}[a, 0]$

### Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

#### Rule 6310

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*(u_), x_Symbol] := With[{v = IntHide[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]
```

#### Rule 6721

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(b^IntPart[p]*(a + b*x^n)^FracPart[p])/(x^(n*FracPart[p])*(1 + a/(x^n*b))^FracPart[p]), Int[u*x^(n*p)*(1 + a/(x^n*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \operatorname{csch}^{-1}(cx))}{(d + ex)^{5/2}} dx &= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{b \int \frac{2(-2d-3ex)}{3e^2\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}} dx}{c} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{(2b) \int \frac{-2d-3ex}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}} dx}{3ce^2} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{(2b\sqrt{1+c^2x^2}) \int \frac{-2d-3ex}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}} dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} + \frac{(2b\sqrt{1+c^2x^2}) \int \left(-\frac{3e}{(d+ex)^{3/2}\sqrt{1+c^2x^2}} - \frac{1}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}}\right) dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{(4bd\sqrt{1+c^2x^2}) \int \frac{1}{x(d+ex)^{3/2}\sqrt{1+c^2x^2}} dx}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&= \frac{4b(1+c^2x^2)}{c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{1}{c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4b(1+c^2x^2)}{c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{1}{c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4b(1+c^2x^2)}{3c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{1}{3c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4b(1+c^2x^2)}{3c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{1}{3c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4b(1+c^2x^2)}{3c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{1}{3c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&= \frac{4b(1+c^2x^2)}{3c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} + \frac{2d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex)^{3/2}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{e^2\sqrt{d + ex}} - \frac{1}{3c(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

**Mathematica [C]** time = 2.51, size = 390, normalized size = 0.99

$$\frac{2}{3} \left( \frac{a(2d + 3ex)}{e^2(d + ex)^{3/2}} + \frac{2bcx\sqrt{\frac{1}{c^2x^2} + 1}}{(c^2d^2 + e^2)\sqrt{d + ex}} + \frac{2ib\sqrt{-\frac{c}{cd-ie}}\sqrt{-\frac{e(cx-i)}{cd+ie}}\sqrt{-\frac{e(cx+i)}{cd-ie}}}{(-cdF(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d + ex}\right)))} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x)^(5/2), x]

[Out] (2\*((2\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x)/((c^2\*d^2 + e^2)\*Sqrt[d + e\*x]) - (a\*(2\*d + 3\*e\*x))/(e^2\*(d + e\*x)^(3/2)) - (b\*(2\*d + 3\*e\*x)\*ArcCsch[c\*x])/(e^2\*(d + e\*x)^(3/2)) + ((2\*I)\*b\*Sqrt[-c/(c\*d - I\*e)])\*Sqrt[-((e\*(-I + c\*x))/(c\*d + I\*e))]\*Sqrt[-((e\*(I + c\*x))/(c\*d - I\*e))]\*(c\*d\*EllipticE[I\*ArcSinh[Sqrt[-c/(c\*d - I\*e)]]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e)] - c\*d\*EllipticF[I\*ArcSinh[Sqrt[-c/(c\*d - I\*e)]]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e)] + 2\*(c\*d - I\*e)\*EllipticPi[1 - (I\*e)/(c\*d), I\*ArcSinh[Sqrt[-c/(c\*d - I\*e)]]\*Sqrt[d + e\*x]], (c\*d - I\*e)/(c\*d + I\*e)))/(c^2\*d\*e^2\*Sqrt[1 + 1/(c^2\*x^2)]\*x))/3

**fricas [F]** time = 8.18, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx \operatorname{arcsch}(cx) + ax)\sqrt{ex + d}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2), x, algorithm="fricas")

[Out] integral((b\*x\*arccsch(c\*x) + a\*x)\*sqrt(e\*x + d)/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x/(e\*x + d)^(5/2), x)

**maple [C]** time = 0.08, size = 2107, normalized size = 5.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2), x)

[Out] 2/e^2\*(a\*(-1/(e\*x+d)^(1/2))+1/3\*d/(e\*x+d)^(3/2))+b\*(-1/(e\*x+d)^(1/2)\*arccsch(c\*x)+1/3\*arccsch(c\*x)\*d/(e\*x+d)^(3/2)+2/3/c\*(2\*I\*(-(I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticPi((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), 1/(I\*e+c\*d)/c\*(c^2\*d^2+e^2)/d, (-I\*e-c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*c^2\*d^2\*e+I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^2\*c^2\*d\*e-((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^2\*c^3\*d^2-2\*(-(I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticPi((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2), 1/(I\*e+c\*d)/c\*(c^2\*d^2+e^2))^(1/2)

$2*d^2+e^2)/d, (-I*e-c*d)*c/(c^2*d^2+e^2))^{(1/2)}/((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)^{(1/2)}*c^3*d^3+(-I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^{(1/2)}*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)^{(1/2)}*c^3*d^3-(-I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^{(1/2)}*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)^{(1/2)}*c^3*d^3-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)*c^2*d^2*e+2*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)*c^3*d^3+2*I*(-I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^{(1/2)}*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*EllipticPi((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}, 1/(I*e+c*d)/c*(c^2*d^2+e^2)/d, (-I*e-c*d)*c/(c^2*d^2+e^2))^{(1/2)}/((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)^{(1/2)}*e^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*c^2*d^3*e-((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*c^3*d^4-2*(-I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^{(1/2)}*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*EllipticPi((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}, 1/(I*e+c*d)/c*(c^2*d^2+e^2)/d, (-I*e-c*d)*c/(c^2*d^2+e^2))^{(1/2)}/((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)^{(1/2)}*c*d*e^2+(-I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^{(1/2)}*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*EllipticF((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)^{(1/2)}*c*d*e^2-(-I*(e*x+d)*c*e+(e*x+d)*c^2*d-c^2*d^2-e^2)/(c^2*d^2+e^2))^{(1/2)}*((I*(e*x+d)*c*e-(e*x+d)*c^2*d+c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*EllipticE((e*x+d)^{(1/2)}*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}, (-2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^{(1/2)}*(e*x+d)^{(1/2)}*c*d*e^2+I*((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*d*e^3-((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}*c*d^2*e^2)/(((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2+e^2)/c^2/x^2/e^2)^{(1/2)}/x/d/((I*e+c*d)*c/(c^2*d^2+e^2))^{(1/2)}/(c^2*d^2+e^2)/(e*x+d)^{(1/2)}/(I*e-c*d)))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}b \left( \frac{2(3ex + 2d) \log(\sqrt{c^2x^2 + 1} + 1)}{(e^3x + de^2)\sqrt{ex + d}} + 3 \int \frac{2(3c^2ex^2 + 2c^2dx)}{3((c^2e^3x^3 + c^2de^2x^2 + e^3x + de^2)\sqrt{c^2x^2 + 1}\sqrt{ex + d} + (c^2e^3x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] -1/3\*b\*(2\*(3\*e\*x + 2\*d)\*log(sqrt(c^2\*x^2 + 1) + 1)/((e^3\*x + d\*e^2)\*sqrt(e\*x + d)) + 3\*integrate(2/3\*(3\*c^2\*e\*x^2 + 2\*c^2\*d\*x)/((c^2\*e^3\*x^3 + c^2\*d\*e^2\*x^2 + e^3\*x + d\*e^2)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x + d) + (c^2\*e^3\*x^3 + c^2\*d\*e^2\*x^2 + e^3\*x + d\*e^2)\*sqrt(e\*x + d)), x) + 3\*integrate(-1/3\*(10\*c^2\*d\*e\*x^2 - 3\*(e^2\*log(c) - 2\*e^2)\*c^2\*x^3 + (4\*c^2\*d^2 - 3\*e^2\*log(c))\*x - 3\*(c^2\*e^2\*x^3 + e^2\*x)\*log(x))/((c^2\*e^4\*x^4 + 2\*c^2\*d\*e^3\*x^3 + 2\*d\*e^3\*x + d^2\*e^2 + (c^2\*d^2\*e^2 + e^4)\*x^2)\*sqrt(e\*x + d)), x) - 2/3\*a\*(3/(sqrt(e\*x + d)\*e^2) - d/((e\*x + d)^(3/2)\*e^2))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(d + ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(5/2),x)

[Out] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{acsch}(cx))}{(d + ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))/(e\*x+d)\*\*(5/2),x)

[Out] Integral(x\*(a + b\*acsch(c\*x))/(d + e\*x)\*\*(5/2), x)

$$3.72 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=369

$$\frac{2(a+b\operatorname{csch}^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4be(c^2x^2+1)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)\sqrt{d+ex}} + \frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\right)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)\sqrt{\frac{d+ex}{\sqrt{-c^2}}+d}}$$

[Out]  $-2/3*(a+b*\operatorname{arccsch}(c*x))/e/(e*x+d)^{(3/2)}-4/3*b*e*(c^2*x^2+1)/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/3*b*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(-c^2)^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)})/((e*x+d)/(d+e/(-c^2)^{(1/2)}))^{(1/2)}+4/3*b*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)}},2,2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*(e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/d/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 0.57, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {6290, 1574, 958, 745, 21, 719, 424, 933, 168, 538, 537}

$$\frac{2(a+b\operatorname{csch}^{-1}(cx))}{3e(d+ex)^{3/2}} - \frac{4be(c^2x^2+1)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)\sqrt{d+ex}} + \frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\right)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)\sqrt{\frac{d+ex}{\sqrt{-c^2}}+d}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])/(d + e*x)^{(5/2)}, x]$

[Out]  $(-4*b*e*(1 + c^2*x^2))/(3*c*d*(c^2*d^2 + e^2)*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x]) - (2*(a + b*\operatorname{ArcCsch}[c*x])/(3*e*(d + e*x)^{(3/2)})) + (4*b*\operatorname{Sqrt}[-c^2]*\operatorname{Sqrt}[d + e*x]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d - \operatorname{Sqrt}[-c^2]*e)))/(3*c*d*(c^2*d^2 + e^2)*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[(d + e*x)/(d + e/\operatorname{Sqrt}[-c^2])]) + (4*b*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d + e*x))/(\operatorname{Sqrt}[-c^2]*d + e)]*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\operatorname{Sqrt}[1 - \operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]], (2*e)/(\operatorname{Sqrt}[-c^2]*d + e)))/(3*c*d*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x*\operatorname{Sqrt}[d + e*x])$

### Rule 21

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*(v_.))^{(m_.)*((c_.) + (d_.)*(v_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c + d*v)^{(m+n)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, n\}, x$  &&  $\operatorname{EqQ}[b*c - a*d, 0]$  &&  $\operatorname{IntegerQ}[m]$  &&  $(\neg \operatorname{IntegerQ}[n] \parallel \operatorname{SimplerQ}[c + d*x, a + b*x])$

### Rule 168

$\operatorname{Int}[1/(((a_.) + (b_.)*(x_.))*\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)]*\operatorname{Sqrt}[(e_.) + (f_.)*(x_.)]*\operatorname{Sqrt}[(g_.) + (h_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/(\operatorname{Simp}[b*c - a*d - b*x^2, x]*\operatorname{Sqrt}[\operatorname{Simp}[(d*e - c*f)/d + (f*x^2)/d, x]]*\operatorname{Sqrt}[\operatorname{Simp}[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, \operatorname{Sqrt}[c + d*x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, g, h\}, x$  &&  $\operatorname{GtQ}[(d*e - c*f)/d, 0]$

### Rule 424

$\operatorname{Int}[\operatorname{Sqrt}[(a_.) + (b_.)*(x_.)^2]/\operatorname{Sqrt}[(c_.) + (d_.)*(x_.)^2], x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[a]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\operatorname{Sqrt}[c]*\operatorname{Rt}[-(d/c$



), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

#### Rule 537

Int[1/(((a\_) + (b\_)\*(x\_)^2)\*Sqrt[(c\_) + (d\_)\*(x\_)^2]\*Sqrt[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Simp[(1\*EllipticPi[(b\*c)/(a\*d), ArcSin[Rt[-(d/c), 2]\*x], (c\*f)/(d\*e)]/(a\*Sqrt[c]\*Sqrt[e]\*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])]

#### Rule 538

Int[1/(((a\_) + (b\_)\*(x\_)^2)\*Sqrt[(c\_) + (d\_)\*(x\_)^2]\*Sqrt[(e\_) + (f\_)\*(x\_)^2]), x\_Symbol] := Dist[Sqrt[1 + (d\*x^2)/c]/Sqrt[c + d\*x^2], Int[1/((a + b\*x^2)\*Sqrt[1 + (d\*x^2)/c]\*Sqrt[e + f\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

#### Rule 719

Int[((d\_) + (e\_)\*(x\_))^(m\_)/Sqrt[(a\_) + (c\_)\*(x\_)^2], x\_Symbol] := Dist[(2\*a\*Rt[-(c/a), 2]\*(d + e\*x)^m\*Sqrt[1 + (c\*x^2)/a])/(c\*Sqrt[a + c\*x^2]\*((c\*(d + e\*x))/(c\*d - a\*e\*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2\*a\*e\*Rt[-(c/a), 2]\*x^2)/(c\*d - a\*e\*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]\*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0] && EqQ[m^2, 1/4]

#### Rule 745

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[(e\*(d + e\*x)^(m + 1)\*(a + c\*x^2)^(p + 1))/((m + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[c/((m + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^(m + 1)\*Simp[d\*(m + 1) - e\*(m + 2\*p + 3)\*x, x]\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2\*p + 3], 0])

#### Rule 933

Int[1/(((d\_) + (e\_)\*(x\_))\*Sqrt[(f\_) + (g\_)\*(x\_)^2]\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c\*x^2)/a]/Sqrt[a + c\*x^2], Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 - q\*x]\*Sqrt[1 + q\*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && !GtQ[a, 0]

#### Rule 958

Int[((f\_) + (g\_)\*(x\_))^(n\_)/(((d\_) + (e\_)\*(x\_))\*Sqrt[(a\_) + (c\_)\*(x\_)^2]), x\_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), (f + g\*x)^(n + 1/2)/(d + e\*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[n + 1/2]

#### Rule 1574

Int[(x\_)^(m\_)\*((a\_) + (c\_)\*(x\_)^(mn2\_))^(p\_)\*((d\_) + (e\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[(x^(2\*n\*FracPart[p])\*(a + c/x^(2\*n))^FracPart[p])/(c + a\*x^(2\*n))^FracPart[p], Int[x^(m - 2\*n\*p)\*(d + e\*x^n)^q\*(c + a\*x^(2\*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2\*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]

## Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
:= Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex)^{5/2}} dx &= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{(2b) \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^2 (d + ex)^{3/2}} dx}{3ce} \\
&= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b \sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{x(d + ex)^{3/2} \sqrt{\frac{1}{c^2} + x^2}} dx}{3ce \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b \sqrt{\frac{1}{c^2} + x^2}\right) \int \left(-\frac{e}{d(d + ex)^{3/2} \sqrt{\frac{1}{c^2} + x^2}} + \frac{1}{dx \sqrt{d + ex} \sqrt{\frac{1}{c^2} + x^2}}\right) dx}{3ce \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(2b \sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d + ex)^{3/2} \sqrt{\frac{1}{c^2} + x^2}} dx}{3cd \sqrt{1 + \frac{1}{c^2 x^2}} x} - \frac{\left(2b \sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{x \sqrt{d + ex} \sqrt{\frac{1}{c^2} + x^2}} dx}{3cde \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{4be(1 + c^2 x^2)}{3cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(4bc \sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{-\frac{d}{2}}{\sqrt{d + ex} \sqrt{\frac{1}{c^2} + x^2}} dx}{3d(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{4be(1 + c^2 x^2)}{3cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(2bc \sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{\sqrt{d + ex}}{\sqrt{\frac{1}{c^2} + x^2}} dx}{3d(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{4be(1 + c^2 x^2)}{3cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(4b \sqrt{-c^2} \sqrt{d + ex} \sqrt{1 + c^2 x^2}\right)}{3cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{4be(1 + c^2 x^2)}{3cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2(a + b \operatorname{csch}^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{4b \sqrt{-c^2} \sqrt{d + ex} \sqrt{1 + c^2 x^2}}{3cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}}}
\end{aligned}$$

**Mathematica [C]** time = 14.10, size = 892, normalized size = 2.42

$$b \left( \frac{2 \left( \frac{d}{x} + e \right)^{5/2} (cx)^{5/2} \left( \frac{i \sqrt{2} cd (cd - ie) \sqrt{icx + 1} \sqrt{\frac{e(cx+i)(cd+cx)}{(icd+e)^2}} \Pi \left( \frac{icd}{e} + 1, \sin^{-1} \left( \sqrt{-\frac{e(cx+i)}{cd-ie}} \right) \frac{icd+e}{2e} \right) + 2e \cosh(2 \operatorname{csch}^{-1}(cx)) \right)}{e \sqrt{1 + \frac{1}{2x^2}} \sqrt{\frac{d}{x} + e} (cx)^{3/2}} \right) \frac{cx \left( cd \sqrt{2icx+2} (cx+i) \sqrt{\frac{cd+cx}{cd-ie}} F \left( \sin^{-1} \left( \sqrt{-\frac{e(cx+i)}{cd-ie}} \right) \right) \right)}{e \sqrt{1 + \frac{1}{2x^2}} \sqrt{\frac{d}{x} + e} (cx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x)^(5/2), x]

[Out]  $(-2*a)/(3*e*(d + e*x)^{(3/2)}) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*\sqrt{1 + 1/(c^2*x^2)})))/(3*c*d*(c^2*d^2 + e^2)) + (2*ArcCsch[c*x])/(3*c^2*d^2*e) + (2*e*ArcCsch[c*x])/(3*c^2*d^2*(e + d/x)^2) - (4*(-(c*d*e*\sqrt{1 + 1/(c^2*x^2)})) + c^2*d^2*ArcCsch[c*x] + e^2*ArcCsch[c*x]))/(3*c^2*d^2*(c^2*d^2 + e^2)*(e + d/x)))/(d + e*x)^{(5/2)} + (2*(e + d/x)^{(5/2)}*(c*x)^{(5/2)}*((I*\sqrt{2}*c*d*(c*d - I*e)*\sqrt{1 + I*c*x}*\sqrt{(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2}*EllipticPi[1 + (I*c*d)/e, ArcSin[\sqrt{-((e*(I + c*x))/(c*d - I*e))}]], (I*c*d + e)/(2*e)))/(e*\sqrt{1 + 1/(c^2*x^2)}*\sqrt{e + d/x}*(c*x)^{(3/2)} + (2*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*\sqrt{2 + (2*I)*c*x}*(I + c*x)*\sqrt{(c*d + c*e*x)/(c*d - I*e)}*EllipticF[ArcSin[\sqrt{-((e*(I + c*x))/(c*d - I*e))}]], (I*c*d + e)/(2*e)] + 2*\sqrt{-((e*(-I + c*x))/(c*d + I*e))}*(I + c*x)*\sqrt{(c*d + c*e*x)/(c*d - I*e)}*((c*d + I*e)*EllipticE[ArcSin[\sqrt{(c*d + c*e*x)/(c*d - I*e)}], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[\sqrt{(c*d + c*e*x)/(c*d - I*e)}], (c*d - I*e)/(c*d + I*e)])) + (I*c*d + e)*\sqrt{2 + (2*I)*c*x}*\sqrt{-((e*(I + c*x))/(c*d - I*e))}*\sqrt{(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2}*EllipticPi[1 + (I*c*d)/e, ArcSin[\sqrt{-((e*(I + c*x))/(c*d - I*e))}]], (I*c*d + e)/(2*e)))/(2*\sqrt{-((e*(I + c*x))/(c*d - I*e))}))/((c*d*\sqrt{1 + 1/(c^2*x^2)}*\sqrt{e + d/x}*\sqrt[c*x]*(2 + c^2*x^2)))/(3*e*(c^2*d^2 + e^2)*(d + e*x)^{(5/2)))/c$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x + d)^(5/2), x)

**maple [C]** time = 0.07, size = 2079, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x)

[Out] 2/e\*(-1/3/(e\*x+d)^(3/2)\*a+b\*(-1/3/(e\*x+d)^(3/2)\*arccsch(c\*x)-2/3/c\*(-I\*(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticPi((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2),1/(I\*e+c\*d)/c\*(c^2\*d^2+e^2)/d,(-I\*e-c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*e^3-((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^2\*c^3\*d^2+I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*d\*e^3+(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticPi((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2),1/(I\*e+c\*d)/c\*(c^2\*d^2+e^2)/d,(-I\*e-c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*c^3\*d^3+(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticF((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2),(-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*c^3\*d^3-(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticE((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2),(-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*c^3\*d^3-2\*I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)\*c^2\*d^2\*e+2\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)\*c^3\*d^3-I\*(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticPi((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2),1/(I\*e+c\*d)/c\*(c^2\*d^2+e^2)/d,(-I\*e-c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*c^2\*d^2\*e-((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*c^3\*d^4+I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*c^2\*d^3\*e+(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticPi((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2),1/(I\*e+c\*d)/c\*(c^2\*d^2+e^2)/d,(-I\*e-c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*c\*d\*e^2+(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticF((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2),(-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*c\*d\*e^2-(-I\*(e\*x+d)\*c\*e+(e\*x+d)\*c^2\*d-c^2\*d^2-e^2)/(c^2\*d^2+e^2))^(1/2)\*((I\*(e\*x+d)\*c\*e-(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2)\*EllipticE((e\*x+d)^(1/2)\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2),(-2\*I\*c\*d\*e-c^2\*d^2+e^2)/(c^2\*d^2+e^2))^(1/2))\*(e\*x+d)^(1/2)\*c\*d\*e^2+I\*((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*(e\*x+d)^2\*c^2\*d\*e-((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)\*c\*d^2\*e^2)/(((e\*x+d)^2\*c^2-2\*(e\*x+d)\*c^2\*d+c^2\*d^2+e^2)/c^2/x^2/e^2)^(1/2)/x/d^2/((I\*e+c\*d)\*c/(c^2\*d^2+e^2))^(1/2)/(c^2\*d^2+e^2)/(e\*x+d)^(1/2)/(I\*e-c\*d))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} \left( 6c^2 \int \frac{x}{3 \left( (c^2e^2x^3 + c^2dex^2 + e^2x + de) \sqrt{c^2x^2 + 1} \sqrt{ex + d} + (c^2e^2x^3 + c^2dex^2 + e^2x + de) \sqrt{ex + d} \right)} dx + \frac{2}{3} \int \frac{1}{(c^2e^2x^3 + c^2dex^2 + e^2x + de) \sqrt{ex + d}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^(5/2),x, algorithm="maxima")

[Out] -1/3\*(6\*c^2\*integrate(1/3\*x/((c^2\*e^2\*x^3 + c^2\*d\*e\*x^2 + e^2\*x + d\*e)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x + d) + (c^2\*e^2\*x^3 + c^2\*d\*e\*x^2 + e^2\*x + d\*e)\*sqrt

```
t(e*x + d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/((e^2*x + d*e)*sqrt(e*x + d)
) + 3*integrate(1/3*((3*e*log(c) - 2*e)*c^2*x^2 - 2*c^2*d*x + 3*e*log(c) +
3*(c^2*e*x^2 + e)*log(x))/((c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2
*e + (c^2*d^2*e + e^3)*x^2)*sqrt(e*x + d)), x))*b - 2/3*a/((e*x + d)^(3/2)*
e)
```

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(d + ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(5/2), x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(5/2), x)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{(d + ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(5/2), x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

$$3.73 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{5/2}} dx$$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{5/2}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x/(e\*x+d)^(5/2), x)

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x)^(5/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x)^(5/2)), x]

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{5/2}} dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{5/2}} dx$$

**Mathematica [A]** time = 28.84, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x)^(5/2)), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x)^(5/2)), x]

**fricas [A]** time = 3.64, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{e^3x^4+3de^2x^3+3d^2ex^2+d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/(e^3\*x^4 + 3\*d\*e^2\*x^3 + 3\*d^2\*e\*x^2 + d^3\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{5}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(5/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x + d)^(5/2)\*x), x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x+d)^(5/2), x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x+d)^(5/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}b \left( \frac{\left( \frac{3e \log\left(\frac{\sqrt{ex+d}-\sqrt{d}}{\sqrt{ex+d}+\sqrt{d}}\right)}{d^{\frac{5}{2}}} + \frac{2(3(ex+d)e+de)}{(ex+d)^{\frac{3}{2}}d^2} \right) \log(c)}{e} + 3 \int \frac{\log(x)}{\sqrt{ex+d}e^2x^3 + 2\sqrt{ex+d}dex^2 + \sqrt{ex+d}d^2x} dx - 3 \int \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x+d)^(5/2), x, algorithm="maxima")

[Out] -1/3\*b\*((3\*e\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(5/2) + 2\*(3\*(e\*x + d)\*e + d\*e)/((e\*x + d)^(3/2)\*d^2))\*log(c)/e + 3\*integrate(log(x)/(sqrt(e\*x + d)\*e^2\*x^3 + 2\*sqrt(e\*x + d)\*d\*e\*x^2 + sqrt(e\*x + d)\*d^2\*x), x) - 3\*integrate(log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x + d)\*e^2\*x^3 + 2\*sqrt(e\*x + d)\*d\*e\*x^2 + sqrt(e\*x + d)\*d^2\*x), x) + 1/3\*a\*(3\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(5/2) + 2\*(3\*e\*x + 4\*d)/((e\*x + d)^(3/2)\*d^2))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x(d + ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x)^(5/2)), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x)^(5/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(e\*x+d)\*\*(5/2), x)

[Out] Timed out

$$3.74 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}} dx$$

Optimal. Leaf size=24

$$\operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(5/2), x)

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

[Out] Defer[Int] [(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}} dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}} dx$$

**Mathematica [A]** time = 28.47, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x)^(5/2)), x]

**fricas [A]** time = 2.04, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{e^3x^5+3de^2x^4+3d^2ex^3+d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x + d)\*(b\*arccsch(c\*x) + a)/(e^3\*x^5 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^3 + d^3\*x^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{5}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(5/2), x, algorithm="giac")



[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x + d)^(5/2)\*x^2), x)

**maple** [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 (ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(5/2), x)

[Out] int((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(5/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} b \left( \frac{2 \left( \frac{15 (ex+d)^2 e^2 - 10 (ex+d) d e^2 - 2 d^2 e^2}{(ex+d)^2 d^3 - (ex+d)^2 d^4} + \frac{15 e^2 \log\left(\frac{\sqrt{ex+d} - \sqrt{d}}{\sqrt{ex+d} + \sqrt{d}}\right)}{d^2} \right) \log(c)}{e} - 6 \int \frac{\log(x)}{\sqrt{ex+d} e^2 x^4 + 2 \sqrt{ex+d} d e x^3 + \sqrt{ex+d} d^2 x^2} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x+d)^(5/2), x, algorithm="maxima")

[Out] 1/6\*b\*((2\*(15\*(e\*x + d)^2\*e^2 - 10\*(e\*x + d)\*d\*e^2 - 2\*d^2\*e^2)/((e\*x + d)^(5/2)\*d^3 - (e\*x + d)^(3/2)\*d^4) + 15\*e^2\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(7/2))\*log(c)/e - 6\*integrate(log(x)/(sqrt(e\*x + d)\*e^2\*x^4 + 2\*sqrt(e\*x + d)\*d\*e\*x^3 + sqrt(e\*x + d)\*d^2\*x^2), x) + 6\*integrate(log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x + d)\*e^2\*x^4 + 2\*sqrt(e\*x + d)\*d\*e\*x^3 + sqrt(e\*x + d)\*d^2\*x^2), x) - 1/6\*a\*(2\*(15\*(e\*x + d)^2\*e - 10\*(e\*x + d)\*d\*e - 2\*d^2\*e)/((e\*x + d)^(5/2)\*d^3 - (e\*x + d)^(3/2)\*d^4) + 15\*e\*log((sqrt(e\*x + d) - sqrt(d))/(sqrt(e\*x + d) + sqrt(d)))/d^(7/2))

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2 (d + ex)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x)^(5/2)), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x)^(5/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*2/(e\*x+d)\*\*(5/2), x)

[Out] Timed out

$$3.75 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}} dx$$

Optimal. Leaf size=648

$$\frac{2(a+b\operatorname{csch}^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{4be(c^2x^2+1)}{5cd^2x\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)\sqrt{d+ex}} - \frac{16bce(c^2x^2+1)}{15x\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)^2\sqrt{d+ex}} - \frac{4b^2e}{15cdx\sqrt{\frac{1}{c^2x^2}+1}}$$

[Out]  $-2/5*(a+b*\operatorname{arccsch}(c*x))/e/(e*x+d)^{(5/2)}-4/15*b*e*(c^2*x^2+1)/c/d/(c^2*d^2+e^2)/x/(e*x+d)^{(3/2)}/(1+1/c^2/x^2)^{(1/2)}-16/15*b*c*e*(c^2*x^2+1)/(c^2*d^2+e^2)^2/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/5*b*e*(c^2*x^2+1)/c/d^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}-4/15*b*c*(7*c^2*d^2+3*e^2)*\operatorname{EllipticE}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)},2^{(1/2)}*(e*(-c^2)^{(1/2)}/(-c^2*d+e*(-c^2)^{(1/2)}))^{(1/2)}*(e*x+d)^{(1/2)}*(c^2*x^2+1)^{(1/2)}/d^2/(c^2*d^2+e^2)^2/x/(-c^2)^{(1/2)}/(1+1/c^2/x^2)^{(1/2)}/((e*x+d)/(d+e/(-c^2)^{(1/2)}))^{(1/2)}-4/15*b*\operatorname{EllipticF}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)},(-2*e*(-c^2)^{(1/2)}/(c^2*d-e*(-c^2)^{(1/2)}))^{(1/2)}*(-c^2)^{(1/2)}*(c^2*x^2+1)^{(1/2)}*((e*x+d)/(d+e/(-c^2)^{(1/2)}))^{(1/2)}/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}+4/5*b*\operatorname{EllipticPi}(1/2*(1-(-c^2)^{(1/2)*x})^{(1/2)*2^{(1/2)},2,2^{(1/2)}*(e/(d*(-c^2)^{(1/2)}+e))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*((e*x+d)*(-c^2)^{(1/2)}/(d*(-c^2)^{(1/2)}+e))^{(1/2)}/c/d^2/e/x/(1+1/c^2/x^2)^{(1/2)}/(e*x+d)^{(1/2)}$

**Rubi [A]** time = 1.04, antiderivative size = 785, normalized size of antiderivative = 1.21, number of steps used = 19, number of rules used = 14, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$ , Rules used = {6290, 1574, 958, 745, 835, 844, 719, 424, 419, 21, 933, 168, 538, 537}

$$\frac{2(a+b\operatorname{csch}^{-1}(cx))}{5e(d+ex)^{5/2}} - \frac{4be(c^2x^2+1)}{5cd^2x\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)\sqrt{d+ex}} - \frac{16bce(c^2x^2+1)}{15x\sqrt{\frac{1}{c^2x^2}+1}(c^2d^2+e^2)^2\sqrt{d+ex}} - \frac{4b^2e}{15cdx\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a+b*\operatorname{ArcCsch}[c*x])/(d+e*x)^{(7/2)},x]$

[Out]  $(-4*b*e*(1+c^2*x^2))/(15*c*d*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*(d+e*x)^{(3/2)} - (16*b*c*e*(1+c^2*x^2))/(15*(c^2*d^2+e^2)^2*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x]) - (4*b*e*(1+c^2*x^2))/(5*c*d^2*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x]) - (2*(a+b*\operatorname{ArcCsch}[c*x]))/(5*e*(d+e*x)^{(5/2)}) + (16*b*c*\operatorname{Sqrt}[-c^2]*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]],(-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)))/(15*(c^2*d^2+e^2)^2*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[(d+e*x)/(d+e/\operatorname{Sqrt}[-c^2])]) + (4*b*\operatorname{Sqrt}[-c^2]*\operatorname{Sqrt}[d+e*x]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]],(-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)))/(5*c*d^2*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[(d+e*x)/(d+e/\operatorname{Sqrt}[-c^2])]) - (4*b*\operatorname{Sqrt}[-c^2]*\operatorname{Sqrt}[(d+e*x)/(d+e/\operatorname{Sqrt}[-c^2])])* \operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]],(-2*\operatorname{Sqrt}[-c^2]*e)/(c^2*d-\operatorname{Sqrt}[-c^2]*e)))/(15*c*d*(c^2*d^2+e^2)*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x]) + (4*b*\operatorname{Sqrt}[(\operatorname{Sqrt}[-c^2]*(d+e*x))/(\operatorname{Sqrt}[-c^2]*d+e)]*\operatorname{Sqrt}[1+c^2*x^2]*\operatorname{EllipticPi}[2,\operatorname{ArcSin}[\operatorname{Sqrt}[1-\operatorname{Sqrt}[-c^2]*x]/\operatorname{Sqrt}[2]],(2*e)/(\operatorname{Sqrt}[-c^2]*d+e)))/(5*c*d^2*e*\operatorname{Sqrt}[1+1/(c^2*x^2)]*x*\operatorname{Sqrt}[d+e*x])$

Rule 21

$\operatorname{Int}[(u_.)*((a_.)+(b_.)*(v_))^{(m_.)}*((c_.)+(d_.)*(v_))^{(n_.)},x\_Symbol] := \operatorname{Dist}[(b/d)^m, \operatorname{Int}[u*(c+d*v)^{(m+n)},x],x] /;$   $\operatorname{FreeQ}[\{a,b,c,d,n\},x]$

$\&\& \text{EqQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[m] \&\& ( !\text{IntegerQ}[n] || \text{SimplerQ}[c + d*x, a + b*x])$

### Rule 168

$\text{Int}[1/(((a\_.) + (b\_.)*(x\_))*\text{Sqrt}[(c\_.) + (d\_.)*(x\_)]*\text{Sqrt}[(e\_.) + (f\_.)*(x\_)]*\text{Sqrt}[(g\_.) + (h\_.)*(x\_)]), x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/(\text{Simp}[b*c - a*d - b*x^2, x]*\text{Sqrt}[\text{Simp}[(d*e - c*f)/d + (f*x^2)/d, x]]*\text{Sqrt}[\text{Simp}[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x] \&\& \text{GtQ}[(d*e - c*f)/d, 0]$

### Rule 419

$\text{Int}[1/(\text{Sqrt}[(a\_.) + (b\_.)*(x\_)]*\text{Sqrt}[(c\_.) + (d\_.)*(x\_)]), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0] \&\& !( \text{NegQ}[b/a] \&\& \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

### Rule 424

$\text{Int}[\text{Sqrt}[(a\_.) + (b\_.)*(x\_)]/\text{Sqrt}[(c\_.) + (d\_.)*(x\_)], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$

### Rule 537

$\text{Int}[1/(((a\_.) + (b\_.)*(x\_)]*\text{Sqrt}[(c\_.) + (d\_.)*(x\_)]*\text{Sqrt}[(e\_.) + (f\_.)*(x\_)]), x\_Symbol] \rightarrow \text{Simp}[(1*\text{EllipticPi}[(b*c)/(a*d), \text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (c*f)/(d*e)])/(a*\text{Sqrt}[c]*\text{Sqrt}[e]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& !\text{GtQ}[d/c, 0] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[e, 0] \&\& !( !\text{GtQ}[f/e, 0] \&\& \text{SimplerSqrtQ}[-(f/e), -(d/c)])$

### Rule 538

$\text{Int}[1/(((a\_.) + (b\_.)*(x\_)]*\text{Sqrt}[(c\_.) + (d\_.)*(x\_)]*\text{Sqrt}[(e\_.) + (f\_.)*(x\_)]), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[1 + (d*x^2)/c]/\text{Sqrt}[c + d*x^2], \text{Int}[1/((a + b*x^2)*\text{Sqrt}[1 + (d*x^2)/c]*\text{Sqrt}[e + f*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& !\text{GtQ}[c, 0]$

### Rule 719

$\text{Int}(((d\_.) + (e\_.)*(x\_)]^m/\text{Sqrt}[(a\_.) + (c\_.)*(x\_)]), x\_Symbol] \rightarrow \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + e*x)^m*\text{Sqrt}[1 + (c*x^2)/a])/((c*\text{Sqrt}[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/a), 2]*x)/2]], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{EqQ}[m^2, 1/4]$

### Rule 745

$\text{Int}(((d\_.) + (e\_.)*(x\_)]^m*((a\_.) + (c\_.)*(x\_)]^p), x\_Symbol] \rightarrow \text{Simp}[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + \text{Dist}[c/((m + 1)*(c*d^2 + a*e^2)), \text{Int}[(d + e*x)^(m + 1)*\text{Simp}[d*(m + 1) - e*(m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, c, d, e, m, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[m, -1] \&\& ((\text{LtQ}[m, -1] \&\& \text{IntQuadraticQ}[a, 0, c, d, e, m, p, x]) || (\text{SumSimplerQ}[m, 1] \&\& \text{IntegerQ}[p]) || \text{ILtQ}[\text{Simplify}[m + 2*p + 3], 0])$

### Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

#### Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

#### Rule 933

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[Sqrt[1 + (c*x^2)/a]/Sqrt[a + c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && !GtQ[a, 0]
```

#### Rule 958

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

#### Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^p)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n]
```

#### Rule 6290

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCsch[c*x]))/(e*(m + 1)), x] + Dist[b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex)^{7/2}} dx &= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{(2b) \int \frac{1}{\sqrt{1 + \frac{1}{c^2 x^2}} x^2 (d + ex)^{5/2}} dx}{5ce} \\
&= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{\left(2b \sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{x(d + ex)^{5/2} \sqrt{\frac{1}{c^2} + x^2}} dx}{5ce \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{\left(2b \sqrt{\frac{1}{c^2} + x^2}\right) \int \left( -\frac{e}{d(d + ex)^{5/2} \sqrt{\frac{1}{c^2} + x^2}} - \frac{e}{d^2(d + ex)^{3/2} \sqrt{\frac{1}{c^2} + x^2}} + \frac{1}{d^2 x \sqrt{\frac{1}{c^2} + x^2}} \right) dx}{5ce \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{2(a + b \operatorname{csch}^{-1}(cx))}{5e(d + ex)^{5/2}} + \frac{\left(2b \sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d + ex)^{3/2} \sqrt{\frac{1}{c^2} + x^2}} dx}{5cd^2 \sqrt{1 + \frac{1}{c^2 x^2}} x} + \frac{\left(2b \sqrt{\frac{1}{c^2} + x^2}\right) \int \frac{1}{(d + ex)^{5/2} \sqrt{\frac{1}{c^2} + x^2}} dx}{5cd \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{4be(1 + c^2 x^2)}{15cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{4be(1 + c^2 x^2)}{5cd^2(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2}{5cd} \\
&= -\frac{4be(1 + c^2 x^2)}{15cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{16bce(1 + c^2 x^2)}{15(c^2 d^2 + e^2)^2 \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2}{5cd} \\
&= -\frac{4be(1 + c^2 x^2)}{15cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{16bce(1 + c^2 x^2)}{15(c^2 d^2 + e^2)^2 \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2}{5cd} \\
&= -\frac{4be(1 + c^2 x^2)}{15cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{16bce(1 + c^2 x^2)}{15(c^2 d^2 + e^2)^2 \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2}{5cd} \\
&= -\frac{4be(1 + c^2 x^2)}{15cd(c^2 d^2 + e^2) \sqrt{1 + \frac{1}{c^2 x^2}} x(d + ex)^{3/2}} - \frac{16bce(1 + c^2 x^2)}{15(c^2 d^2 + e^2)^2 \sqrt{1 + \frac{1}{c^2 x^2}} x \sqrt{d + ex}} - \frac{2}{5cd}
\end{aligned}$$

**Mathematica [C]** time = 14.49, size = 1217, normalized size = 1.88

$$b \left( 2 \left( \frac{d}{x} + e \right)^{7/2} (cx)^{7/2} \left[ \frac{\sqrt{2}(e^3 + c^2 d^2 e) \sqrt{icx+1} (cx+i) \sqrt{\frac{cd+cex}{cd-ie}} F\left(\sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right), \frac{icd+e}{2e}\right)}{\sqrt{1+\frac{1}{c^2 x^2}} \sqrt{\frac{d}{x}+e} (cx)^{3/2} \sqrt{\frac{e(1-icx)}{icd+e}}} + \frac{i \sqrt{2}(cd-ie)(3c^3 d^3 - cde^2) \sqrt{icx+1} \sqrt{\frac{e(cx+i)(cd+cex)}{(icd+e)^2}} \Pi\left(\frac{icd}{e}+1; \sin^{-1}\left(\sqrt{-\frac{e(cx+i)}{cd-ie}}\right)\right)}{e \sqrt{1+\frac{1}{c^2 x^2}} \sqrt{\frac{d}{x}+e} (cx)^{3/2}} \right] \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2), x]
```

```
[Out] (-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((-4*(7*c^2*d^2 + 3*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(15*c^2*d^2*(c^2*d^2 + e^2)^2) + (2*ArcCsch[c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsch[c*x])/(5*c^3*d^3*(e + d/x)^3) + (2*(-2*c*d*e^2*Sqrt[1 + 1/(c^2*x^2)] + 9*c^2*d^2*e*ArcCsch[c*x] + 9*e^3*ArcCsch[c*x]))/(15*c^3*d^3*(c^2*d^2 + e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[1 + 1/(c^2*x^2)] - 8*c*d*e^3*Sqrt[1 + 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsch[c*x] + 18*c^2*d^2*e^2*ArcCsch[c*x] + 9*e^4*ArcCsch[c*x]))/(15*c^3*d^3*(c^2*d^2 + e^2)^2*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*(-(Sqrt[2]*(c^2*d^2*e + e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e)))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(3*c^3*d^3 - c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e]^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e)))/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-7*c^2*d^2*e - 3*e^3)*Cos h[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*(c*d + I*e)*Elliptic E[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e]^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]]], (I*c*d + e)/(2*e)))/(2*Sqrt[-((e*(I + c*x))/(c*d - I*e))]))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^2)))/(15*c*d*e*(c^2*d^2 + e^2)^2*(d + e*x)^(7/2)))/c
```

**fricas [F]** time = 9.39, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex + d} (b \operatorname{arcsch}(cx) + a)}{e^4 x^4 + 4 d e^3 x^3 + 6 d^2 e^2 x^2 + 4 d^3 e x + d^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2), x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex + d)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^(7/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x + d)^(7/2), x)

**maple** [C] time = 0.11, size = 3782, normalized size = 5.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x+d)^(7/2),x)

[Out] 
$$\frac{2}{e} \left( -\frac{1}{5} \frac{a}{(ex+d)^{5/2}} + b \left( -\frac{1}{5} \frac{1}{(ex+d)^{5/2}} \operatorname{arccsch}(cx) - \frac{2}{15} \frac{1}{c} \left( 3 \left( -\left( (ex+d) \right) \frac{c \cdot e + (ex+d) \cdot c^2 d - c^2 d^2 - e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \left( \frac{(ex+d) \cdot c \cdot e - (ex+d) \cdot c^2 d + c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \operatorname{EllipticPi} \left( \frac{(ex+d)^{1/2}}{(ex+d)^{1/2}} \right) \right. \right. \right.$$

$$\left. \left. \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}}, \frac{1}{(ex+d) \cdot c} \frac{c^2 d^2 + e^2}{d}, \left( -\frac{(ex-cd) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{1}{(ex+d)^{3/2}} \frac{c^5 d^5 + I \left( -\left( (ex+d) \right) \frac{c \cdot e + (ex+d) \cdot c^2 d - c^2 d^2 - e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \left( \frac{(ex+d) \cdot c \cdot e - (ex+d) \cdot c^2 d + c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \operatorname{EllipticF} \left( \frac{(ex+d)^{1/2}}{(ex+d)^{1/2}} \right) \right. \right.$$

$$\left. \left. \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}}, \left( -\frac{2 \cdot I \cdot c \cdot d \cdot e - c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{1}{(ex+d)^{3/2}} \frac{c^2 d^2 \cdot e^3 - 6 \cdot I \cdot \left( -\left( (ex+d) \right) \frac{c \cdot e + (ex+d) \cdot c^2 d - c^2 d^2 - e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \left( \frac{(ex+d) \cdot c \cdot e - (ex+d) \cdot c^2 d + c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \operatorname{EllipticPi} \left( \frac{(ex+d)^{1/2}}{(ex+d)^{1/2}} \right) \right. \right.$$

$$\left. \left. \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}}, \frac{1}{(ex+d) \cdot c} \frac{c^2 d^2 + e^2}{d}, \left( -\frac{(ex-cd) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{1}{(ex+d)^{3/2}} \frac{c^2 d^2 \cdot e^3 - 3 \cdot I \cdot \left( -\left( (ex+d) \right) \frac{c \cdot e + (ex+d) \cdot c^2 d - c^2 d^2 - e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \left( \frac{(ex+d) \cdot c \cdot e - (ex+d) \cdot c^2 d + c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \operatorname{EllipticPi} \left( \frac{(ex+d)^{1/2}}{(ex+d)^{1/2}} \right) \right. \right.$$

$$\left. \left. \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}}, \frac{1}{(ex+d) \cdot c} \frac{c^2 d^2 + e^2}{d}, \left( -\frac{(ex-cd) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{1}{(ex+d)^{3/2}} \frac{c^4 d^4 \cdot e + I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{c^4 d^6 \cdot e + 2 \cdot I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{c^2 d^4 \cdot e^3 - 2 \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{c^3 d^5 \cdot e^2 + I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{d^2 \cdot e^5 - \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{c \cdot d^3 \cdot e^4 - \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{c^5 d^7 + 7 \cdot I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{c^2 d^2 + e^2}{(ex+d)^3} \frac{c^4 d^3 \cdot e - 13 \cdot I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d)^2 \cdot c^4 d^4 \cdot e + 3 \cdot I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d)^3 \cdot c^2 d \cdot e^3 + 5 \cdot I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d) \cdot c^4 d^5 \cdot e - 5 \cdot I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d)^2 \cdot c^2 d^2 \cdot e^3 + 8 \cdot I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d) \cdot c^2 d^3 \cdot e^3 - 3 \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d) \cdot c \cdot d^2 \cdot e^4 + 3 \cdot I \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d) \cdot d \cdot e^5 - 3 \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{c^2 d^2 + e^2}{(ex+d)^3} \frac{c^3 d^2 \cdot e^2 + I \cdot \left( -\left( (ex+d) \right) \frac{c \cdot e + (ex+d) \cdot c^2 d - c^2 d^2 - e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \left( \frac{(ex+d) \cdot c \cdot e - (ex+d) \cdot c^2 d + c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \operatorname{EllipticF} \left( \frac{(ex+d)^{1/2}}{(ex+d)^{1/2}} \right) \right. \right.$$

$$\left. \left. \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}}, \left( -\frac{2 \cdot I \cdot c \cdot d \cdot e - c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{1}{(ex+d)^{3/2}} \frac{c^4 d^4 \cdot e + 5 \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d)^2 \cdot c^3 d^3 \cdot e^2 - 8 \cdot \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{(ex+d) \cdot c^3 d^4 \cdot e^2 + 6 \cdot \left( -\left( (ex+d) \right) \frac{c \cdot e + (ex+d) \cdot c^2 d - c^2 d^2 - e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \left( \frac{(ex+d) \cdot c \cdot e - (ex+d) \cdot c^2 d + c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \operatorname{EllipticF} \left( \frac{(ex+d)^{1/2}}{(ex+d)^{1/2}} \right) \right. \right.$$

$$\left. \left. \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}}, \left( -\frac{2 \cdot I \cdot c \cdot d \cdot e - c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{1}{(ex+d)^{3/2}} \frac{c^5 d^5 \cdot e - 7 \cdot \left( -\left( (ex+d) \right) \frac{c \cdot e + (ex+d) \cdot c^2 d - c^2 d^2 - e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \left( \frac{(ex+d) \cdot c \cdot e - (ex+d) \cdot c^2 d + c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \operatorname{EllipticE} \left( \frac{(ex+d)^{1/2}}{(ex+d)^{1/2}} \right) \right. \right.$$

$$\left. \left. \left( \frac{(ex+d) \cdot c}{(c^2 d^2 + e^2)^{1/2}}, \left( -\frac{2 \cdot I \cdot c \cdot d \cdot e - c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \frac{1}{(ex+d)^3} \frac{c^5 d^5 \cdot e - 3 \cdot I \cdot \left( -\left( (ex+d) \right) \frac{c \cdot e + (ex+d) \cdot c^2 d - c^2 d^2 - e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \left( \frac{(ex+d) \cdot c \cdot e - (ex+d) \cdot c^2 d + c^2 d^2 + e^2}{(c^2 d^2 + e^2)^{1/2}} \right) \right. \right.$$

$d^2+e^2)^{(1/2)} * \text{EllipticPi}((e*x+d)^{(1/2)} * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)}, 1/(I*e+c*d) / c * (c^2*d^2+e^2) / d, (-I*e-c*d) * c / (c^2*d^2+e^2))^{(1/2)} / ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(3/2)} * e^{-5-7} * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^3 * c^5 * d^4 + 13 * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^2 * c^5 * d^5 - 5 * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d) * c^5 * d^6 + 9 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticF}((e*x+d)^{(1/2)} * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(3/2)} * c^3 * d^3 * e^2 - 10 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticE}((e*x+d)^{(1/2)} * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(3/2)} * c^3 * d^3 * e^2 + 3 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticPi}((e*x+d)^{(1/2)} * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)}, 1/(I*e+c*d) / c * (c^2*d^2+e^2) / d, (-I*e-c*d) * c / (c^2*d^2+e^2))^{(1/2)} / ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(3/2)} * c * d * e^4 + 3 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticF}((e*x+d)^{(1/2)} * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(3/2)} * c * d * e^4 - 3 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticE}((e*x+d)^{(1/2)} * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)}, (-2 * I * c * d * e - c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(3/2)} * c * d * e^4 + 6 * (-I * (e*x+d) * c * e + (e*x+d) * c^2 * d - c^2 * d^2 - e^2) / (c^2*d^2+e^2))^{(1/2)} * ((I * (e*x+d) * c * e - (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / (c^2*d^2+e^2))^{(1/2)} * \text{EllipticPi}((e*x+d)^{(1/2)} * ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)}, 1/(I*e+c*d) / c * (c^2*d^2+e^2) / d, (-I*e-c*d) * c / (c^2*d^2+e^2))^{(1/2)} / ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)} * (e*x+d)^{(3/2)} * c^3 * d^3 * e^2) / (((e*x+d)^2 * c^2 - 2 * (e*x+d) * c^2 * d + c^2 * d^2 + e^2) / c^2 / x^2 / e^2)^{(1/2)} / x / d^3 / ((I*e+c*d) * c / (c^2*d^2+e^2))^{(1/2)} / (c^2*d^2+e^2)^2 / (e*x+d)^{(3/2)} / (I*e-c*d))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{5} \left( 10c^2 \int \frac{x}{5 \left( (c^2e^3x^4 + 2c^2de^2x^3 + 2de^2x + d^2e + (c^2d^2e + e^3)x^2 \right) \sqrt{c^2x^2 + 1} \sqrt{ex + d} + (c^2e^3x^4 + 2c^2de^2x^3 + 2de^2x + d^2e + (c^2d^2e + e^3)x^2) \sqrt{c^2x^2 + 1} \sqrt{ex + d} \right)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x+d)^(7/2),x, algorithm="maxima")  
 [Out] -1/5\*(10\*c^2\*integrate(1/5\*x/((c^2\*e^3\*x^4 + 2\*c^2\*d\*e^2\*x^3 + 2\*d\*e^2\*x + d^2\*e + (c^2\*d^2\*e + e^3)\*x^2)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x + d) + (c^2\*e^3\*x^4 + 2\*c^2\*d\*e^2\*x^3 + 2\*d\*e^2\*x + d^2\*e + (c^2\*d^2\*e + e^3)\*x^2)\*sqrt(e\*x + d)), x) + 2\*log(sqrt(c^2\*x^2 + 1) + 1)/((e^3\*x^2 + 2\*d\*e^2\*x + d^2\*e)\*sqrt(e\*x + d)) + 5\*integrate(1/5\*((5\*e\*log(c) - 2\*e)\*c^2\*x^2 - 2\*c^2\*d\*x + 5\*e\*log(c) + 5\*(c^2\*e\*x^2 + e)\*log(x))/((c^2\*e^4\*x^5 + 3\*c^2\*d\*e^3\*x^4 + 3\*d^2\*e^2\*x + d^3\*e + (3\*c^2\*d^2\*e^2 + e^4)\*x^3 + (c^2\*d^3\*e + 3\*d\*e^3)\*x^2)\*sqrt(e\*x + d)), x))\*b - 2/5\*a/((e\*x + d)^(5/2)\*e)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(d + ex)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(d + e\*x)^(7/2),x)  
 [Out] int((a + b\*asinh(1/(c\*x)))/(d + e\*x)^(7/2), x)



sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/(e\*x+d)\*\*(7/2), x)

[Out] Timed out

### 3.76 $\int x^4 (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=214

$$\frac{1}{5}dx^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7}ex^7 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^6 \sqrt{-c^2x^2 - 1}}{42c \sqrt{-c^2x^2}} - \frac{bx (42c^2d - 25e) \tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2 - 1}}\right)}{560c^6 \sqrt{-c^2x^2}} - \frac{bx^2 \sqrt{-c^2x^2 - 1}}{560c^6 \sqrt{-c^2x^2}}$$

[Out]  $\frac{1}{5}d*x^5*(a+b*\operatorname{arccsch}(c*x))+\frac{1}{7}*e*x^7*(a+b*\operatorname{arccsch}(c*x))-1/560*b*(42*c^2*d-25*e)*x*\operatorname{arctan}(c*x/(-c^2*x^2-1)^{(1/2)})/c^6/(-c^2*x^2)^{(1/2)}-1/560*b*(42*c^2*d-25*e)*x^2*(-c^2*x^2-1)^{(1/2)}/c^5/(-c^2*x^2)^{(1/2)}+1/840*b*(42*c^2*d-25*e)*x^4*(-c^2*x^2-1)^{(1/2)}/c^3/(-c^2*x^2)^{(1/2)}+1/42*b*e*x^6*(-c^2*x^2-1)^{(1/2)}/c/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {14, 6302, 12, 459, 321, 217, 203}

$$\frac{1}{5}dx^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7}ex^7 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^4 \sqrt{-c^2x^2 - 1} (42c^2d - 25e)}{840c^3 \sqrt{-c^2x^2}} - \frac{bx^2 \sqrt{-c^2x^2 - 1} (42c^2d - 25e)}{560c^5 \sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] `Int[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]`

[Out]  $-(b*(42*c^2*d - 25*e)*x^2*\operatorname{Sqrt}[-1 - c^2*x^2])/(560*c^5*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*(42*c^2*d - 25*e)*x^4*\operatorname{Sqrt}[-1 - c^2*x^2])/(840*c^3*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*e*x^6*\operatorname{Sqrt}[-1 - c^2*x^2])/(42*c*\operatorname{Sqrt}[-(c^2*x^2)]) + (d*x^5*(a + b*\operatorname{ArcCsch}[c*x]))/5 + (e*x^7*(a + b*\operatorname{ArcCsch}[c*x]))/7 - (b*(42*c^2*d - 25*e)*x*\operatorname{ArcTan}[(c*x)/\operatorname{Sqrt}[-1 - c^2*x^2]])/(560*c^6*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 14

`Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

#### Rule 203

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])`

#### Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

#### Rule 321

`Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p`

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 6302

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned} \int x^4 (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{5} dx^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(bcx) \int \frac{x^4(7d+5ex^2)}{35\sqrt{-1-c^2x^2}}}{\sqrt{-c^2x^2}} \\ &= \frac{1}{5} dx^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(bcx) \int \frac{x^4(7d+5ex^2)}{\sqrt{-1-c^2x^2}}}{35\sqrt{-c^2x^2}} \\ &= \frac{bcx^6 \sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}} + \frac{1}{5} dx^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \operatorname{csch}^{-1}(cx)) - \\ &= \frac{b(42c^2d - 25e)x^4 \sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}} + \frac{bcx^6 \sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}} + \frac{1}{5} dx^5 (a + b \operatorname{csch}^{-1}(cx)) - \\ &= -\frac{b(42c^2d - 25e)x^2 \sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}} + \frac{b(42c^2d - 25e)x^4 \sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}} + \frac{bcx^6 \sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}} + \\ &= -\frac{b(42c^2d - 25e)x^2 \sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}} + \frac{b(42c^2d - 25e)x^4 \sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}} + \frac{bcx^6 \sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}} + \\ &= -\frac{b(42c^2d - 25e)x^2 \sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}} + \frac{b(42c^2d - 25e)x^4 \sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}} + \frac{bcx^6 \sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}} + \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 138, normalized size = 0.64

$$\frac{48ac^7x^5(7d + 5ex^2) + 48bc^7x^5\operatorname{csch}^{-1}(cx)(7d + 5ex^2) + 3b(42c^2d - 25e)\log\left(x\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right)\right) + bc^2x^2\sqrt{\frac{1}{c^2x^2} + 1}}{1680c^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] (48\*a\*c^7\*x^5\*(7\*d + 5\*e\*x^2) + b\*c^2\*Sqrt[1 + 1/(c^2\*x^2)]\*x^2\*(75\*e - 2\*c^2\*(63\*d + 25\*e\*x^2) + c^4\*(84\*d\*x^2 + 40\*e\*x^4)) + 48\*b\*c^7\*x^5\*(7\*d + 5\*e

$x^2) \cdot \text{ArcCsch}[c \cdot x] + 3 \cdot b \cdot (42 \cdot c^2 \cdot d - 25 \cdot e) \cdot \text{Log}[(1 + \text{Sqrt}[1 + 1/(c^2 \cdot x^2)]) \cdot x]) / (1680 \cdot c^7)$

**fricas** [A] time = 1.95, size = 295, normalized size = 1.38

$$240 ac^7 ex^7 + 336 ac^7 dx^5 + 48(7bc^7d + 5bc^7e) \log\left(cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 3(42bc^2d - 25be) \log\left(cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 1/1680\*(240\*a\*c^7\*e\*x^7 + 336\*a\*c^7\*d\*x^5 + 48\*(7\*b\*c^7\*d + 5\*b\*c^7\*e)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - 3\*(42\*b\*c^2\*d - 25\*b\*e)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) - 48\*(7\*b\*c^7\*d + 5\*b\*c^7\*e)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + 48\*(5\*b\*c^7\*e\*x^7 + 7\*b\*c^7\*d\*x^5 - 7\*b\*c^7\*d - 5\*b\*c^7\*e)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (40\*b\*c^6\*e\*x^6 + 2\*(42\*b\*c^6\*d - 25\*b\*c^4\*e)\*x^4 - 3\*(42\*b\*c^4\*d - 25\*b\*c^2\*e)\*x^2)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^7

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)(b \operatorname{arcsch}(cx) + a)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*x^4, x)

**maple** [A] time = 0.05, size = 211, normalized size = 0.99

$$\frac{a\left(\frac{1}{7}e c^7 x^7 + \frac{1}{5}c^7 x^5 d\right)}{c^2} + \frac{b\left(\frac{\operatorname{arccsch}(cx)e c^7 x^7}{7} + \frac{\operatorname{arccsch}(cx)c^7 x^5 d}{5} + \frac{\sqrt{c^2x^2+1}\left(40e^5x^5\sqrt{c^2x^2+1} + 84c^5dx^3\sqrt{c^2x^2+1} - 50e^3x^3\sqrt{c^2x^2+1} - 126c^3dx\sqrt{c^2x^2+1} + 126c^2d \operatorname{arcsinh}(cx)\right)}{1680\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx}\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c^5\*(a/c^2\*(1/7\*e\*c^7\*x^7+1/5\*c^7\*x^5\*d)+b/c^2\*(1/7\*arccsch(c\*x)\*e\*c^7\*x^7+1/5\*arccsch(c\*x)\*c^7\*x^5\*d+1/1680\*(c^2\*x^2+1)^(1/2)\*(40\*e\*c^5\*x^5\*(c^2\*x^2+1)^(1/2)+84\*c^5\*d\*x^3\*(c^2\*x^2+1)^(1/2)-50\*e\*c^3\*x^3\*(c^2\*x^2+1)^(1/2)-126\*c^3\*d\*x\*(c^2\*x^2+1)^(1/2)+126\*c^2\*d\*arcsinh(c\*x)+75\*e\*c\*x\*(c^2\*x^2+1)^(1/2)-75\*e\*arcsinh(c\*x)))/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x)

**maxima** [A] time = 0.37, size = 289, normalized size = 1.35

$$\frac{1}{7} aex^7 + \frac{1}{5} adx^5 + \frac{1}{80} \left( 16x^5 \operatorname{arcsch}(cx) - \frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2c^4\left(\frac{1}{c^2x^2}+1\right)+c^4} - \frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4} \right) bd + \frac{1}{672}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{7}aex^7 + \frac{1}{5}adx^5 + \frac{1}{80}(16x^5\operatorname{arccsch}(cx) - (2(3(1/(c^2x^2) + 1)^{3/2} - 5\sqrt{1/(c^2x^2) + 1}))/c^4(1/(c^2x^2) + 1)^2 - 2c^4(1/(c^2x^2) + 1) + c^4) - 3\log(\sqrt{1/(c^2x^2) + 1} + 1)/c^4 + 3\log(\sqrt{1/(c^2x^2) + 1} - 1)/c^4)/c * b * d + \frac{1}{672}(96x^7\operatorname{arccsch}(cx) + (2(15(1/(c^2x^2) + 1)^{5/2} - 40(1/(c^2x^2) + 1)^{3/2} + 33\sqrt{1/(c^2x^2) + 1}))/c^6(1/(c^2x^2) + 1)^3 - 3c^6(1/(c^2x^2) + 1)^2 + 3c^6(1/(c^2x^2) + 1) - c^6) - 15\log(\sqrt{1/(c^2x^2) + 1} + 1)/c^6 + 15\log(\sqrt{1/(c^2x^2) + 1} - 1)/c^6)/c * b * e$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 (ex^2 + d) \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x^4\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + b \operatorname{acsch}(cx)) (d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x\*\*2+d)\*(a+b\*acsch(c\*x)),x)

[Out] Integral(x\*\*4\*(a + b\*acsch(c\*x))\*(d + e\*x\*\*2), x)

### 3.77 $\int x^2 (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=167

$$\frac{1}{3}dx^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{5}ex^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^4 \sqrt{-c^2x^2 - 1}}{20c \sqrt{-c^2x^2}} + \frac{bx (20c^2d - 9e) \tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2 - 1}}\right)}{120c^4 \sqrt{-c^2x^2}} + \frac{bx^2 \sqrt{-c^2x^2 - 1}}{120c^3 \sqrt{-c^2x^2}}$$

[Out]  $\frac{1}{3}d*x^3*(a+b*\operatorname{arccsch}(c*x))+\frac{1}{5}e*x^5*(a+b*\operatorname{arccsch}(c*x))+\frac{1}{120}b*(20*c^2*d-9*e)*x*\operatorname{arctan}(c*x/(-c^2*x^2-1)^{(1/2)})/c^4/(-c^2*x^2)^{(1/2)}+\frac{1}{120}b*(20*c^2*d-9*e)*x^2*(-c^2*x^2-1)^{(1/2)}/c^3/(-c^2*x^2)^{(1/2)}+\frac{1}{20}b*e*x^4*(-c^2*x^2-1)^{(1/2)}/c/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {14, 6302, 12, 459, 321, 217, 203}

$$\frac{1}{3}dx^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{5}ex^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^2 \sqrt{-c^2x^2 - 1} (20c^2d - 9e)}{120c^3 \sqrt{-c^2x^2}} + \frac{bx (20c^2d - 9e) \tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2 - 1}}\right)}{120c^4 \sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2*(d + e*x^2)*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $(b*(20*c^2*d - 9*e)*x^2*\operatorname{Sqrt}[-1 - c^2*x^2])/(120*c^3*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*e*x^4*\operatorname{Sqrt}[-1 - c^2*x^2])/(20*c*\operatorname{Sqrt}[-(c^2*x^2)]) + (d*x^3*(a + b*\operatorname{ArcCsch}[c*x]))/3 + (e*x^5*(a + b*\operatorname{ArcCsch}[c*x]))/5 + (b*(20*c^2*d - 9*e)*x*\operatorname{ArcTan}[(c*x)/\operatorname{Sqrt}[-1 - c^2*x^2]])/(120*c^4*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{MatchQ}[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 14

$\operatorname{Int}[(u_*)((c_*)*(x_))^{(m_)}], x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*u, x], x] /; \operatorname{FreeQ}[\{c, m\}, x] \&\& \operatorname{SumQ}[u] \&\& !\operatorname{LinearQ}[u, x] \&\& !\operatorname{MatchQ}[u, (a_*) + (b_*)*(v_)] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{InverseFunctionQ}[v]$

#### Rule 203

$\operatorname{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*x]/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \|\| \operatorname{GtQ}[b, 0])$

#### Rule 217

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_*) + (b_*)*(x_)^2], x\_Symbol] := \operatorname{Subst}[\operatorname{Int}[1/(1 - b*x^2), x], x, x/\operatorname{Sqrt}[a + b*x^2]] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& !\operatorname{GtQ}[a, 0]$

#### Rule 321

$\operatorname{Int}[(c_*)*(x_))^{(m_)*((a_*) + (b_*)*(x_))^{(n_))^{(p_)}], x\_Symbol] := \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \operatorname{Dist}[(a*c^{(n-1)})/(b*(m+n*p+1)), \operatorname{Int}[(c*x)^{(m-n)}*(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}[\{a, b, c, p\}, x] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{GtQ}[m, n-1] \&\& \operatorname{NeQ}[m+n*p+1, 0] \&\& \operatorname{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned} \int x^2 (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{3} dx^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(bcx) \int \frac{x^2(5d+3ex^2)}{15\sqrt{-1-c^2x^2}}}{\sqrt{-c^2x^2}} \\ &= \frac{1}{3} dx^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(bcx) \int \frac{x^2(5d+3ex^2)}{\sqrt{-1-c^2x^2}}}{15\sqrt{-c^2x^2}} \\ &= \frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + \frac{1}{3} dx^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{csch}^{-1}(cx)) - \\ &= \frac{b(20c^2d - 9e)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + \frac{1}{3} dx^3 (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{b(20c^2d - 9e)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + \frac{1}{3} dx^3 (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{b(20c^2d - 9e)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + \frac{1}{3} dx^3 (a + b \operatorname{csch}^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 119, normalized size = 0.71

$$\frac{c^2x^2 \left( 8ac^3x(5d + 3ex^2) + b\sqrt{\frac{1}{c^2x^2} + 1} (c^2(20d + 6ex^2) - 9e) \right) + 8bc^5x^3 \operatorname{csch}^{-1}(cx)(5d + 3ex^2) + b(9e - 20c^2d)}{120c^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] (c^2\*x^2\*(8\*a\*c^3\*x\*(5\*d + 3\*e\*x^2) + b\*Sqrt[1 + 1/(c^2\*x^2)]\*(-9\*e + c^2\*(20\*d + 6\*e\*x^2))) + 8\*b\*c^5\*x^3\*(5\*d + 3\*e\*x^2)\*ArcCsch[c\*x] + b\*(-20\*c^2\*d + 9\*e)\*Log[(1 + Sqrt[1 + 1/(c^2\*x^2)])\*x])/(120\*c^5)

**fricas [A]** time = 3.02, size = 273, normalized size = 1.63

$$24ac^5ex^5 + 40ac^5dx^3 + 8(5bc^5d + 3bc^5e) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) + (20bc^2d - 9be) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(5*b*c^5*d + 3*b*c^5*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (20*b*c^2*d - 9*b*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 8*(5*b*c^5*d + 3*b*c^5*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3 - 5*b*c^5*d - 3*b*c^5*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*e*x^4 + (20*b*c^4*d - 9*b*c^2*e)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/c^5
```

```
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (ex^2 + d)(b \operatorname{arcsch}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

```
maple [A] time = 0.05, size = 171, normalized size = 1.02
```

$$\frac{a\left(\frac{1}{5}c^5x^5e + \frac{1}{3}c^5x^3d\right)}{c^2} + \frac{b\left(\frac{\operatorname{arcsch}(cx)c^5x^5e}{5} + \frac{\operatorname{arcsch}(cx)c^5x^3d}{3} - \frac{\sqrt{c^2x^2+1}\left(-6ec^3x^3\sqrt{c^2x^2+1} - 20c^3dx\sqrt{c^2x^2+1} + 20c^2d\operatorname{arcsinh}(cx) + 9ecx\sqrt{c^2x^2+1} - 9e\operatorname{arcsinh}(cx)\right)}{120\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] 1/c^3*(a/c^2*(1/5*c^5*x^5*e+1/3*c^5*x^3*d)+b/c^2*(1/5*arccsch(c*x)*c^5*x^5*e+1/3*arccsch(c*x)*c^5*x^3*d-1/120*(c^2*x^2+1)^(1/2)*(-6*e*c^3*x^3*(c^2*x^2+1)^(1/2)-20*c^3*d*x*(c^2*x^2+1)^(1/2)+20*c^2*d*arcsinh(c*x)+9*e*c*x*(c^2*x^2+1)^(1/2)-9*e*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x)
```

```
maxima [A] time = 0.38, size = 227, normalized size = 1.36
```

$$\frac{1}{5} aex^5 + \frac{1}{3} adx^3 + \frac{1}{12} \left( 4x^3 \operatorname{arcsch}(cx) + \frac{2\sqrt{\frac{1}{c^2x^2}+1} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} \right) \left( bd + \frac{1}{80} 16x^5 \operatorname{arcsch}(cx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1)))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```



**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (e x^2 + d) \left( a + b \operatorname{asinh} \left( \frac{1}{c x} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)`

[Out] `int(x^2*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{acsch}(c x)) (d + e x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x**2+d)*(a+b*acsch(c*x)),x)`

[Out] `Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2), x)`

### 3.78 $\int (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=115

$$dx (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3} ex^3 (a + b \operatorname{csch}^{-1}(cx)) - \frac{bx(6c^2d - e) \tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2 - 1}}\right)}{6c^2\sqrt{-c^2x^2}} + \frac{bex^2\sqrt{-c^2x^2 - 1}}{6c\sqrt{-c^2x^2}}$$

[Out] d\*x\*(a+b\*arccsch(c\*x))+1/3\*e\*x^3\*(a+b\*arccsch(c\*x))-1/6\*b\*(6\*c^2\*d-e)\*x\*arc tan(c\*x/(-c^2\*x^2-1)^(1/2))/c^2/(-c^2\*x^2)^(1/2)+1/6\*b\*e\*x^2\*(-c^2\*x^2-1)^(1/2)/c/(-c^2\*x^2)^(1/2)

**Rubi [A]** time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {6292, 12, 388, 217, 203}

$$dx (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3} ex^3 (a + b \operatorname{csch}^{-1}(cx)) - \frac{bx(6c^2d - e) \tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2 - 1}}\right)}{6c^2\sqrt{-c^2x^2}} + \frac{bex^2\sqrt{-c^2x^2 - 1}}{6c\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]),x]

[Out] (b\*e\*x^2\*Sqrt[-1 - c^2\*x^2])/(6\*c\*Sqrt[-(c^2\*x^2)]) + d\*x\*(a + b\*ArcCsch[c\*x]) + (e\*x^3\*(a + b\*ArcCsch[c\*x]))/3 - (b\*(6\*c^2\*d - e)\*x\*ArcTan[(c\*x)/Sqrt[-1 - c^2\*x^2]])/(6\*c^2\*Sqrt[-(c^2\*x^2)])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 6292

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])

#### Rubi steps

$$\begin{aligned}
\int (d + ex^2)(a + bcsch^{-1}(cx)) dx &= dx(a + bcsch^{-1}(cx)) + \frac{1}{3}ex^3(a + bcsch^{-1}(cx)) - \frac{(bcx) \int \frac{3d+ex^2}{3\sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\
&= dx(a + bcsch^{-1}(cx)) + \frac{1}{3}ex^3(a + bcsch^{-1}(cx)) - \frac{(bcx) \int \frac{3d+ex^2}{\sqrt{-1-c^2x^2}} dx}{3\sqrt{-c^2x^2}} \\
&= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} + dx(a + bcsch^{-1}(cx)) + \frac{1}{3}ex^3(a + bcsch^{-1}(cx)) - \frac{(bcx) \int \frac{3d+ex^2}{\sqrt{-1-c^2x^2}} dx}{3\sqrt{-c^2x^2}} \\
&= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} + dx(a + bcsch^{-1}(cx)) + \frac{1}{3}ex^3(a + bcsch^{-1}(cx)) - \frac{(bcx) \int \frac{3d+ex^2}{\sqrt{-1-c^2x^2}} dx}{3\sqrt{-c^2x^2}} \\
&= \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} + dx(a + bcsch^{-1}(cx)) + \frac{1}{3}ex^3(a + bcsch^{-1}(cx)) - \frac{b(6d+ex^2)}{6c\sqrt{-c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.26, size = 135, normalized size = 1.17

$$adx + \frac{1}{3}aex^3 + \frac{bdx\sqrt{\frac{1}{c^2x^2} + 1} \sinh^{-1}(cx)}{\sqrt{c^2x^2 + 1}} + \frac{bex^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{6c} - \frac{be \log\left(x\left(\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1\right)\right)}{6c^3} + bdxcsch^{-1}(cx) + \frac{1}{3}bex^3csch^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] a\*d\*x + (a\*e\*x^3)/3 + (b\*e\*x^2\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)])/(6\*c) + b\*d\*x\*ArcCsch[c\*x] + (b\*e\*x^3\*ArcCsch[c\*x])/3 + (b\*d\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*ArcSinh[c\*x])/Sqrt[1 + c^2\*x^2] - (b\*e\*Log[x\*(1 + Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)])])/6\*c^3

**fricas [B]** time = 1.01, size = 245, normalized size = 2.13

$$2ac^3ex^3 + bc^2ex^2\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 6ac^3dx + 2(3bc^3d + bc^3e) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - (6bc^2d - be) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] 1/6\*(2\*a\*c^3\*e\*x^3 + b\*c^2\*e\*x^2\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 6\*a\*c^3\*d\*x + 2\*(3\*b\*c^3\*d + b\*c^3\*e)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - (6\*b\*c^2\*d - b\*e)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) - 2\*(3\*b\*c^3\*d + b\*c^3\*e)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + 2\*(b\*c^3\*e\*x^3 + 3\*b\*c^3\*d\*x - 3\*b\*c^3\*d - b\*c^3\*e)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/c^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)(b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a), x)

**maple** [A] time = 0.06, size = 126, normalized size = 1.10

$$\frac{a\left(\frac{1}{3}e c^3 x^3 + x c^3 d\right)}{c^2} + \frac{b\left(\frac{\operatorname{arccsch}(cx)e c^3 x^3}{3} + \operatorname{arccsch}(cx)c^3 dx + \frac{\sqrt{c^2 x^2 + 1}\left(6c^2 d \operatorname{arcsinh}(cx) + e c x \sqrt{c^2 x^2 + 1} - e \operatorname{arcsinh}(cx)\right)}{6\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} cx}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c\*(a/c^2\*(1/3\*e\*c^3\*x^3+x\*c^3\*d)+b/c^2\*(1/3\*arccsch(c\*x)\*e\*c^3\*x^3+arccsch(c\*x)\*c^3\*d\*x+1/6\*(c^2\*x^2+1)^(1/2)\*(6\*c^2\*d\*arcsinh(c\*x)+e\*c\*x\*(c^2\*x^2+1)^(1/2)-e\*arcsinh(c\*x)))/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x)

**maxima** [A] time = 0.35, size = 148, normalized size = 1.29

$$\frac{1}{3} a e x^3 + \frac{1}{12} \left( 4 x^3 \operatorname{arcsch}(c x) + \frac{2 \sqrt{\frac{1}{c^2 x^2} + 1} - \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right) + \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} - 1\right)}{c^2 \left(\frac{1}{c^2 x^2} + 1\right) - c^2} \right) b e + a d x + \frac{(2 c x \operatorname{arcsch}(c x) + \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} - 1\right))}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/3\*a\*e\*x^3 + 1/12\*(4\*x^3\*arccsch(c\*x) + (2\*sqrt(1/(c^2\*x^2) + 1)/(c^2\*(1/(c^2\*x^2) + 1) - c^2) - log(sqrt(1/(c^2\*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*b\*e + a\*d\*x + 1/2\*(2\*c\*x\*arccsch(c\*x) + log(sqrt(1/(c^2\*x^2) + 1) + 1) - log(sqrt(1/(c^2\*x^2) + 1) - 1))\*b\*d/c

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e x^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{c x}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(c x)) (d + e x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsch(c\*x)),x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2), x)

$$3.79 \quad \int \frac{(d+ex^2)(a+bcsch^{-1}(cx))}{x^2} dx$$

**Optimal.** Leaf size=91

$$-\frac{d(a+bcsch^{-1}(cx))}{x} + ex(a+bcsch^{-1}(cx)) + \frac{bcd\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}} - \frac{bex \tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}}$$

[Out]  $-d*(a+b*arccsch(c*x))/x+e*x*(a+b*arccsch(c*x))-b*e*x*arctan(c*x/(-c^2*x^2-1)^{(1/2)})/(-c^2*x^2)^{(1/2)}+b*c*d*(-c^2*x^2-1)^{(1/2)}/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 6302, 451, 217, 203}

$$-\frac{d(a+bcsch^{-1}(cx))}{x} + ex(a+bcsch^{-1}(cx)) + \frac{bcd\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}} - \frac{bex \tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^2,x]

[Out]  $(b*c*d*\text{Sqrt}[-1 - c^2*x^2])/ \text{Sqrt}[-(c^2*x^2)] - (d*(a + b*\text{ArcCsch}[c*x]))/x + e*x*(a + b*\text{ArcCsch}[c*x]) - (b*e*x*\text{ArcTan}[(c*x)/\text{Sqrt}[-1 - c^2*x^2]])/\text{Sqrt}[-(c^2*x^2)]$

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_)+(b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 451

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(c\*(e\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[d/e^n, Int[(e\*x)^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n\*(p+1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))

#### Rule 6302

Int[((a\_) + ArcCsch[(c\_)\*(x\_)])\*(b\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m-1)/2, 0] && GtQ[m + 2\*p + 3, 0]))

|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + bcsch^{-1}(cx))}{x^2} dx &= -\frac{d(a + bcsch^{-1}(cx))}{x} + ex(a + bcsch^{-1}(cx)) - \frac{(bcx) \int \frac{-d+ex^2}{x^2\sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{x} + ex(a + bcsch^{-1}(cx)) - \frac{(bcx) \int \frac{-d+ex^2}{x^2\sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{x} + ex(a + bcsch^{-1}(cx)) - \frac{(bcx) \text{Subst}(\int \frac{-d+ex^2}{x^2\sqrt{-1-c^2x^2}} dx, \sqrt{-1-c^2x^2})}{\sqrt{-c^2x^2}} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{x} + ex(a + bcsch^{-1}(cx)) - \frac{bex \tan^{-1}\left(\frac{\sqrt{-1-c^2x^2}}{cx}\right)}{\sqrt{-c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 89, normalized size = 0.98

$$-\frac{ad}{x} + aex + bcd\sqrt{\frac{c^2x^2+1}{c^2x^2}} + \frac{bex\sqrt{\frac{1}{c^2x^2}+1}\sinh^{-1}(cx)}{\sqrt{c^2x^2+1}} - \frac{bdcsch^{-1}(cx)}{x} + bexcsch^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^2,x]

[Out] -((a\*d)/x) + a\*e\*x + b\*c\*d\*Sqrt[(1 + c^2\*x^2)/(c^2\*x^2)] - (b\*d\*ArcCsch[c\*x])/x + b\*e\*x\*ArcCsch[c\*x] + (b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*ArcSinh[c\*x])/Sqrt[1 + c^2\*x^2]

**fricas [B]** time = 0.80, size = 222, normalized size = 2.44

$$bc^2dx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + bc^2dx + acex^2 - bex \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) - acd - (bcd - bce)x \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) + (bcd$$

cx

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="fricas")

[Out] (b\*c^2\*d\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + b\*c^2\*d\*x + a\*c\*e\*x^2 - b\*e\*x\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) - a\*c\*d - (b\*c\*d - b\*c\*e)\*x\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) + (b\*c\*d - b\*c\*e)\*x\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + (b\*c\*e\*x^2 - b\*c\*d + (b\*c\*d - b\*c\*e)\*x)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)))/(c\*x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(b \operatorname{arcsch}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^2, x)

**maple** [A] time = 0.07, size = 107, normalized size = 1.18

$$c \left( \frac{a \left( cxe - \frac{cd}{x} \right)}{c^2} + \frac{b \left( \operatorname{arccsch}(cx) cxe - \frac{\operatorname{arccsch}(cx) cd}{x} + \frac{\sqrt{c^2x^2+1} \left( c^2d\sqrt{c^2x^2+1} + e \operatorname{arcsinh}(cx)cx \right)}{c^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}} \right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^2,x)

[Out] c\*(a/c^2\*(c\*x\*e-c\*d/x)+b/c^2\*(arccsch(c\*x)\*c\*x\*e-arccsch(c\*x)\*c\*d/x+(c^2\*x^2+1)^(1/2)\*(c^2\*d\*(c^2\*x^2+1)^(1/2)+e\*arcsinh(c\*x)\*c\*x)/c^2/x^2/((c^2\*x^2+1)/c^2/x^2)^(1/2)))

**maxima** [A] time = 0.37, size = 84, normalized size = 0.92

$$\left( c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsch}(cx)}{x} \right) bd + aex + \frac{\left( 2cx \operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right) \right) be}{2c} - \frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="maxima")

[Out] (c\*sqrt(1/(c^2\*x^2) + 1) - arccsch(c\*x)/x)\*b\*d + a\*e\*x + 1/2\*(2\*c\*x\*arccsch(c\*x) + log(sqrt(1/(c^2\*x^2) + 1) + 1) - log(sqrt(1/(c^2\*x^2) + 1) - 1))\*b\*e/c - a\*d/x

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^2,x)

[Out] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsch(c\*x))/x\*\*2,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)/x\*\*2, x)

$$3.80 \quad \int \frac{(d+ex^2)(a+bcsch^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=109

$$-\frac{d(a+bcsch^{-1}(cx))}{3x^3} - \frac{e(a+bcsch^{-1}(cx))}{x} - \frac{bc\sqrt{-c^2x^2-1}(2c^2d-9e)}{9\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}$$

[Out]  $-1/3*d*(a+b*arccsch(c*x))/x^3-e*(a+b*arccsch(c*x))/x-1/9*b*c*(2*c^2*d-9*e)*(-c^2*x^2-1)^{(1/2)/(-c^2*x^2)^{(1/2)}+1/9*b*c*d*(-c^2*x^2-1)^{(1/2)}/x^2/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 6302, 12, 453, 264}

$$-\frac{d(a+bcsch^{-1}(cx))}{3x^3} - \frac{e(a+bcsch^{-1}(cx))}{x} - \frac{bc\sqrt{-c^2x^2-1}(2c^2d-9e)}{9\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^4, x]

[Out]  $-(b*c*(2*c^2*d - 9*e)*\text{Sqrt}[-1 - c^2*x^2])/(9*\text{Sqrt}[-(c^2*x^2)]) + (b*c*d*\text{Sqrt}[-1 - c^2*x^2])/(9*x^2*\text{Sqrt}[-(c^2*x^2)]) - (d*(a + b*\text{ArcCsch}[c*x]))/(3*x^3) - (e*(a + b*\text{ArcCsch}[c*x]))/x$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_)+(b\_)\*(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_)+(b\_)\*(x\_))^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

#### Rule 453

Int[((e\_)\*(x\_))^(m\_)\*((a\_)+(b\_)\*(x\_))^(n\_))^(p\_)\*((c\_)+(d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[(a\*d\*(m+1)-b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c-a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

#### Rule 6302

Int[((a\_)+ArcCsch[(c\_)\*(x\_)]\*(b\_))\*((f\_)\*(x\_))^(m\_)\*((d\_)+(e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d+e\*x^2)^p, x]}, Dist[a+b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1-c^2\*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !ILtQ[(m-1)/2, 0] && GtQ[m+2\*p+3, 0]))



|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + bcsch^{-1}(cx))}{x^4} dx &= -\frac{d(a + bcsch^{-1}(cx))}{3x^3} - \frac{e(a + bcsch^{-1}(cx))}{x} - \frac{(bcx) \int \frac{-d-3ex^2}{3x^4\sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\ &= -\frac{d(a + bcsch^{-1}(cx))}{3x^3} - \frac{e(a + bcsch^{-1}(cx))}{x} - \frac{(bcx) \int \frac{-d-3ex^2}{x^4\sqrt{-1-c^2x^2}} dx}{3\sqrt{-c^2x^2}} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{3x^3} - \frac{e(a + bcsch^{-1}(cx))}{x} - \frac{(bc(2c^2d - 9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{3x^3} - \frac{e(a + bcsch^{-1}(cx))}{x} \end{aligned}$$

**Mathematica** [A] time = 0.08, size = 68, normalized size = 0.62

$$\frac{-3a(d + 3ex^2) + bcx\sqrt{\frac{1}{c^2x^2} + 1}(-2c^2dx^2 + d + 9ex^2) - 3bcsch^{-1}(cx)(d + 3ex^2)}{9x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^4, x]

[Out] (-3\*a\*(d + 3\*e\*x^2) + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(d - 2\*c^2\*d\*x^2 + 9\*e\*x^2) - 3\*b\*(d + 3\*e\*x^2)\*ArcCsch[c\*x])/(9\*x^3)

**fricas** [A] time = 0.95, size = 105, normalized size = 0.96

$$\frac{9aex^2 + 3ad + 3(3bex^2 + bd) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}+1}}{cx}\right) - (bcdx - (2bc^3d - 9bce)x^3)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^4, x, algorithm="fricas")

[Out] -1/9\*(9\*a\*e\*x^2 + 3\*a\*d + 3\*(3\*b\*e\*x^2 + b\*d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - (b\*c\*d\*x - (2\*b\*c^3\*d - 9\*b\*c\*e)\*x^3)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/x^3

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(b \operatorname{arcsch}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^4, x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^4, x)

**maple** [A] time = 0.07, size = 122, normalized size = 1.12

$$c^3 \left( \frac{a \left( -\frac{e}{cx} - \frac{d}{3cx^3} \right)}{c^2} + \frac{b \left( -\frac{\operatorname{arcsch}(cx)e}{cx} - \frac{\operatorname{arcsch}(cx)d}{3cx^3} - \frac{(c^2x^2+1)(2c^4dx^2-9c^2x^2e-c^2d)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}} c^4x^4} \right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^4,x)

[Out]  $c^3 * (a/c^2 * (-e/c/x - 1/3 * c*d/x^3) + b/c^2 * (-\operatorname{arccsch}(c*x) * e/c/x - 1/3 * \operatorname{arccsch}(c*x) / c*d/x^3 - 1/9 * (c^2*x^2+1) * (2*c^4*d*x^2 - 9*c^2*e*x^2 - c^2*d) / ((c^2*x^2+1)/c^2/x^2)^{(1/2)} / c^4/x^4))$

**maxima** [A] time = 0.35, size = 91, normalized size = 0.83

$$\left( c\sqrt{\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsch}(cx)}{x} \right) be + \frac{1}{9} bd \left( \frac{c^4 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{\frac{1}{c^2x^2} + 1}}{c} - \frac{3 \operatorname{arcsch}(cx)}{x^3} \right) - \frac{ae}{x} - \frac{ad}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^4,x, algorithm="maxima")

[Out]  $(c*\sqrt{1/(c^2*x^2) + 1} - \operatorname{arccsch}(c*x)/x)*b*e + 1/9*b*d*((c^4*(1/(c^2*x^2) + 1)^{(3/2)} - 3*c^4*\sqrt{1/(c^2*x^2) + 1})/c - 3*\operatorname{arccsch}(c*x)/x^3) - a*e/x - 1/3*a*d/x^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^4,x)

[Out] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsch(c\*x))/x\*\*4,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)/x\*\*4, x)

$$3.81 \quad \int \frac{(d+ex^2)(a+bcsch^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=158

$$\frac{d(a+bcsch^{-1}(cx))}{5x^5} - \frac{e(a+bcsch^{-1}(cx))}{3x^3} - \frac{bc\sqrt{-c^2x^2-1}(12c^2d-25e)}{225x^2\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{25x^4\sqrt{-c^2x^2}} + \frac{2bc^3\sqrt{-c^2x^2-1}}{225\sqrt{-c^2x^2}}$$

[Out]  $-1/5*d*(a+b*arccsch(c*x))/x^5-1/3*e*(a+b*arccsch(c*x))/x^3+2/225*b*c^3*(12*c^2*d-25*e)*(-c^2*x^2-1)^{(1/2)}/(-c^2*x^2)^{(1/2)}+1/25*b*c*d*(-c^2*x^2-1)^{(1/2)}/x^4/(-c^2*x^2)^{(1/2)}-1/225*b*c*(12*c^2*d-25*e)*(-c^2*x^2-1)^{(1/2)}/x^2/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {14, 6302, 12, 453, 271, 264}

$$\frac{d(a+bcsch^{-1}(cx))}{5x^5} - \frac{e(a+bcsch^{-1}(cx))}{3x^3} + \frac{2bc^3\sqrt{-c^2x^2-1}(12c^2d-25e)}{225\sqrt{-c^2x^2}} - \frac{bc\sqrt{-c^2x^2-1}(12c^2d-25e)}{225x^2\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{25x^4\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^6, x]

[Out]  $(2*b*c^3*(12*c^2*d - 25*e)*\text{Sqrt}[-1 - c^2*x^2])/(225*\text{Sqrt}[-(c^2*x^2)]) + (b*c*d*\text{Sqrt}[-1 - c^2*x^2])/(25*x^4*\text{Sqrt}[-(c^2*x^2)]) - (b*c*(12*c^2*d - 25*e)*\text{Sqrt}[-1 - c^2*x^2])/(225*x^2*\text{Sqrt}[-(c^2*x^2)]) - (d*(a + b*\text{ArcCsch}[c*x]))/(5*x^5) - (e*(a + b*\text{ArcCsch}[c*x]))/(3*x^3)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_.)\*(v\_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

#### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m+1)\*(a+b\*x^n)^(p+1))/(a\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*(m+1)), Int[x^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

#### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (

LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

### Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + bcsch^{-1}(cx))}{x^6} dx &= -\frac{d(a + bcsch^{-1}(cx))}{5x^5} - \frac{e(a + bcsch^{-1}(cx))}{3x^3} - \frac{(bcx) \int \frac{-3d-5ex^2}{15x^6\sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\ &= -\frac{d(a + bcsch^{-1}(cx))}{5x^5} - \frac{e(a + bcsch^{-1}(cx))}{3x^3} - \frac{(bcx) \int \frac{-3d-5ex^2}{x^6\sqrt{-1-c^2x^2}} dx}{15\sqrt{-c^2x^2}} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{5x^5} - \frac{e(a + bcsch^{-1}(cx))}{3x^3} - \frac{bc(12c^2d - 25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{bc(12c^2d - 25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{5x^5} - \frac{e(a + bcsch^{-1}(cx))}{3x^3} \\ &= \frac{2bc^3(12c^2d - 25e)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{bc(12c^2d - 25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 93, normalized size = 0.59

$$\frac{-15a(3d + 5ex^2) + bcx\sqrt{\frac{1}{c^2x^2} + 1} (25ex^2(1 - 2c^2x^2) + 3d(8c^4x^4 - 4c^2x^2 + 3)) - 15bcsch^{-1}(cx)(3d + 5ex^2)}{225x^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^6, x]

[Out] (-15\*a\*(3\*d + 5\*e\*x^2) + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(25\*e\*x^2\*(1 - 2\*c^2\*x^2) + 3\*d\*(3 - 4\*c^2\*x^2 + 8\*c^4\*x^4)) - 15\*b\*(3\*d + 5\*e\*x^2)\*ArcCsch[c\*x])/(225\*x^5)

**fricas [A]** time = 0.95, size = 127, normalized size = 0.80

$$\frac{75 a e x^2 + 45 a d + 15 (5 b e x^2 + 3 b d) \log \left( \frac{c x \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2} + 1}}{c x} \right) - (2 (12 b c^5 d - 25 b c^3 e) x^5 + 9 b c d x - (12 b c^3 d - 25 b c e) x)}{225 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^6, x, algorithm="fricas")

[Out] -1/225\*(75\*a\*e\*x^2 + 45\*a\*d + 15\*(5\*b\*e\*x^2 + 3\*b\*d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - (2\*(12\*b\*c^5\*d - 25\*b\*c^3\*e)\*x^5 + 9\*b\*c\*d\*x - (12\*b\*c^3\*d - 25\*b\*c\*e)\*x^3)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/x^5

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(b \operatorname{arcsch}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^6,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^6, x)

**maple** [A] time = 0.06, size = 140, normalized size = 0.89

$$c^5 \left( \frac{a \left( -\frac{e}{3c^3x^3} - \frac{d}{5c^3x^5} \right)}{c^2} + \frac{b \left( -\frac{\operatorname{arcsch}(cx)e}{3c^3x^3} - \frac{\operatorname{arcsch}(cx)d}{5c^3x^5} + \frac{(c^2x^2+1)(24c^6dx^4-50c^4ex^4-12c^4dx^2+25c^2x^2e+9c^2d)}{225\sqrt{\frac{c^2x^2+1}{c^2x^2}}c^6x^6} \right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^6,x)

[Out] c^5\*(a/c^2\*(-1/3\*e/c^3/x^3-1/5/c^3\*d/x^5)+b/c^2\*(-1/3\*arccsch(c\*x)\*e/c^3/x^3-1/5\*arccsch(c\*x)/c^3\*d/x^5+1/225\*(c^2\*x^2+1)\*(24\*c^6\*d\*x^4-50\*c^4\*e\*x^4-12\*c^4\*d\*x^2+25\*c^2\*x^2\*e+9\*c^2\*d)/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6))

**maxima** [A] time = 0.32, size = 132, normalized size = 0.84

$$\frac{1}{75} bd \left( \frac{3c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} - 10c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 15c^6 \sqrt{\frac{1}{c^2x^2} + 1}}{c} - \frac{15 \operatorname{arcsch}(cx)}{x^5} \right) + \frac{1}{9} be \left( \frac{c^4 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{\frac{1}{c^2x^2} + 1}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^6,x, algorithm="maxima")

[Out] 1/75\*b\*d\*((3\*c^6\*(1/(c^2\*x^2) + 1)^(5/2) - 10\*c^6\*(1/(c^2\*x^2) + 1)^(3/2) + 15\*c^6\*sqrt(1/(c^2\*x^2) + 1))/c - 15\*arccsch(c\*x)/x^5) + 1/9\*b\*e\*((c^4\*(1/(c^2\*x^2) + 1)^(3/2) - 3\*c^4\*sqrt(1/(c^2\*x^2) + 1))/c - 3\*arccsch(c\*x)/x^3) - 1/3\*a\*e/x^3 - 1/5\*a\*d/x^5

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^6,x)

[Out] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^6, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))(d + ex^2)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsch(c\*x))/x\*\*6,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)/x\*\*6, x)

$$3.82 \quad \int \frac{(d+ex^2)(a+bcsch^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=205

$$\frac{d(a+bcsch^{-1}(cx))}{7x^7} - \frac{e(a+bcsch^{-1}(cx))}{5x^5} - \frac{bc\sqrt{-c^2x^2-1}(30c^2d-49e)}{1225x^4\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-c^2x^2-1}}{49x^6\sqrt{-c^2x^2}} - \frac{8bc^5\sqrt{-c^2x^2-1}(30c^2d-49e)}{3675\sqrt{-c^2x^2}}$$

[Out]  $-1/7*d*(a+b*arccsch(c*x))/x^7-1/5*e*(a+b*arccsch(c*x))/x^5-8/3675*b*c^5*(30*c^2*d-49*e)*(-c^2*x^2-1)^{(1/2)}/(-c^2*x^2)^{(1/2)}+1/49*b*c*d*(-c^2*x^2-1)^{(1/2)}/x^6/(-c^2*x^2)^{(1/2)}-1/1225*b*c*(30*c^2*d-49*e)*(-c^2*x^2-1)^{(1/2)}/x^4/(-c^2*x^2)^{(1/2)}+4/3675*b*c^3*(30*c^2*d-49*e)*(-c^2*x^2-1)^{(1/2)}/x^2/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.12, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {14, 6302, 12, 453, 271, 264}

$$\frac{d(a+bcsch^{-1}(cx))}{7x^7} - \frac{e(a+bcsch^{-1}(cx))}{5x^5} - \frac{8bc^5\sqrt{-c^2x^2-1}(30c^2d-49e)}{3675\sqrt{-c^2x^2}} + \frac{4bc^3\sqrt{-c^2x^2-1}(30c^2d-49e)}{3675x^2\sqrt{-c^2x^2}} - \frac{bcd\sqrt{-c^2x^2-1}}{49x^6\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^8, x]

[Out]  $(-8*b*c^5*(30*c^2*d - 49*e)*\text{Sqrt}[-1 - c^2*x^2])/(3675*\text{Sqrt}[-(c^2*x^2)]) + (b*c*d*\text{Sqrt}[-1 - c^2*x^2])/(49*x^6*\text{Sqrt}[-(c^2*x^2)]) - (b*c*(30*c^2*d - 49*e)*\text{Sqrt}[-1 - c^2*x^2])/(1225*x^4*\text{Sqrt}[-(c^2*x^2)]) + (4*b*c^3*(30*c^2*d - 49*e)*\text{Sqrt}[-1 - c^2*x^2])/(3675*x^2*\text{Sqrt}[-(c^2*x^2)]) - (d*(a + b*\text{ArcCsch}[c*x]))/(7*x^7) - (e*(a + b*\text{ArcCsch}[c*x]))/(5*x^5)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m+1)\*(a + b\*x^n)^(p+1))/(a\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*(m+1)), Int[x^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n + p + 1], 0] && NeQ[m, -1]

#### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*

$x)^{(m+n)}(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{IntegerQ}[n] \parallel \text{GtQ}[e, 0]) \&\& ((\text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1]) \parallel (\text{LtQ}[n, 0] \&\& \text{GtQ}[m + n, -1])) \&\& !\text{ILtQ}[p, -1]$

### Rule 6302

$\text{Int}[(a + \text{ArcSch}[c*x])(b*x)^m * (d + e*x^2)^p, x\_Symbol] := \text{With}[\{u = \text{IntHide}[(f*x)^m * (d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcSch}[c*x], u, x] - \text{Dist}[(b*c*x)/\text{Sqrt}[-(c^2*x^2)], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[-1 - c^2*x^2]), x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m, p\}, x] \&\& ((\text{IGtQ}[p, 0] \&\& !(\text{ILtQ}[(m - 1)/2, 0] \&\& \text{GtQ}[m + 2*p + 3, 0])) \parallel (\text{IGtQ}[(m + 1)/2, 0] \&\& !(\text{ILtQ}[p, 0] \&\& \text{GtQ}[m + 2*p + 3, 0])) \parallel (\text{ILtQ}[(m + 2*p + 1)/2, 0] \&\& !\text{ILtQ}[(m - 1)/2, 0]))$

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + bcsch^{-1}(cx))}{x^8} dx &= -\frac{d(a + bcsch^{-1}(cx))}{7x^7} - \frac{e(a + bcsch^{-1}(cx))}{5x^5} - \frac{(bcx) \int \frac{-5d-7ex^2}{35x^8\sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\ &= -\frac{d(a + bcsch^{-1}(cx))}{7x^7} - \frac{e(a + bcsch^{-1}(cx))}{5x^5} - \frac{(bcx) \int \frac{-5d-7ex^2}{x^8\sqrt{-1-c^2x^2}} dx}{35\sqrt{-c^2x^2}} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{7x^7} - \frac{e(a + bcsch^{-1}(cx))}{5x^5} - \frac{(bc(30c^2d - 49e)\sqrt{-1-c^2x^2})}{1225x^4\sqrt{-c^2x^2}} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d - 49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} - \frac{d(a + bcsch^{-1}(cx))}{7x^7} - \frac{e(a + bcsch^{-1}(cx))}{5x^5} \\ &= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d - 49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{4bc^3(30c^2d - 49e)\sqrt{-1-c^2x^2}}{3675x^2\sqrt{-c^2x^2}} \\ &= -\frac{8bc^5(30c^2d - 49e)\sqrt{-1-c^2x^2}}{3675\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d - 49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 109, normalized size = 0.53

$$\frac{-105a(5d + 7ex^2) + bcx\sqrt{\frac{1}{c^2x^2} + 1}(49ex^2(8c^4x^4 - 4c^2x^2 + 3) - 15d(16c^6x^6 - 8c^4x^4 + 6c^2x^2 - 5)) - 105bcsc^{-1}(cx)}{3675x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcSch[c\*x]))/x^8, x]

[Out] (-105\*a\*(5\*d + 7\*e\*x^2) + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(49\*e\*x^2\*(3 - 4\*c^2\*x^2 + 8\*c^4\*x^4) - 15\*d\*(-5 + 6\*c^2\*x^2 - 8\*c^4\*x^4 + 16\*c^6\*x^6)) - 105\*b\*(5\*d + 7\*e\*x^2)\*ArcSch[c\*x])/(3675\*x^7)

**fricas [A]** time = 1.04, size = 146, normalized size = 0.71

$$\frac{735aex^2 + 525ad + 105(7bex^2 + 5bd) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2} + 1}}{cx}\right) + (8(30bc^7d - 49bc^5e)x^7 - 4(30bc^5d - 49bc^3e))}{3675x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^8,x, algorithm="fricas")

[Out]  $-1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)) + (8*(30*b*c^7*d - 49*b*c^5*e)*x^7 - 4*(30*b*c^5*d - 49*b*c^3*e)*x^5 - 75*b*c*d*x + 3*(30*b*c^3*d - 49*b*c*e)*x^3)*\sqrt{rt((c^2*x^2 + 1)/(c^2*x^2))}/x^7$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(b \operatorname{arcsch}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^8,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^8, x)

**maple** [A] time = 0.06, size = 158, normalized size = 0.77

$$c^7 \left( \frac{a \left( -\frac{d}{7c^5x^7} - \frac{e}{5c^5x^5} \right)}{c^2} + \frac{b \left( -\frac{\operatorname{arccsch}(cx)d}{7c^5x^7} - \frac{\operatorname{arccsch}(cx)e}{5c^5x^5} - \frac{(c^2x^2+1)(240c^8dx^6-392c^6ex^6-120c^6dx^4+196c^4ex^4+90c^4dx^2-147c^2x^2e-75c^2d)}{3675\sqrt{\frac{c^2x^2+1}{c^2x^2}}c^8x^8} \right)}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^8,x)

[Out]  $c^7*(a/c^2*(-1/7/c^5*d/x^7-1/5*e/c^5/x^5)+b/c^2*(-1/7*arccsch(c*x)/c^5*d/x^7-1/5*arccsch(c*x)*e/c^5/x^5-1/3675*(c^2*x^2+1)*(240*c^8*d*x^6-392*c^6*e*x^6-120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2-147*c^2*e*x^2-75*c^2*d)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^8/x^8)$

**maxima** [A] time = 0.33, size = 165, normalized size = 0.80

$$\frac{1}{245} bd \left( \frac{5c^8 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{7}{2}} - 21c^8 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} + 35c^8 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 35c^8 \sqrt{\frac{1}{c^2x^2} + 1}}{c} - \frac{35 \operatorname{arcsch}(cx)}{x^7} \right) + \frac{1}{75} be \left( \frac{3c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{7}{2}} - 21c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} + 35c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 35c^6 \sqrt{\frac{1}{c^2x^2} + 1}}{c} - \frac{35 \operatorname{arcsch}(cx)}{x^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^8,x, algorithm="maxima")

[Out]  $1/245*b*d*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/2) + 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*\sqrt{1/(c^2*x^2) + 1})/c - 35*\operatorname{arcsch}(c*x)/x^7) + 1/75*b*e*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) + 15*c^6*\sqrt{1/(c^2*x^2) + 1})/c - 15*\operatorname{arccsch}(c*x)/x^5) - 1/5*a*e/x^5 - 1/7*a*d/x^7$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^8,x)



[Out] `int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^8, x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))(d + ex^2)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)*(a+b*acsch(c*x))/x**8, x)`

[Out] `Integral((a + b*acsch(c*x))*(d + e*x**2)/x**8, x)`

### 3.83 $\int x^5 (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=204

$$\frac{1}{6} dx^6 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx(-c^2x^2-1)^{5/2}(4c^2d-9e)}{120c^7\sqrt{-c^2x^2}} + \frac{bx(-c^2x^2-1)^{3/2}(8c^2d-9e)}{72c^7\sqrt{-c^2x^2}} + \dots$$

[Out]  $\frac{1}{6}d*x^6*(a+b*\operatorname{arccsch}(c*x))+\frac{1}{8}*e*x^8*(a+b*\operatorname{arccsch}(c*x))+\frac{1}{72}*b*(8*c^2*d-9*e)*x*(-c^2*x^2-1)^{(3/2)}/c^7/(-c^2*x^2)^{(1/2)}+\frac{1}{120}*b*(4*c^2*d-9*e)*x*(-c^2*x^2-1)^{(5/2)}/c^7/(-c^2*x^2)^{(1/2)}-\frac{1}{56}*b*e*x*(-c^2*x^2-1)^{(7/2)}/c^7/(-c^2*x^2)^{(1/2)}+\frac{1}{24}*b*(4*c^2*d-3*e)*x*(-c^2*x^2-1)^{(1/2)}/c^7/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.16, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 6302, 12, 446, 77}

$$\frac{1}{6} dx^6 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \operatorname{csch}^{-1}(cx)) + \frac{bx(-c^2x^2-1)^{5/2}(4c^2d-9e)}{120c^7\sqrt{-c^2x^2}} + \frac{bx(-c^2x^2-1)^{3/2}(8c^2d-9e)}{72c^7\sqrt{-c^2x^2}} + \dots$$

Antiderivative was successfully verified.

[In] `Int[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]`

[Out]  $(b*(4*c^2*d - 3*e)*x*\operatorname{Sqrt}[-1 - c^2*x^2])/(24*c^7*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*(8*c^2*d - 9*e)*x*(-1 - c^2*x^2)^{(3/2)})/(72*c^7*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*(4*c^2*d - 9*e)*x*(-1 - c^2*x^2)^{(5/2)})/(120*c^7*\operatorname{Sqrt}[-(c^2*x^2)]) - (b*e*x*(-1 - c^2*x^2)^{(7/2)})/(56*c^7*\operatorname{Sqrt}[-(c^2*x^2)]) + (d*x^6*(a + b*\operatorname{ArcCsch}[c*x]))/6 + (e*x^8*(a + b*\operatorname{ArcCsch}[c*x]))/8$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 14

`Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]`

#### Rule 77

`Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0]) || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))`

#### Rule 446

`Int[(x_)^m_)*((a_) + (b_)*(x_)^n_))^(p_)*((c_) + (d_)*(x_)^n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

#### Rule 6302

`Int[((a_) + ArcCsch[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di`

```

st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int x^5 (d + ex^2) (a + bcsch^{-1}(cx)) dx &= \frac{1}{6} dx^6 (a + bcsch^{-1}(cx)) + \frac{1}{8} ex^8 (a + bcsch^{-1}(cx)) - \frac{(bcx) \int \frac{x^5(4d+3ex^2)}{24\sqrt{-1-c^2x^2}}}{\sqrt{-c^2x^2}} \\
&= \frac{1}{6} dx^6 (a + bcsch^{-1}(cx)) + \frac{1}{8} ex^8 (a + bcsch^{-1}(cx)) - \frac{(bcx) \int \frac{x^5(4d+3ex^2)}{\sqrt{-1-c^2x^2}}}{24\sqrt{-c^2x^2}} \\
&= \frac{1}{6} dx^6 (a + bcsch^{-1}(cx)) + \frac{1}{8} ex^8 (a + bcsch^{-1}(cx)) - \frac{(bcx) \text{Subst}\left(\int \frac{x^2}{\sqrt{-c^2x^2}}\right)}{48\sqrt{-c^2x^2}} \\
&= \frac{1}{6} dx^6 (a + bcsch^{-1}(cx)) + \frac{1}{8} ex^8 (a + bcsch^{-1}(cx)) - \frac{(bcx) \text{Subst}\left(\int \left(\frac{1}{c^6}\right)\right)}{48\sqrt{-c^2x^2}} \\
&= \frac{b(4c^2d - 3e)x\sqrt{-1 - c^2x^2}}{24c^7\sqrt{-c^2x^2}} + \frac{b(8c^2d - 9e)x(-1 - c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}} + \frac{b(4c^2d - 3e)}{24c^7}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 114, normalized size = 0.56

$$x \left( 105ax^5(4d + 3ex^2) + \frac{b\sqrt{\frac{1}{c^2x^2} + 1}(c^6(84dx^4 + 45ex^6) - 2c^4(56dx^2 + 27ex^4) + 8c^2(28d + 9ex^2) - 144e)}{c^7} + 105bx^5csch^{-1}(cx)(4d + 3ex^2) \right)$$


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Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] (x\*(105\*a\*x^5\*(4\*d + 3\*e\*x^2) + (b\*Sqrt[1 + 1/(c^2\*x^2)]\*(-144\*e + 8\*c^2\*(2\*8\*d + 9\*e\*x^2) - 2\*c^4\*(56\*d\*x^2 + 27\*e\*x^4) + c^6\*(84\*d\*x^4 + 45\*e\*x^6))))/c^7 + 105\*b\*x^5\*(4\*d + 3\*e\*x^2)\*ArcCsch[c\*x])/2520

**fricas [A]** time = 0.82, size = 165, normalized size = 0.81

$$315 ac^7 ex^8 + 420 ac^7 dx^6 + 105 (3 bc^7 ex^8 + 4 bc^7 dx^6) \log \left( \frac{cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx} \right) + (45 bc^6 ex^7 + 6 (14 bc^6 d - 9 bc^4 e) x^5)$$


---

2520 c^7

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] 1/2520\*(315\*a\*c^7\*e\*x^8 + 420\*a\*c^7\*d\*x^6 + 105\*(3\*b\*c^7\*e\*x^8 + 4\*b\*c^7\*d\*x^6)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (45\*b\*c^6\*e\*x^7 + 6\*(14\*b\*c^6\*d - 9\*b\*c^4\*e)\*x^5 - 8\*(14\*b\*c^4\*d - 9\*b\*c^2\*e)\*x^3 + 16\*(14\*b\*c^2\*d - 9\*b\*e)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2))/c^7

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)(b \operatorname{arcsch}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*x^5, x)

**maple** [A] time = 0.06, size = 152, normalized size = 0.75

$$\frac{a\left(\frac{1}{8}ec^8x^8 + \frac{1}{6}c^8dx^6\right)}{c^2} + \frac{b\left(\frac{\operatorname{arccsch}(cx)ec^8x^8}{8} + \frac{\operatorname{arccsch}(cx)c^8x^6d}{6} + \frac{(c^2x^2+1)(45c^6ex^6+84c^6dx^4-54c^4ex^4-112c^4dx^2+72c^2x^2e+224c^2d-144e)}{2520\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx}\right)}{c^2}}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x)

[Out]  $\frac{1}{c^6} \left( \frac{a}{c^2} \left( \frac{1}{8} e c^8 x^8 + \frac{1}{6} c^8 d x^6 \right) + b \left( \frac{1}{8} \operatorname{arccsch}(c x) e c^8 x^8 + \frac{1}{6} \operatorname{arccsch}(c x) c^8 x^6 d + \frac{1}{2520} (c^2 x^2 + 1) (45 c^6 e x^6 + 84 c^6 d x^4 - 54 c^4 e x^4 - 112 c^4 d x^2 + 72 c^2 e x^2 + 224 c^2 d - 144 e) \right) \right) / \left( (c^2 x^2 + 1) / c^2 / x^2 \right)^{(1/2)} / c / x$

**maxima** [A] time = 0.34, size = 176, normalized size = 0.86

$$\frac{1}{8} a e x^8 + \frac{1}{6} a d x^6 + \frac{1}{90} \left( 15 x^6 \operatorname{arcsch}(c x) + \frac{3 c^4 x^5 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} - 10 c^2 x^3 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 15 x \sqrt{\frac{1}{c^2 x^2} + 1}}{c^5} \right) b d + \frac{1}{280} \left( 35 x^8 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out]  $\frac{1}{8} a e x^8 + \frac{1}{6} a d x^6 + \frac{1}{90} \left( 15 x^6 \operatorname{arccsch}(c x) + \frac{3 c^4 x^5 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{5}{2}} - 10 c^2 x^3 \left( \frac{1}{c^2 x^2} + 1 \right)^{\frac{3}{2}} + 15 x \sqrt{\frac{1}{c^2 x^2} + 1}}{c^5} \right) b d + \frac{1}{280} \left( 35 x^8 \right)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 (e x^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{c x}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x^5\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + b \operatorname{acsch}(c x)) (d + e x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(e\*x\*\*2+d)\*(a+b\*acsch(c\*x)),x)

[Out] Integral(x\*\*5\*(a + b\*acsch(c\*x))\*(d + e\*x\*\*2), x)

### 3.84 $\int x^3 (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=159

$$\frac{1}{4} dx^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{csch}^{-1}(cx)) - \frac{bx(-c^2x^2 - 1)^{3/2} (3c^2d - 4e)}{36c^5 \sqrt{-c^2x^2}} - \frac{bx\sqrt{-c^2x^2 - 1} (3c^2d - 2e)}{12c^5 \sqrt{-c^2x^2}} + \dots$$

[Out]  $\frac{1}{4} d x^4 (a + b \operatorname{arccsch}(c x)) + \frac{1}{6} e x^6 (a + b \operatorname{arccsch}(c x)) - \frac{1}{36} b (3 c^2 d - 4 e) x (-c^2 x^2 - 1)^{3/2} / c^5 / (-c^2 x^2)^{1/2} + \frac{1}{30} b e x (-c^2 x^2 - 1)^{5/2} / c^5 / (-c^2 x^2)^{1/2} - \frac{1}{12} b (3 c^2 d - 2 e) x (-c^2 x^2 - 1)^{1/2} / c^5 / (-c^2 x^2)^{1/2}$

**Rubi [A]** time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {14, 6302, 12, 446, 77}

$$\frac{1}{4} dx^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{csch}^{-1}(cx)) - \frac{bx(-c^2x^2 - 1)^{3/2} (3c^2d - 4e)}{36c^5 \sqrt{-c^2x^2}} - \frac{bx\sqrt{-c^2x^2 - 1} (3c^2d - 2e)}{12c^5 \sqrt{-c^2x^2}} + \dots$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3(d + e x^2)(a + b \operatorname{ArcCsch}[c x]), x]$

[Out]  $-(b(3c^2d - 2e)x\sqrt{-1 - c^2x^2}) / (12c^5\sqrt{-(c^2x^2)}) - (b(3c^2d - 4e)x(-1 - c^2x^2)^{3/2}) / (36c^5\sqrt{-(c^2x^2)}) + (bex(-1 - c^2x^2)^{5/2}) / (30c^5\sqrt{-(c^2x^2)}) + (dx^4(a + b \operatorname{ArcCsch}[c x])) / 4 + (ex^6(a + b \operatorname{ArcCsch}[c x])) / 6$

#### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_\*)(v\_)] /; FreeQ[b, x]

#### Rule 14

$\operatorname{Int}[(u_*)((c_*)(x_))^{(m_)}), x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(c x)^m u, x], x] /;$  FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_\*) + (b\_\*)(v\_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

#### Rule 77

$\operatorname{Int}[(a_*) + (b_*)(x_)*((c_*) + (d_*)(x_))^{(n_)}*((e_*) + (f_*)(x_))^{(p_)}), x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b x)(c + d x)^n (e + f x)^p, x], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

#### Rule 446

$\operatorname{Int}[(x_)^{(m_)}*((a_*) + (b_*)(x_))^{(n_)}*((c_*) + (d_*)(x_))^{(q_)}), x\_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1)}(a + b x)^p (c + d x)^q, x], x, x^n], x] /;$  FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 6302

$\operatorname{Int}[(a_*) + \operatorname{ArcCsch}[(c_*)(x_)]*(b_)*((f_*)(x_))^{(m_)}*((d_*) + (e_*)(x_)^2)^{(p_)}), x\_Symbol] := \operatorname{With}[\{u = \operatorname{IntHide}[(f x)^m (d + e x^2)^p, x]\}, \operatorname{Di}$

```
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int x^3 (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{4} dx^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(bcx) \int \frac{x^3(3d+2ex^2)}{12\sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\ &= \frac{1}{4} dx^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(bcx) \int \frac{x^3(3d+2ex^2)}{\sqrt{-1-c^2x^2}} dx}{12\sqrt{-c^2x^2}} \\ &= \frac{1}{4} dx^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(bcx) \operatorname{Subst}\left(\int \frac{x(3d+2ex^2)}{\sqrt{-1-c^2x^2}} dx\right)}{24\sqrt{-c^2x^2}} \\ &= \frac{1}{4} dx^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{csch}^{-1}(cx)) - \frac{(bcx) \operatorname{Subst}\left(\int \left(\frac{-3c^2}{c^4\sqrt{-1-c^2x^2}}\right) dx\right)}{24\sqrt{-c^2x^2}} \\ &= -\frac{b(3c^2d - 2e)x\sqrt{-1 - c^2x^2}}{12c^5\sqrt{-c^2x^2}} - \frac{b(3c^2d - 4e)x(-1 - c^2x^2)^{3/2}}{36c^5\sqrt{-c^2x^2}} + \frac{bex(-1 - c^2x^2)^{3/2}}{30c^5\sqrt{-c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.21, size = 97, normalized size = 0.61

$$\frac{1}{180} x \left( 15ax^3(3d + 2ex^2) + \frac{b\sqrt{\frac{1}{c^2x^2} + 1} (3c^4(5dx^2 + 2ex^4) - 2c^2(15d + 4ex^2) + 16e)}{c^5} + 15bx^3 \operatorname{csch}^{-1}(cx)(3d + 2ex^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] (x\*(15\*a\*x^3\*(3\*d + 2\*e\*x^2) + (b\*Sqrt[1 + 1/(c^2\*x^2)]\*(16\*e - 2\*c^2\*(15\*d + 4\*e\*x^2) + 3\*c^4\*(5\*d\*x^2 + 2\*e\*x^4))))/c^5 + 15\*b\*x^3\*(3\*d + 2\*e\*x^2)\*ArcCsch[c\*x])/180

**fricas [A]** time = 0.89, size = 144, normalized size = 0.91

$$\frac{30ac^5ex^6 + 45ac^5dx^4 + 15(2bc^5ex^6 + 3bc^5dx^4) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx}\right) + (6bc^4ex^5 + (15bc^4d - 8bc^2e)x^3 - 2(15bc^2d - 8bc^2e)x)}{180c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] 1/180\*(30\*a\*c^5\*e\*x^6 + 45\*a\*c^5\*d\*x^4 + 15\*(2\*b\*c^5\*e\*x^6 + 3\*b\*c^5\*d\*x^4)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (6\*b\*c^4\*e\*x^5 + (15\*b\*c^4\*d - 8\*b\*c^2\*e)\*x^3 - 2\*(15\*b\*c^2\*d - 8\*b\*e)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^5

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)(b \operatorname{arcsch}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*x^3, x)

**maple** [A] time = 0.05, size = 134, normalized size = 0.84

$$\frac{a\left(\frac{1}{6}c^6ex^6+\frac{1}{4}c^6dx^4\right)}{c^2} + \frac{b\left(\frac{\operatorname{arccsch}(cx)c^6x^6e}{6} + \frac{\operatorname{arccsch}(cx)c^6x^4d}{4} + \frac{(c^2x^2+1)(6c^4ex^4+15c^4dx^2-8c^2x^2e-30c^2d+16e)}{180\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx}\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c^4\*(a/c^2\*(1/6\*c^6\*e\*x^6+1/4\*c^6\*d\*x^4)+b/c^2\*(1/6\*arccsch(c\*x)\*c^6\*x^6\*e+1/4\*arccsch(c\*x)\*c^6\*x^4\*d+1/180\*(c^2\*x^2+1)\*(6\*c^4\*e\*x^4+15\*c^4\*d\*x^2-8\*c^2\*e\*x^2-30\*c^2\*d+16\*e)/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x))

**maxima** [A] time = 0.31, size = 137, normalized size = 0.86

$$\frac{1}{6}aex^6 + \frac{1}{4}adx^4 + \frac{1}{12}\left(3x^4 \operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2} + 1}}{c^3}\right)bd + \frac{1}{90}\left(15x^6 \operatorname{arcsch}(cx) + \frac{3c^4x^5\left(\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 3cx\sqrt{\frac{1}{c^2x^2} + 1}}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/6\*a\*e\*x^6 + 1/4\*a\*d\*x^4 + 1/12\*(3\*x^4\*arccsch(c\*x) + (c^2\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) - 3\*x\*sqrt(1/(c^2\*x^2) + 1))/c^3)\*b\*d + 1/90\*(15\*x^6\*arccsch(c\*x) + (3\*c^4\*x^5\*(1/(c^2\*x^2) + 1)^(5/2) - 10\*c^2\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) + 15\*x\*sqrt(1/(c^2\*x^2) + 1))/c^5)\*b\*e

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 (ex^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x^3\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{acsch}(cx)) (d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x\*\*2+d)\*(a+b\*acsch(c\*x)),x)

[Out] Integral(x\*\*3\*(a + b\*acsch(c\*x))\*(d + e\*x\*\*2), x)

### 3.85 $\int x (d + ex^2) (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=146

$$\frac{(d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx))}{4e} - \frac{bcd^2 x \tan^{-1}\left(\sqrt{-c^2 x^2 - 1}\right)}{4e\sqrt{-c^2 x^2}} + \frac{bx\sqrt{-c^2 x^2 - 1} (2c^2 d - e)}{4c^3\sqrt{-c^2 x^2}} - \frac{bex(-c^2 x^2 - 1)^{3/2}}{12c^3\sqrt{-c^2 x^2}}$$

[Out] 1/4\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/e-1/12\*b\*e\*x\*(-c^2\*x^2-1)^(3/2)/c^3/(-c^2\*x^2)^(1/2)-1/4\*b\*c\*d^2\*x\*arctan((-c^2\*x^2-1)^(1/2))/e/(-c^2\*x^2)^(1/2)+1/4\*b\*(2\*c^2\*d-e)\*x\*(-c^2\*x^2-1)^(1/2)/c^3/(-c^2\*x^2)^(1/2)

**Rubi [A]** time = 0.11, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {6300, 446, 88, 63, 205}

$$\frac{(d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx))}{4e} - \frac{bcd^2 x \tan^{-1}\left(\sqrt{-c^2 x^2 - 1}\right)}{4e\sqrt{-c^2 x^2}} + \frac{bx\sqrt{-c^2 x^2 - 1} (2c^2 d - e)}{4c^3\sqrt{-c^2 x^2}} - \frac{bex(-c^2 x^2 - 1)^{3/2}}{12c^3\sqrt{-c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]),x]

[Out] (b\*(2\*c^2\*d - e)\*x\*Sqrt[-1 - c^2\*x^2])/(4\*c^3\*Sqrt[-(c^2\*x^2)]) - (b\*e\*x\*(-1 - c^2\*x^2)^(3/2))/(12\*c^3\*Sqrt[-(c^2\*x^2)]) + ((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/(4\*e) - (b\*c\*d^2\*x\*ArcTan[Sqrt[-1 - c^2\*x^2]])/(4\*e\*Sqrt[-(c^2\*x^2)])

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 88

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 6300

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsch[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[-(c^2\*x^2)]), Int[(d + e\*x^2)^(p + 1)/



$(x*\text{Sqrt}[-1 - c^2*x^2]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{NeQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int x(d+ex^2)(a+b\text{csch}^{-1}(cx)) dx &= \frac{(d+ex^2)^2(a+b\text{csch}^{-1}(cx))}{4e} - \frac{(bcx) \int \frac{(d+ex^2)^2}{x\sqrt{-1-c^2x^2}} dx}{4e\sqrt{-c^2x^2}} \\ &= \frac{(d+ex^2)^2(a+b\text{csch}^{-1}(cx))}{4e} - \frac{(bcx) \text{Subst}\left(\int \frac{(d+ex^2)^2}{x\sqrt{-1-c^2x}} dx, x, x^2\right)}{8e\sqrt{-c^2x^2}} \\ &= \frac{(d+ex^2)^2(a+b\text{csch}^{-1}(cx))}{4e} - \frac{(bcx) \text{Subst}\left(\int \left(-\frac{e(-2c^2d+e)}{c^2\sqrt{-1-c^2x}} + \frac{d^2}{x\sqrt{-1-c^2x}}\right) dx, x, x^2\right)}{8e\sqrt{-c^2x^2}} \\ &= \frac{b(2c^2d-e)x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}} - \frac{bcx(-1-c^2x^2)^{3/2}}{12c^3\sqrt{-c^2x^2}} + \frac{(d+ex^2)^2(a+b\text{csch}^{-1}(cx))}{4e} \\ &= \frac{b(2c^2d-e)x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}} - \frac{bcx(-1-c^2x^2)^{3/2}}{12c^3\sqrt{-c^2x^2}} + \frac{(d+ex^2)^2(a+b\text{csch}^{-1}(cx))}{4e} \\ &= \frac{b(2c^2d-e)x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}} - \frac{bcx(-1-c^2x^2)^{3/2}}{12c^3\sqrt{-c^2x^2}} + \frac{(d+ex^2)^2(a+b\text{csch}^{-1}(cx))}{4e} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 77, normalized size = 0.53

$$\frac{x\left(3ac^3x(2d+ex^2) + 3bc^3x\text{csch}^{-1}(cx)(2d+ex^2) + b\sqrt{\frac{1}{c^2x^2}+1}(c^2(6d+ex^2)-2e)\right)}{12c^3}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]), x]

[Out]  $(x*(3*a*c^3*x*(2*d + e*x^2) + b*\text{Sqrt}[1 + 1/(c^2*x^2)]*(-2*e + c^2*(6*d + e*x^2)) + 3*b*c^3*x*(2*d + e*x^2)*\text{ArcCsch}[c*x]))/(12*c^3)$

**fricas [A]** time = 1.13, size = 123, normalized size = 0.84

$$\frac{3ac^3ex^4 + 6ac^3dx^2 + 3(bc^3ex^4 + 2bc^3dx^2) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}+1}}{cx}\right) + (bc^2ex^3 + 2(3bc^2d - be)x)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out]  $1/12*(3*a*c^3*e*x^4 + 6*a*c^3*d*x^2 + 3*(b*c^3*e*x^4 + 2*b*c^3*d*x^2)*\log((c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*e*x^3 + 2*(3*b*c^2*d - b*e)*x)*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)))/c^3$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)(b \text{arcsch}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*x, x)

**maple** [A] time = 0.05, size = 115, normalized size = 0.79

$$\frac{a\left(\frac{1}{4}c^4ex^4+\frac{1}{2}c^4dx^2\right)}{c^2} + \frac{b\left(\frac{\operatorname{arccsch}(cx)c^4x^4e}{4} + \frac{\operatorname{arccsch}(cx)c^4x^2d}{2} + \frac{(c^2x^2+1)(c^2x^2e+6c^2d-2e)}{12\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx}\right)}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c^2\*(a/c^2\*(1/4\*c^4\*e\*x^4+1/2\*c^4\*d\*x^2)+b/c^2\*(1/4\*arccsch(c\*x)\*c^4\*x^4\*e+1/2\*arccsch(c\*x)\*c^4\*x^2\*d+1/12\*(c^2\*x^2+1)\*(c^2\*e\*x^2+6\*c^2\*d-2\*e)/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x))

**maxima** [A] time = 0.34, size = 95, normalized size = 0.65

$$\frac{1}{4}aex^4 + \frac{1}{2}adx^2 + \frac{1}{2}\left(x^2 \operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2} + 1}}{c}\right)bd + \frac{1}{12}\left(3x^4 \operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2} + 1}}{c^3}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/4\*a\*e\*x^4 + 1/2\*a\*d\*x^2 + 1/2\*(x^2\*arccsch(c\*x) + x\*sqrt(1/(c^2\*x^2) + 1)/c)\*b\*d + 1/12\*(3\*x^4\*arccsch(c\*x) + (c^2\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) - 3\*x\*sqrt(1/(c^2\*x^2) + 1))/c^3)\*b\*e

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x (ex^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{acsch}(cx)) (d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x\*\*2+d)\*(a+b\*acsch(c\*x)),x)

[Out] Integral(x\*(a + b\*acsch(c\*x))\*(d + e\*x\*\*2), x)

$$3.86 \quad \int \frac{(d+ex^2)(a+bcsch^{-1}(cx))}{x} dx$$

Optimal. Leaf size=115

$$-d \log\left(\frac{1}{x}\right) \left(a + bcsch^{-1}(cx)\right) + \frac{1}{2}ex^2 \left(a + bcsch^{-1}(cx)\right) + \frac{bex\sqrt{\frac{1}{c^2x^2} + 1}}{2c} - \frac{1}{2}bdLi_2\left(e^{2csch^{-1}(cx)}\right) + \frac{1}{2}bdcsch^{-1}(cx)^2$$

[Out]  $1/2*b*d*arccsch(c*x)^2 + 1/2*e*x^2*(a+b*arccsch(c*x)) - b*d*arccsch(c*x)*\ln(1 - (1/c/x + (1+1/c^2/x^2)^{1/2}))^2 + b*d*arccsch(c*x)*\ln(1/x) - d*(a+b*arccsch(c*x))*\ln(1/x) - 1/2*b*d*polylog(2, (1/c/x + (1+1/c^2/x^2)^{1/2}))^2 + 1/2*b*e*x*(1+1/c^2/x^2)^{1/2}/c$

Rubi [A] time = 0.29, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$ , Rules used = {6304, 14, 5789, 6742, 264, 2325, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}bdPolyLog\left(2, e^{2csch^{-1}(cx)}\right) - d \log\left(\frac{1}{x}\right) \left(a + bcsch^{-1}(cx)\right) + \frac{1}{2}ex^2 \left(a + bcsch^{-1}(cx)\right) + \frac{bex\sqrt{\frac{1}{c^2x^2} + 1}}{2c} + \frac{1}{2}bdcsch^{-1}(cx)^2$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x, x]

[Out]  $(b*e*\text{Sqrt}[1 + 1/(c^2*x^2)]*x)/(2*c) + (b*d*\text{ArcCsch}[c*x]^2)/2 + (e*x^2*(a + b*\text{ArcCsch}[c*x]))/2 - b*d*\text{ArcCsch}[c*x]*\text{Log}[1 - E^{(2*\text{ArcCsch}[c*x])}] + b*d*\text{ArcCsch}[c*x]*\text{Log}[x^{(-1)}] - d*(a + b*\text{ArcCsch}[c*x])* \text{Log}[x^{(-1)}] - (b*d*\text{PolyLog}[2, E^{(2*\text{ArcCsch}[c*x])}])/2$

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 2190

Int[(((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2325

Int[(a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_)]/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] := Simp[(ArcSinh[(Rt[e, 2]\*x)/Sqrt[d]]\*(a + b\*Log[c\*x^n]))/Rt[e, 2], x]

- Dist[(b\*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]\*x)/Sqrt[d]]/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))]/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 5659

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tanh[x], x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 5789

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))\*((f\_.)\*(x\_)^(m\_.))\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSinh[c\*x], u, x] - Dist[b\*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2\*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2\*d] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

#### Rule 6304

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)(a + b\operatorname{csch}^{-1}(cx))}{x} dx &= -\operatorname{Subst}\left(\int \frac{(e + dx^2)(a + b\sinh^{-1}\left(\frac{x}{c}\right))}{x^3} dx, x, \frac{1}{x}\right) \\
&= \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) - d(a + b\operatorname{csch}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\operatorname{Subst}\left(\int \frac{-\frac{e}{2x^2} + \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}}{2x^2} dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) - d(a + b\operatorname{csch}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{b\operatorname{Subst}\left(\int \left(-\frac{e}{2x^2} + \frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\right) dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) - d(a + b\operatorname{csch}^{-1}(cx))\log\left(\frac{1}{x}\right) + \frac{(bd)\operatorname{Subst}\left(\int \frac{\log\left(\frac{1}{\sqrt{1-\frac{x^2}{c^2}}}\right)}{\sqrt{1-\frac{x^2}{c^2}}} dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{be\sqrt{1 + \frac{1}{c^2x^2}}}{2c}x + \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) + bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a + b\operatorname{csch}^{-1}(cx)) \\
&= \frac{be\sqrt{1 + \frac{1}{c^2x^2}}}{2c}x + \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) + bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a + b\operatorname{csch}^{-1}(cx)) \\
&= \frac{be\sqrt{1 + \frac{1}{c^2x^2}}}{2c}x + \frac{1}{2}bd\operatorname{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) + bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a + b\operatorname{csch}^{-1}(cx)) \\
&= \frac{be\sqrt{1 + \frac{1}{c^2x^2}}}{2c}x + \frac{1}{2}bd\operatorname{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) - bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a + b\operatorname{csch}^{-1}(cx)) \\
&= \frac{be\sqrt{1 + \frac{1}{c^2x^2}}}{2c}x + \frac{1}{2}bd\operatorname{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) - bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a + b\operatorname{csch}^{-1}(cx)) \\
&= \frac{be\sqrt{1 + \frac{1}{c^2x^2}}}{2c}x + \frac{1}{2}bd\operatorname{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2(a + b\operatorname{csch}^{-1}(cx)) - bd\operatorname{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d(a + b\operatorname{csch}^{-1}(cx))
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 93, normalized size = 0.81

$$\frac{2acd \log(x) + acex^2 + bex\sqrt{\frac{1}{c^2x^2} + 1} + bccsch^{-1}(cx)\left(ex^2 - 2d \log\left(1 - e^{-2csch^{-1}(cx)}\right)\right) + bcd\operatorname{Li}_2\left(e^{-2csch^{-1}(cx)}\right) - d(a + bcsch^{-1}(cx))}{2c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x,x]

[Out] (b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x + a\*c\*e\*x^2 - b\*c\*d\*ArcCsch[c\*x]^2 + b\*c\*ArcCsch[c\*x]\*(e\*x^2 - 2\*d\*Log[1 - E^(-2\*ArcCsch[c\*x])]) + 2\*a\*c\*d\*Log[x] + b\*c\*d\*PolyLog[2, E^(-2\*ArcCsch[c\*x])])/(2\*c)

**fricas [F]** time = 0.95, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{aex^2 + ad + (bex^2 + bd)\operatorname{arcsch}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(b \operatorname{arcsch}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x, x)

**maple** [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(a + b \operatorname{arccsch}(cx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x,x)

[Out] int((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$2bc^2d \int \frac{x \log(x)}{2(\sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1)} dx - \frac{1}{2} bex^2 \log(c) - \frac{1}{2} bex^2 \log(x) + \frac{1}{2} aex^2 - bd \log(c) \log(x) - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x,x, algorithm="maxima")

[Out] 2\*b\*c^2\*d\*integrate(1/2\*x\*log(x)/(sqrt(c^2\*x^2 + 1)\*c^2\*x^2 + c^2\*x^2 + sqrt(c^2\*x^2 + 1) + 1), x) - 1/2\*b\*e\*x^2\*log(c) - 1/2\*b\*e\*x^2\*log(x) + 1/2\*a\*e\*x^2 - b\*d\*log(c)\*log(x) - 1/2\*b\*d\*log(x)^2 - 1/4\*(2\*log(c^2\*x^2 + 1)\*log(x) + dilog(-c^2\*x^2))\*b\*d + a\*d\*log(x) + 1/2\*(b\*e\*x^2 + 2\*b\*d\*log(x))\*log(sqrt(c^2\*x^2 + 1) + 1) + 1/4\*b\*e\*(2\*sqrt(c^2\*x^2 + 1) - log(c^2\*x^2 + 1))/c^2 + 1/4\*b\*e\*log(c^2\*x^2 + 1)/c^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x,x)

[Out] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))(d + ex^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsch(c\*x))/x,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)/x, x)

$$3.87 \quad \int \frac{(d+ex^2)(a+bcsch^{-1}(cx))}{x^3} dx$$

**Optimal.** Leaf size=128

$$-\frac{d(a+bcsch^{-1}(cx))}{2x^2} - e \log\left(\frac{1}{x}\right)(a+bcsch^{-1}(cx)) + \frac{bcd\sqrt{\frac{1}{c^2x^2}+1}}{4x} - \frac{1}{4}bc^2dcsch^{-1}(cx) - \frac{1}{2}beLi_2\left(e^{2csch^{-1}(cx)}\right) + \frac{1}{2}$$

[Out]  $-1/4*b*c^2*d*arccsch(c*x)+1/2*b*e*arccsch(c*x)^2-1/2*d*(a+b*arccsch(c*x))/x^2-b*e*arccsch(c*x)*\ln(1-(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)+b*e*arccsch(c*x)*\ln(1/x)-e*(a+b*arccsch(c*x))*\ln(1/x)-1/2*b*e*polylog(2,(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)+1/4*b*c*d*(1+1/c^2/x^2)^{(1/2)}/x$

**Rubi [A]** time = 0.30, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$ , Rules used = {6304, 14, 5789, 12, 6742, 321, 215, 2325, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}bePolyLog\left(2, e^{2csch^{-1}(cx)}\right) - \frac{d(a+bcsch^{-1}(cx))}{2x^2} - e \log\left(\frac{1}{x}\right)(a+bcsch^{-1}(cx)) + \frac{bcd\sqrt{\frac{1}{c^2x^2}+1}}{4x} - \frac{1}{4}bc^2dcsch^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^3, x]

[Out]  $(b*c*d*\text{Sqrt}[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*d*\text{ArcCsch}[c*x])/4 + (b*e*\text{ArcCsch}[c*x]^2)/2 - (d*(a + b*\text{ArcCsch}[c*x]))/(2*x^2) - b*e*\text{ArcCsch}[c*x]*\text{Log}[1 - E^{(2*\text{ArcCsch}[c*x])}] + b*e*\text{ArcCsch}[c*x]*\text{Log}[x^{-1}] - e*(a + b*\text{ArcCsch}[c*x])* \text{Log}[x^{-1}] - (b*e*\text{PolyLog}[2, E^{(2*\text{ArcCsch}[c*x])}])/2$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 14

Int[(u\_)\*((c\_)\*(x\_))^(m\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_)\*(v\_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

#### Rule 215

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSinh[(Rt[b, 2]\*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

#### Rule 321

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Di

st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2325

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)]/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symbol] :> Simp[(ArcSinh[Rt[e, 2]\*x]/Sqrt[d]]\*(a + b\*Log[c\*x^n])/Rt[e, 2], x] - Dist[(b\*n)/Rt[e, 2], Int[ArcSinh[Rt[e, 2]\*x]/Sqrt[d]/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 3716

Int[((c\_) + (d\_)\*(x\_)^(m\_))\*tan[(e\_) + Pi\*(k\_) + (Complex[0, fz\_])\*(f\_)\*(x\_)], x\_Symbol] :> -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))]/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 5659

Int[((a\_) + ArcSinh[(c\_)\*(x\_)])\*(b\_)]^(n\_)/(x\_), x\_Symbol] :> Subst[Int[(a + b\*x)^n/Tanh[x], x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 5789

Int[((a\_) + ArcSinh[(c\_)\*(x\_)])\*(b\_))\*((f\_)\*(x\_)^(m\_))\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSinh[c\*x], u, x] - Dist[b\*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2\*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2\*d] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

#### Rule 6304

Int[((a\_) + ArcCsch[(c\_)\*(x\_)])\*(b\_)]^(n\_)\*(x\_)^(m\_))\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m, p]

#### Rule 6742

Int[u\_, x\_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rubi steps



$$\begin{aligned}
\int \frac{(d + ex^2)(a + bcsch^{-1}(cx))}{x^3} dx &= -\text{Subst} \left( \int \frac{(e + dx^2)(a + b \sinh^{-1}(\frac{x}{c}))}{x} dx, x, \frac{1}{x} \right) \\
&= -\frac{d(a + bcsch^{-1}(cx))}{2x^2} - e(a + bcsch^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \text{Subst} \left( \int \frac{dx^2 + 2e1}{2\sqrt{1+}}}{c} \right)}{c} \\
&= -\frac{d(a + bcsch^{-1}(cx))}{2x^2} - e(a + bcsch^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \text{Subst} \left( \int \frac{dx^2 + 2e1}{\sqrt{1+}}}{2c} \right)}{2c} \\
&= -\frac{d(a + bcsch^{-1}(cx))}{2x^2} - e(a + bcsch^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \text{Subst} \left( \int \left( \frac{dx^2}{\sqrt{1+\frac{x^2}{c^2}}} \right) \right)}{2c} \\
&= -\frac{d(a + bcsch^{-1}(cx))}{2x^2} - e(a + bcsch^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{(bd) \text{Subst} \left( \int \frac{x^2}{\sqrt{1+}} \right)}{2c} \\
&= \frac{bcd\sqrt{1 + \frac{1}{c^2x^2}}}{4x} - \frac{d(a + bcsch^{-1}(cx))}{2x^2} + becsch^{-1}(cx) \log\left(\frac{1}{x}\right) - e(a + bcsch^{-1}(cx)) \\
&= \frac{bcd\sqrt{1 + \frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2dcsch^{-1}(cx) - \frac{d(a + bcsch^{-1}(cx))}{2x^2} + becsch^{-1}(cx) \\
&= \frac{bcd\sqrt{1 + \frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2dcsch^{-1}(cx) + \frac{1}{2}becsch^{-1}(cx)^2 - \frac{d(a + bcsch^{-1}(cx))}{2x^2} \\
&= \frac{bcd\sqrt{1 + \frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2dcsch^{-1}(cx) + \frac{1}{2}becsch^{-1}(cx)^2 - \frac{d(a + bcsch^{-1}(cx))}{2x^2} \\
&= \frac{bcd\sqrt{1 + \frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2dcsch^{-1}(cx) + \frac{1}{2}becsch^{-1}(cx)^2 - \frac{d(a + bcsch^{-1}(cx))}{2x^2} \\
&= \frac{bcd\sqrt{1 + \frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2dcsch^{-1}(cx) + \frac{1}{2}becsch^{-1}(cx)^2 - \frac{d(a + bcsch^{-1}(cx))}{2x^2}
\end{aligned}$$

**Mathematica [A]** time = 0.57, size = 138, normalized size = 1.08

$$\frac{1}{4} \left( -\frac{2ad}{x^2} + 4ae \log(x) - \frac{bd \left( -c^2x^2 + c^2x^2 \sqrt{c^2x^2 + 1} \tanh^{-1} \left( \sqrt{c^2x^2 + 1} \right) - 1 \right)}{cx^3 \sqrt{\frac{1}{c^2x^2} + 1}} - \frac{2bdcsch^{-1}(cx)}{x^2} + 2be\text{Li}_2 \left( e^{-2c} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/x^3, x]

[Out] ((-2\*a\*d)/x^2 - (2\*b\*d\*ArcCsch[c\*x])/x^2 - (b\*d\*(-1 - c^2\*x^2 + c^2\*x^2\*Sqrt[1 + c^2\*x^2])\*ArcTanh[Sqrt[1 + c^2\*x^2]])/(c\*Sqrt[1 + 1/(c^2\*x^2)]\*x^3) - 2\*b\*e\*ArcCsch[c\*x]\*(ArcCsch[c\*x] + 2\*Log[1 - E^(-2\*ArcCsch[c\*x])]) + 4\*a\*e\*Log[x] + 2\*b\*e\*PolyLog[2, E^(-2\*ArcCsch[c\*x])])/4

**fricas** [F] time = 2.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{aex^2 + ad + (bex^2 + bd) \operatorname{arcsch}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))/x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(b \operatorname{arcsch}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^3, x)

**maple** [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)(a + b \operatorname{arccsch}(cx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^3,x)

[Out] int((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( 4c^2 \int \frac{x^2 \log(x)}{c^2 x^3 + x} dx - 2c^2 \int \frac{x \log(x)}{c^2 x^2 + (c^2 x^2 + 1)^{\frac{3}{2}} + 1} dx - (\log(c^2 x^2 + 1) - 2 \log(x)) \log(c) + \log(c^2 x^2 + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="maxima")

[Out] -1/2\*(4\*c^2\*integrate(x^2\*log(x)/(c^2\*x^3 + x), x) - 2\*c^2\*integrate(x\*log(x)/(c^2\*x^2 + (c^2\*x^2 + 1)^(3/2) + 1), x) - (log(c^2\*x^2 + 1) - 2\*log(x))\*log(c) + log(c^2\*x^2 + 1)\*log(c) - 2\*log(x)\*log(sqrt(c^2\*x^2 + 1) + 1) + 2\*integrate(log(x)/(c^2\*x^3 + x), x))\*b\*e + 1/8\*b\*d\*((2\*c^4\*x\*sqrt(1/(c^2\*x^2) + 1)/(c^2\*x^2\*(1/(c^2\*x^2) + 1) - 1) - c^3\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) + 1) + c^3\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) - 1))/c - 4\*arccsch(c\*x)/x^2) + a\*e\*log(x) - 1/2\*a\*d/x^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^3,x)

[Out] int(((d + e\*x^2)\*(a + b\*asinh(1/(c\*x))))/x^3, x)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx))(d + ex^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*(a+b\*acsch(c\*x))/x\*\*3,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)/x\*\*3, x)

### 3.88 $\int x^2 (d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=260

$$\frac{1}{3}d^2x^3(a + b \operatorname{csch}^{-1}(cx)) + \frac{2}{5}dex^5(a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b \operatorname{csch}^{-1}(cx)) + \frac{be^2x^6\sqrt{-c^2x^2-1}}{42c\sqrt{-c^2x^2}} + \frac{bex^4\sqrt{-c^2x^2-1}}{840c^3\sqrt{-c^2x^2}}$$

[Out]  $\frac{1}{3}d^2x^3(a+b\operatorname{arccsch}(cx))+\frac{2}{5}d^2ex^5(a+b\operatorname{arccsch}(cx))+\frac{1}{7}e^2x^7(a+b\operatorname{arccsch}(cx))+\frac{be^2x^6\sqrt{-c^2x^2-1}}{42c\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-c^2x^2-1}}{840c^3\sqrt{-c^2x^2}}$

**Rubi [A]** time = 0.26, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {270, 6302, 12, 1267, 459, 321, 217, 203}

$$\frac{1}{3}d^2x^3(a + b \operatorname{csch}^{-1}(cx)) + \frac{2}{5}dex^5(a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b \operatorname{csch}^{-1}(cx)) + \frac{bx^2\sqrt{-c^2x^2-1}(280c^4d^2 - 252c^2d^2e + 75e^2)}{1680c^5\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2(d + ex^2)^2(a + b \operatorname{ArcCsch}[cx]), x]$

[Out]  $(b*(280*c^4*d^2 - 252*c^2*d^2*e + 75*e^2)*x^2*\operatorname{Sqrt}[-1 - c^2*x^2])/(1680*c^5*\operatorname{qrt}[-(c^2*x^2)]) + (b*(84*c^2*d - 25*e)*e*x^4*\operatorname{Sqrt}[-1 - c^2*x^2])/(840*c^3*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*e^2*x^6*\operatorname{Sqrt}[-1 - c^2*x^2])/(42*c*\operatorname{Sqrt}[-(c^2*x^2)]) + (d^2*x^3*(a + b*\operatorname{ArcCsch}[c*x]))/3 + (2*d*e*x^5*(a + b*\operatorname{ArcCsch}[c*x]))/5 + (e^2*x^7*(a + b*\operatorname{ArcCsch}[c*x]))/7 + (b*(280*c^4*d^2 - 252*c^2*d^2*e + 75*e^2)*x*\operatorname{ArcTan}[(c*x)/\operatorname{Sqrt}[-1 - c^2*x^2]])/(1680*c^6*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{Match}Q[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 203

$\operatorname{Int}[(a_*) + (b_*)(x_)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTan}[\operatorname{Rt}[b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{PosQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \parallel \operatorname{GtQ}[b, 0])$

#### Rule 217

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_*) + (b_*)(x_)^2], x\_Symbol] := \operatorname{Subst}[\operatorname{Int}[1/(1 - b*x^2), x], x, x/\operatorname{Sqrt}[a + b*x^2]] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& !\operatorname{GtQ}[a, 0]$

#### Rule 270

$\operatorname{Int}[(c_*)(x_)^{(m_*)}((a_*) + (b_*)(x_)^{(n_*)})^{(p_*)}, x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}[\{a, b, c, m, n\}, x] \&\& \operatorname{IGtQ}[p, 0]$

#### Rule 321

$\operatorname{Int}[(c_*)(x_)^{(m_*)}((a_*) + (b_*)(x_)^{(n_*)})^{(p_*)}, x\_Symbol] := \operatorname{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a + b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \operatorname{Dist}[\dots]$

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

### Rule 1267

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^(
q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
+ 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x]
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]
&& !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

### Rule 6302

```
Int[((a_) + ArcSch[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(
x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int x^2 (d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{3} d^2 x^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{2}{5} dex^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7} e^2 x^7 (a + b \operatorname{csch}^{-1}(cx)) \\
&= \frac{1}{3} d^2 x^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{2}{5} dex^5 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{7} e^2 x^7 (a + b \operatorname{csch}^{-1}(cx)) \\
&= \frac{be^2 x^6 \sqrt{-1 - c^2 x^2}}{42c \sqrt{-c^2 x^2}} + \frac{1}{3} d^2 x^3 (a + b \operatorname{csch}^{-1}(cx)) + \frac{2}{5} dex^5 (a + b \operatorname{csch}^{-1}(cx)) \\
&= \frac{b(84c^2 d - 25e) ex^4 \sqrt{-1 - c^2 x^2}}{840c^3 \sqrt{-c^2 x^2}} + \frac{be^2 x^6 \sqrt{-1 - c^2 x^2}}{42c \sqrt{-c^2 x^2}} + \frac{1}{3} d^2 x^3 (a + b \operatorname{csch}^{-1}(cx)) \\
&= \frac{b(280c^4 d^2 - 252c^2 de + 75e^2) x^2 \sqrt{-1 - c^2 x^2}}{1680c^5 \sqrt{-c^2 x^2}} + \frac{b(84c^2 d - 25e) ex^4 \sqrt{-1 - c^2 x^2}}{840c^3 \sqrt{-c^2 x^2}} \\
&= \frac{b(280c^4 d^2 - 252c^2 de + 75e^2) x^2 \sqrt{-1 - c^2 x^2}}{1680c^5 \sqrt{-c^2 x^2}} + \frac{b(84c^2 d - 25e) ex^4 \sqrt{-1 - c^2 x^2}}{840c^3 \sqrt{-c^2 x^2}} \\
&= \frac{b(280c^4 d^2 - 252c^2 de + 75e^2) x^2 \sqrt{-1 - c^2 x^2}}{1680c^5 \sqrt{-c^2 x^2}} + \frac{b(84c^2 d - 25e) ex^4 \sqrt{-1 - c^2 x^2}}{840c^3 \sqrt{-c^2 x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.36, size = 182, normalized size = 0.70

$$c^2x^2 \left( 16ac^5x(35d^2 + 42dex^2 + 15e^2x^4) + b\sqrt{\frac{1}{c^2x^2} + 1} (8c^4(35d^2 + 21dex^2 + 5e^2x^4) - 2c^2e(126d + 25ex^2) + 75e^2) \right)$$

1680c

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]), x]

[Out] (c^2\*x^2\*(16\*a\*c^5\*x\*(35\*d^2 + 42\*d\*e\*x^2 + 15\*e^2\*x^4) + b\*Sqrt[1 + 1/(c^2\*x^2)]\*(75\*e^2 - 2\*c^2\*e\*(126\*d + 25\*e\*x^2) + 8\*c^4\*(35\*d^2 + 21\*d\*e\*x^2 + 5\*e^2\*x^4))) + 16\*b\*c^7\*x^3\*(35\*d^2 + 42\*d\*e\*x^2 + 15\*e^2\*x^4)\*ArcCsch[c\*x] + b\*(-280\*c^4\*d^2 + 252\*c^2\*d\*e - 75\*e^2)\*Log[(1 + Sqrt[1 + 1/(c^2\*x^2)])\*x])/(1680\*c^7)

**fricas [A]** time = 1.33, size = 388, normalized size = 1.49

$$240ac^7e^2x^7 + 672ac^7dex^5 + 560ac^7d^2x^3 + 16(35bc^7d^2 + 42bc^7de + 15bc^7e^2) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) + (280$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] 1/1680\*(240\*a\*c^7\*e^2\*x^7 + 672\*a\*c^7\*d\*e\*x^5 + 560\*a\*c^7\*d^2\*x^3 + 16\*(35\*b\*c^7\*d^2 + 42\*b\*c^7\*d\*e + 15\*b\*c^7\*e^2)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) + (280\*b\*c^4\*d^2 - 252\*b\*c^2\*d\*e + 75\*b\*e^2)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) - 16\*(35\*b\*c^7\*d^2 + 42\*b\*c^7\*d\*e + 15\*b\*c^7\*e^2)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + 16\*(15\*b\*c^7\*e^2\*x^7 + 42\*b\*c^7\*d\*e\*x^5 + 35\*b\*c^7\*d^2\*x^3 - 35\*b\*c^7\*d^2 - 42\*b\*c^7\*d\*e - 15\*b\*c^7\*e^2)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (40\*b\*c^6\*e^2\*x^6 + 2\*(84\*b\*c^6\*d\*e - 25\*b\*c^4\*e^2)\*x^4 + (280\*b\*c^6\*d^2 - 252\*b\*c^4\*d\*e + 75\*b\*c^2\*e^2)\*x^2)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^7

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsch(c\*x) + a)\*x^2, x)

**maple [A]** time = 0.06, size = 286, normalized size = 1.10

$$\frac{a\left(\frac{1}{7}e^2c^7x^7 + \frac{2}{5}c^7dex^5 + \frac{1}{3}x^3c^7d^2\right)}{c^4} + \frac{b\left(\frac{\operatorname{arcsch}(cx)e^2c^7x^7}{7} + \frac{2\operatorname{arcsch}(cx)c^7dex^5}{5} + \frac{\operatorname{arcsch}(cx)c^7x^3d^2}{3} + \frac{\sqrt{c^2x^2+1}\left(40e^2c^5x^5\sqrt{c^2x^2+1} + 168c^5dex^3\sqrt{c^2x^2+1} + 280d^2c^5x\sqrt{c^2x^2+1}\right)}{3}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)), x)

[Out] 1/c^3\*(a/c^4\*(1/7\*e^2\*c^7\*x^7+2/5\*c^7\*d\*e\*x^5+1/3\*x^3\*c^7\*d^2)+b/c^4\*(1/7\*arccsch(c\*x)\*e^2\*c^7\*x^7+2/5\*arccsch(c\*x)\*c^7\*d\*e\*x^5+1/3\*arccsch(c\*x)\*c^7\*x^3\*d^2+1/1680\*(c^2\*x^2+1)^(1/2)\*(40\*e^2\*c^5\*x^5\*(c^2\*x^2+1)^(1/2)+168\*c^5\*d

$*e*x^3*(c^2*x^2+1)^{(1/2)}+280*d^2*c^5*x*(c^2*x^2+1)^{(1/2)}-280*d^2*c^4*\operatorname{arcsinh}(c*x)-50*e^2*c^3*x^3*(c^2*x^2+1)^{(1/2)}-252*c^3*d*e*x*(c^2*x^2+1)^{(1/2)}+252*c^2*d*e*\operatorname{arcsinh}(c*x)+75*e^2*c*x*(c^2*x^2+1)^{(1/2)}-75*e^2*\operatorname{arcsinh}(c*x))/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/c/x))$

**maxima** [A] time = 0.33, size = 396, normalized size = 1.52

$$\frac{1}{7}ae^2x^7 + \frac{2}{5}adex^5 + \frac{1}{3}ad^2x^3 + \frac{1}{12} \left( 4x^3 \operatorname{arcsch}(cx) + \frac{2\sqrt{\frac{1}{c^2x^2}+1} - \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2} \right) bd^2 + \frac{1}{40} \left( 16x^5 \operatorname{arcsch}(cx) - (2*(3*(1/(c^2*x^2) + 1))^{3/2} - 5*\sqrt{1/(c^2*x^2) + 1}) / (c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*\log(\sqrt{1/(c^2*x^2) + 1} + 1)/c^4 + 3*\log(\sqrt{1/(c^2*x^2) + 1} - 1)/c^4)/c * b*d*e + 1/672*(96*x^7*\operatorname{arcsch}(c*x) + (2*(15*(1/(c^2*x^2) + 1))^{5/2} - 40*(1/(c^2*x^2) + 1)^{3/2} + 33*\sqrt{1/(c^2*x^2) + 1})/(c^6*(1/(c^2*x^2) + 1)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*\log(\sqrt{1/(c^2*x^2) + 1} + 1)/c^6 + 15*\log(\sqrt{1/(c^2*x^2) + 1} - 1)/c^6)/c * b*e^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/7\*a\*e^2\*x^7 + 2/5\*a\*d\*e\*x^5 + 1/3\*a\*d^2\*x^3 + 1/12\*(4\*x^3\*arccsch(c\*x) + (2\*sqrt(1/(c^2\*x^2) + 1)/(c^2\*(1/(c^2\*x^2) + 1) - c^2) - log(sqrt(1/(c^2\*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2\*x^2) + 1) - 1)/c^2)/c)\*b\*d^2 + 1/40\*(16\*x^5\*arccsch(c\*x) - (2\*(3\*(1/(c^2\*x^2) + 1))^{3/2} - 5\*sqrt(1/(c^2\*x^2) + 1)) / (c^4\*(1/(c^2\*x^2) + 1)^2 - 2\*c^4\*(1/(c^2\*x^2) + 1) + c^4) - 3\*log(sqrt(1/(c^2\*x^2) + 1) + 1)/c^4 + 3\*log(sqrt(1/(c^2\*x^2) + 1) - 1)/c^4)/c)\*b\*d\*e + 1/672\*(96\*x^7\*arccsch(c\*x) + (2\*(15\*(1/(c^2\*x^2) + 1))^{5/2} - 40\*(1/(c^2\*x^2) + 1)^{3/2} + 33\*sqrt(1/(c^2\*x^2) + 1))/(c^6\*(1/(c^2\*x^2) + 1)^3 - 3\*c^6\*(1/(c^2\*x^2) + 1)^2 + 3\*c^6\*(1/(c^2\*x^2) + 1) - c^6) - 15\*log(sqrt(1/(c^2\*x^2) + 1) + 1)/c^6 + 15\*log(sqrt(1/(c^2\*x^2) + 1) - 1)/c^6)/c)\*b\*e^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (ex^2 + d)^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x^2\*(d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{acsch}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x\*\*2+d)\*\*2\*(a+b\*acsch(c\*x)),x)

[Out] Integral(x\*\*2\*(a + b\*acsch(c\*x))\*(d + e\*x\*\*2)\*\*2, x)

### 3.89 $\int (d + ex^2)^2 (a + b\operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=197

$$d^2x(a + b\operatorname{csch}^{-1}(cx)) + \frac{2}{3}dex^3(a + b\operatorname{csch}^{-1}(cx)) + \frac{1}{5}e^2x^5(a + b\operatorname{csch}^{-1}(cx)) + \frac{be^2x^4\sqrt{-c^2x^2-1}}{20c\sqrt{-c^2x^2}} - \frac{bx(120c^4d^2 - 40c^2de + 9e^2)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{120c^4\sqrt{-c^2x^2}}$$

[Out]  $d^2*x*(a+b*\operatorname{arccsch}(c*x))+2/3*d*e*x^3*(a+b*\operatorname{arccsch}(c*x))+1/5*e^2*x^5*(a+b*\operatorname{arccsch}(c*x))-1/120*b*(120*c^4*d^2-40*c^2*d*e+9*e^2)*x*\operatorname{arctan}(c*x/(-c^2*x^2-1)^{(1/2)})/c^4/(-c^2*x^2)^{(1/2)}+1/120*b*(40*c^2*d-9*e)*e*x^2*(-c^2*x^2-1)^{(1/2)}/c^3/(-c^2*x^2)^{(1/2)}+1/20*b*e^2*x^4*(-c^2*x^2-1)^{(1/2)}/c/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {194, 6292, 12, 1159, 388, 217, 203}

$$d^2x(a + b\operatorname{csch}^{-1}(cx)) + \frac{2}{3}dex^3(a + b\operatorname{csch}^{-1}(cx)) + \frac{1}{5}e^2x^5(a + b\operatorname{csch}^{-1}(cx)) - \frac{bx(120c^4d^2 - 40c^2de + 9e^2)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{120c^4\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]), x]

[Out]  $(b*(40*c^2*d - 9*e)*e*x^2*\operatorname{Sqrt}[-1 - c^2*x^2])/(120*c^3*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*e^2*x^4*\operatorname{Sqrt}[-1 - c^2*x^2])/(20*c*\operatorname{Sqrt}[-(c^2*x^2)]) + d^2*x*(a + b*\operatorname{ArcCsch}[c*x]) + (2*d*e*x^3*(a + b*\operatorname{ArcCsch}[c*x]))/3 + (e^2*x^5*(a + b*\operatorname{ArcCsch}[c*x]))/5 - (b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*x*\operatorname{ArcTan}[(c*x)/\operatorname{Sqrt}[-1 - c^2*x^2]])/(120*c^4*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 194

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]



Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]
```

Rule 6292

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
1/2, 0])
```

Rubi steps

$$\begin{aligned} \int (d + ex^2)^2 (a + bcsch^{-1}(cx)) dx &= d^2x (a + bcsch^{-1}(cx)) + \frac{2}{3}dex^3 (a + bcsch^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + bcsch^{-1}(cx)) \\ &= d^2x (a + bcsch^{-1}(cx)) + \frac{2}{3}dex^3 (a + bcsch^{-1}(cx)) + \frac{1}{5}e^2x^5 (a + bcsch^{-1}(cx)) \\ &= \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + d^2x (a + bcsch^{-1}(cx)) + \frac{2}{3}dex^3 (a + bcsch^{-1}(cx)) + \\ &= \frac{b(40c^2d - 9e)ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + d^2x (a + bcsch^{-1}(cx)) \\ &= \frac{b(40c^2d - 9e)ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + d^2x (a + bcsch^{-1}(cx)) \\ &= \frac{b(40c^2d - 9e)ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} + \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + d^2x (a + bcsch^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 149, normalized size = 0.76

$$\frac{c^2x \left( 8ac^3 (15d^2 + 10dex^2 + 3e^2x^4) + bex\sqrt{\frac{1}{c^2x^2} + 1} (c^2(40d + 6ex^2) - 9e) \right) + 8bc^5xcsch^{-1}(cx) (15d^2 + 10dex^2)}{120c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]), x]

[Out] (c^2\*x\*(8\*a\*c^3\*(15\*d^2 + 10\*d\*e\*x^2 + 3\*e^2\*x^4) + b\*e\*Sqrt[1 + 1/(c^2\*x^2)])\*x\*(-9\*e + c^2\*(40\*d + 6\*e\*x^2))) + 8\*b\*c^5\*x\*(15\*d^2 + 10\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsch[c\*x] + b\*(120\*c^4\*d^2 - 40\*c^2\*d\*e + 9\*e^2)\*Log[(1 + Sqrt[1 + 1/(c^2\*x^2)])\*x]/(120\*c^5)

**fricas [B]** time = 1.15, size = 353, normalized size = 1.79

$$24ac^5e^2x^5 + 80ac^5dex^3 + 120ac^5d^2x + 8(15bc^5d^2 + 10bc^5de + 3bc^5e^2) \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - (120bc^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] 1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - (120*b*c^4*d^2 - 40*b*c^2*d*e + 9*b*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*e^2*x^4 + (40*b*c^4*d*e - 9*b*c^2*e^2)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^5
giac [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a), x)
maple [A] time = 0.05, size = 217, normalized size = 1.10
```

$$\frac{a\left(\frac{1}{5}e^2c^5x^5 + \frac{2}{3}c^5dex^3 + xc^5d^2\right)}{c^4} + \frac{b\left(\frac{\operatorname{arccsch}(cx)e^2c^5x^5}{5} + \frac{2\operatorname{arccsch}(cx)c^5dex^3}{3} + \operatorname{arccsch}(cx)c^5xd^2 + \frac{\sqrt{c^2x^2+1}\left(120d^2c^4\operatorname{arcsinh}(cx) + 6e^2c^3x^3\sqrt{c^2x^2+1} + 40c^3dex\sqrt{c^2x^2+1}\right)}{120\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^4}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x)),x)
[Out] 1/c*(a/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b/c^4*(1/5*arccsch(c*x)*e^2*c^5*x^5+2/3*arccsch(c*x)*c^5*d*e*x^3+arccsch(c*x)*c^5*x*d^2+1/120*(c^2*x^2+1)^(1/2)*(120*d^2*c^4*arcsinh(c*x)+6*e^2*c^3*x^3*(c^2*x^2+1)^(1/2)+40*c^3*d*e*x*(c^2*x^2+1)^(1/2)-40*c^2*d*e*arcsinh(c*x)-9*e^2*c*x*(c^2*x^2+1)^(1/2)+9*e^2*arcsinh(c*x))/(c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
maxima [A] time = 0.36, size = 287, normalized size = 1.46
```

$$\frac{1}{5}ae^2x^5 + \frac{2}{3}adex^3 + \frac{1}{6}\left(4x^3 \operatorname{arcsch}(cx) + \frac{2\sqrt{\frac{1}{c^2x^2}+1} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}\right) bde + \frac{1}{80}\left(16x^5 \operatorname{arcsch}(cx) + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1)))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x + 1/2*(2*c
```

$*x*\operatorname{arccsch}(c*x) + \log(\sqrt{1/(c^2*x^2) + 1} + 1) - \log(\sqrt{1/(c^2*x^2) + 1} - 1)) * b * d^2 / c$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (ex^2 + d)^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)`

[Out] `int((d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(a+b*acsch(c*x)), x)`

[Out] `Integral((a + b*acsch(c*x))*(d + e*x**2)**2, x)`

$$3.90 \quad \int \frac{(d+ex^2)^2 (a+bcsch^{-1}(cx))}{x^2} dx$$

**Optimal.** Leaf size=170

$$-\frac{d^2 (a + bcsch^{-1}(cx))}{x} + 2dex (a + bcsch^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + bcsch^{-1}(cx)) + \frac{bcd^2\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}} - \frac{bex(12c^2d-e)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{6c^2\sqrt{-c^2x^2-1}}$$

[Out]  $-d^2*(a+b*arccsch(c*x))/x+2*d*e*x*(a+b*arccsch(c*x))+1/3*e^2*x^3*(a+b*arccsch(c*x))-1/6*b*(12*c^2*d-e)*e*x*arctan(c*x/(-c^2*x^2-1)^{(1/2)})/c^2/(-c^2*x^2)^{(1/2)}+b*c*d^2*(-c^2*x^2-1)^{(1/2)}/(-c^2*x^2)^{(1/2)}+1/6*b*e^2*x^2*(-c^2*x^2-1)^{(1/2)}/c/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 6302, 12, 1265, 388, 217, 203}

$$-\frac{d^2 (a + bcsch^{-1}(cx))}{x} + 2dex (a + bcsch^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + bcsch^{-1}(cx)) + \frac{bcd^2\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}} - \frac{bex(12c^2d-e)\tan^{-1}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{6c^2\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x^2,x]

[Out]  $(b*c*d^2*\text{Sqrt}[-1 - c^2*x^2])/\text{Sqrt}[-(c^2*x^2)] + (b*e^2*x^2*\text{Sqrt}[-1 - c^2*x^2])/(6*c*\text{Sqrt}[-(c^2*x^2)]) - (d^2*(a + b*\text{ArcCsch}[c*x]))/x + 2*d*e*x*(a + b*\text{ArcCsch}[c*x]) + (e^2*x^3*(a + b*\text{ArcCsch}[c*x]))/3 - (b*(12*c^2*d - e)*e*x*\text{ArcTan}[(c*x)/\text{Sqrt}[-1 - c^2*x^2]])/(6*c^2*\text{Sqrt}[-(c^2*x^2)])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 270

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

Rule 1265

```
Int[((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]}, Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^(m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx))}{x^2} dx &= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{x} + 2dex (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \operatorname{csch}^{-1}(cx)) \\ &= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{x} + 2dex (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{bcd^2\sqrt{-1 - c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{x} + 2dex (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{bcd^2\sqrt{-1 - c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1 - c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{x} + 2dex (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{bcd^2\sqrt{-1 - c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1 - c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{x} + 2dex (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{bcd^2\sqrt{-1 - c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1 - c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{x} + 2dex (a + b \operatorname{csch}^{-1}(cx)) \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 134, normalized size = 0.79

$$\frac{c^2 \left( 2ac (-3d^2 + 6dex^2 + e^2x^4) + bx \sqrt{\frac{1}{c^2x^2} + 1} (6c^2d^2 + e^2x^2) \right) + 2bc^3 \operatorname{csch}^{-1}(cx) (-3d^2 + 6dex^2 + e^2x^4) + bex^3}{6c^3x}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x^2,x]

[Out] (c^2\*(b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(6\*c^2\*d^2 + e^2\*x^2) + 2\*a\*c\*(-3\*d^2 + 6\*d\*e\*x^2 + e^2\*x^4)) + 2\*b\*c^3\*(-3\*d^2 + 6\*d\*e\*x^2 + e^2\*x^4)\*ArcCsch[c\*x] + b\*(12\*c^2\*d - e)\*e\*x\*Log[(1 + Sqrt[1 + 1/(c^2\*x^2)]\*x)]/(6\*c^3\*x)

**fricas** [B] time = 1.22, size = 347, normalized size = 2.04

$$2ac^3e^2x^4 + 6bc^4d^2x + 12ac^3dex^2 - 6ac^3d^2 - 2(3bc^3d^2 - 6bc^3de - bc^3e^2)x \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - (12bc^2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="fricas")

[Out] 1/6\*(2\*a\*c^3\*e^2\*x^4 + 6\*b\*c^4\*d^2\*x + 12\*a\*c^3\*d\*e\*x^2 - 6\*a\*c^3\*d^2 - 2\*(3\*b\*c^3\*d^2 - 6\*b\*c^3\*d\*e - b\*c^3\*e^2)\*x\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) - (12\*b\*c^2\*d\*e - b\*e^2)\*x\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) + 2\*(3\*b\*c^3\*d^2 - 6\*b\*c^3\*d\*e - b\*c^3\*e^2)\*x\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) + 2\*(b\*c^3\*e^2\*x^4 + 6\*b\*c^3\*d\*e\*x^2 - 3\*b\*c^3\*d^2 + (3\*b\*c^3\*d^2 - 6\*b\*c^3\*d\*e - b\*c^3\*e^2)\*x)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (6\*b\*c^4\*d^2\*x + b\*c^2\*e^2\*x^3)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^3\*x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsch(c\*x) + a)/x^2, x)

**maple** [A] time = 0.06, size = 189, normalized size = 1.11

$$c \left( \frac{a \left( \frac{c^3 x^3 e^2}{3} + 2c^3 dex - \frac{d^2 c^3}{x} \right)}{c^4} + \frac{b \left( \frac{e^2 \operatorname{arccsch}(cx) c^3 x^3}{3} + 2 \operatorname{arccsch}(cx) c^3 dex - \frac{\operatorname{arccsch}(cx) d^2 c^3}{x} + \frac{\sqrt{c^2 x^2 + 1} (6d^2 c^4 \sqrt{c^2 x^2 + 1} + 12c^3}{c^4} \right)}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^2,x)

[Out] c\*(a/c^4\*(1/3\*c^3\*x^3\*e^2+2\*c^3\*d\*e\*x-d^2\*c^3/x)+b/c^4\*(1/3\*e^2\*arccsch(c\*x)\*c^3\*x^3+2\*arccsch(c\*x)\*c^3\*d\*e\*x-arccsch(c\*x)\*d^2\*c^3/x+1/6\*(c^2\*x^2+1)^(1/2)\*(6\*d^2\*c^4\*(c^2\*x^2+1)^(1/2)+12\*c^3\*d\*e\*arcsinh(c\*x)\*x+(c^2\*x^2+1)^(1/2)\*c^2\*x^2\*e^2-arcsinh(c\*x)\*c\*x\*e^2)/c^2/x^2/((c^2\*x^2+1)/c^2/x^2)^(1/2))

**maxima** [A] time = 0.41, size = 191, normalized size = 1.12

$$\frac{1}{3} ae^2x^3 + \left( c\sqrt{\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsch}(cx)}{x} \right) bd^2 + \frac{1}{12} \left( 4x^3 \operatorname{arcsch}(cx) + \frac{2\sqrt{\frac{1}{c^2x^2} + 1} - \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right)}{c^2\left(\frac{1}{c^2x^2} + 1\right) - c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} - 1\right)}{c^2} \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="maxima")

```
[Out] 1/3*a*e^2*x^3 + (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d^2 + 1/12*(4*
x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) -
log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c
)*b*e^2 + 2*a*d*e*x + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1)
- log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d*e/c - a*d^2/x
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d)^2 \left( a + b \operatorname{arsinh}\left(\frac{1}{cx}\right) \right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^2, x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^2, x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**2, x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**2, x)
```

$$3.91 \quad \int \frac{(d+ex^2)^2 (a+bcsch^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=164

$$\frac{d^2 (a + bcsch^{-1}(cx))}{3x^3} - \frac{2de (a + bcsch^{-1}(cx))}{x} + e^2 x (a + bcsch^{-1}(cx)) + \frac{bcd^2 \sqrt{-c^2 x^2 - 1}}{9x^2 \sqrt{-c^2 x^2}} - \frac{2bcd \sqrt{-c^2 x^2 - 1} (c^2 d - 1)}{9 \sqrt{-c^2 x^2}}$$

[Out]  $-1/3*d^2*(a+b*arccsch(c*x))/x^3-2*d*e*(a+b*arccsch(c*x))/x+e^2*x*(a+b*arccsch(c*x))-b*e^2*x*arctan(c*x/(-c^2*x^2-1)^{(1/2)})/(-c^2*x^2)^{(1/2)}-2/9*b*c*d*(c^2*d-9*e)*(-c^2*x^2-1)^{(1/2)}/(-c^2*x^2)^{(1/2)}+1/9*b*c*d^2*(-c^2*x^2-1)^{(1/2)}/x^2/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 6302, 12, 1265, 451, 217, 203}

$$\frac{d^2 (a + bcsch^{-1}(cx))}{3x^3} - \frac{2de (a + bcsch^{-1}(cx))}{x} + e^2 x (a + bcsch^{-1}(cx)) + \frac{bcd^2 \sqrt{-c^2 x^2 - 1}}{9x^2 \sqrt{-c^2 x^2}} - \frac{2bcd \sqrt{-c^2 x^2 - 1} (c^2 d - 1)}{9 \sqrt{-c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x^4,x]

[Out]  $(-2*b*c*d*(c^2*d - 9*e)*\text{Sqrt}[-1 - c^2*x^2])/(9*\text{Sqrt}[-(c^2*x^2)]) + (b*c*d^2*\text{Sqrt}[-1 - c^2*x^2])/(9*x^2*\text{Sqrt}[-(c^2*x^2)]) - (d^2*(a + b*\text{ArcCsch}[c*x]))/(3*x^3) - (2*d*e*(a + b*\text{ArcCsch}[c*x]))/x + e^2*x*(a + b*\text{ArcCsch}[c*x]) - (b*e^2*x*\text{ArcTan}[(c*x)/\text{Sqrt}[-1 - c^2*x^2]])/\text{Sqrt}[-(c^2*x^2)]$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 270

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

### Rule 451

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[d/e^n, Int[(e\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n\*(p+1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m, 0]))



Q[m + n, -1]))

### Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]}, Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

### Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^(m+1)*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx))}{x^4} dx &= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3x^3} - \frac{2de (a + b \operatorname{csch}^{-1}(cx))}{x} + e^2 x (a + b \operatorname{csch}^{-1}(cx)) \\ &= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3x^3} - \frac{2de (a + b \operatorname{csch}^{-1}(cx))}{x} + e^2 x (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{9x^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3x^3} - \frac{2de (a + b \operatorname{csch}^{-1}(cx))}{x} + e^2 x \\ &= -\frac{2bcd (c^2 d - 9e) \sqrt{-1 - c^2 x^2}}{9 \sqrt{-c^2 x^2}} + \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{9x^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3x^3} \\ &= -\frac{2bcd (c^2 d - 9e) \sqrt{-1 - c^2 x^2}}{9 \sqrt{-c^2 x^2}} + \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{9x^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3x^3} \\ &= -\frac{2bcd (c^2 d - 9e) \sqrt{-1 - c^2 x^2}}{9 \sqrt{-c^2 x^2}} + \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{9x^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3x^3} \end{aligned}$$

**Mathematica [A]** time = 0.27, size = 123, normalized size = 0.75

$$\frac{bcdx \sqrt{\frac{1}{c^2 x^2} + 1} (-2c^2 dx^2 + d + 18ex^2) - 3a (d^2 + 6dex^2 - 3e^2 x^4)}{9x^3} + \frac{be^2 \log \left( x \left( \sqrt{\frac{1}{c^2 x^2} + 1} + 1 \right) \right)}{c} - \frac{b \operatorname{csch}^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x^4,x]

[Out] (b\*c\*d\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(d - 2\*c^2\*d\*x^2 + 18\*e\*x^2) - 3\*a\*(d^2 + 6\*d\*e\*x^2 - 3\*e^2\*x^4))/(9\*x^3) - (b\*(d^2 + 6\*d\*e\*x^2 - 3\*e^2\*x^4)\*ArcCsch[c\*x])/(3\*x^3) + (b\*e^2\*Log[(1 + Sqrt[1 + 1/(c^2\*x^2)])\*x])/c

**fricas** [B] time = 0.92, size = 334, normalized size = 2.04

$$9ace^2x^4 - 9be^2x^3 \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) - 18acdex^2 - 3(bcd^2 + 6bcde - 3bce^2)x^3 \log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) + 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^4,x, algorithm="fricas")

[Out] 1/9\*(9\*a\*c\*e^2\*x^4 - 9\*b\*e^2\*x^3\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x) - 18\*a\*c\*d\*e\*x^2 - 3\*(b\*c\*d^2 + 6\*b\*c\*d\*e - 3\*b\*c\*e^2)\*x^3\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) + 3\*(b\*c\*d^2 + 6\*b\*c\*d\*e - 3\*b\*c\*e^2)\*x^3\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x - 1) - 3\*a\*c\*d^2 - 2\*(b\*c^4\*d^2 - 9\*b\*c^2\*d\*e)\*x^3 + 3\*(3\*b\*c\*e^2\*x^4 - 6\*b\*c\*d\*e\*x^2 - b\*c\*d^2 + (b\*c\*d^2 + 6\*b\*c\*d\*e - 3\*b\*c\*e^2)\*x^3)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (b\*c^2\*d^2\*x - 2\*(b\*c^4\*d^2 - 9\*b\*c^2\*d\*e)\*x^3)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c\*x^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^4,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsch(c\*x) + a)/x^4, x)

**maple** [A] time = 0.06, size = 190, normalized size = 1.16

$$c^3 \left( \frac{a \left( cx e^2 - \frac{2cde}{x} - \frac{d^2c}{3x^3} \right)}{c^4} + \frac{b \left( \operatorname{arcsch}(cx) cx e^2 - \frac{2 \operatorname{arcsch}(cx) cde}{x} - \frac{\operatorname{arcsch}(cx) d^2c}{3x^3} + \frac{\sqrt{c^2x^2+1} \left( -2\sqrt{c^2x^2+1} c^6x^2d^2 + 18c^4de\sqrt{c^2x^2+1} - 2\sqrt{c^2x^2+1} c^6x^2d^2 + 18c^4de\sqrt{c^2x^2+1} \right)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}}}{c^4} \right)}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^4,x)

[Out] c^3\*(a/c^4\*(c\*x\*e^2-2\*c\*d\*e/x-1/3\*d^2\*c/x^3)+b/c^4\*(arccsch(c\*x)\*c\*x\*e^2-2\*arccsch(c\*x)\*c\*d\*e/x-1/3\*arccsch(c\*x)\*d^2\*c/x^3+1/9\*(c^2\*x^2+1)^(1/2)\*(-2\*(c^2\*x^2+1)^(1/2)\*c^6\*x^2\*d^2+18\*c^4\*d\*e\*(c^2\*x^2+1)^(1/2)\*x^2+d^2\*c^4\*(c^2\*x^2+1)^(1/2)+9\*e^2\*arcsinh(c\*x)\*c^3\*x^3)/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c^4/x^4)

**maxima** [A] time = 0.33, size = 152, normalized size = 0.93

$$2 \left( c \sqrt{\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsch}(cx)}{x} \right) bde + ae^2x + \frac{1}{9} bd^2 \left( \frac{c^4 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 3c^4 \sqrt{\frac{1}{c^2x^2} + 1}}{c} - \frac{3 \operatorname{arcsch}(cx)}{x^3} \right) + \frac{(2cx \operatorname{arcsch}(cx) + \dots)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^4,x, algorithm="maxima")

[Out]  $2*(c*\sqrt{1/(c^2*x^2) + 1} - \operatorname{arccsch}(c*x)/x)*b*d*e + a*e^{2*x} + 1/9*b*d^2*((c^4*(1/(c^2*x^2) + 1)^{(3/2)} - 3*c^4*\sqrt{1/(c^2*x^2) + 1})/c - 3*\operatorname{arccsch}(c*x)/x^3) + 1/2*(2*c*x*\operatorname{arccsch}(c*x) + \log(\sqrt{1/(c^2*x^2) + 1} + 1) - \log(\sqrt{1/(c^2*x^2) + 1} - 1))*b*e^{2/c} - 2*a*d*e/x - 1/3*a*d^2/x^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d)^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^4, x)`

[Out] `int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^4, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**4, x)`

[Out] `Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**4, x)`

$$3.92 \quad \int \frac{(d+ex^2)^2 (a+bcsch^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=189

$$\frac{d^2 (a + bcsch^{-1}(cx))}{5x^5} - \frac{2de (a + bcsch^{-1}(cx))}{3x^3} - \frac{e^2 (a + bcsch^{-1}(cx))}{x} + \frac{bcd^2 \sqrt{-c^2 x^2 - 1}}{25x^4 \sqrt{-c^2 x^2}} - \frac{2bcd \sqrt{-c^2 x^2 - 1} (6c^2 d - 25e^2)}{225x^2 \sqrt{-c^2 x^2}}$$

[Out]  $-1/5*d^2*(a+b*arccsch(c*x))/x^5-2/3*d*e*(a+b*arccsch(c*x))/x^3-e^2*(a+b*arccsch(c*x))/x+1/225*b*c*(24*c^4*d^2-100*c^2*d*e+225*e^2)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/25*b*c*d^2*(-c^2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2)-2/225*b*c*d*(6*c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/x^2/(-c^2*x^2)^(1/2)$

**Rubi [A]** time = 0.16, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {270, 6302, 12, 1265, 453, 264}

$$\frac{d^2 (a + bcsch^{-1}(cx))}{5x^5} - \frac{2de (a + bcsch^{-1}(cx))}{3x^3} - \frac{e^2 (a + bcsch^{-1}(cx))}{x} + \frac{bc\sqrt{-c^2 x^2 - 1} (24c^4 d^2 - 100c^2 de + 225e^2)}{225\sqrt{-c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x^6,x]

[Out]  $(b*c*(24*c^4*d^2 - 100*c^2*d*e + 225*e^2)*\text{Sqrt}[-1 - c^2*x^2])/(225*\text{Sqrt}[-(c^2*x^2)]) + (b*c*d^2*\text{Sqrt}[-1 - c^2*x^2])/(25*x^4*\text{Sqrt}[-(c^2*x^2)]) - (2*b*c*d*(6*c^2*d - 25*e)*\text{Sqrt}[-1 - c^2*x^2])/(225*x^2*\text{Sqrt}[-(c^2*x^2)]) - (d^2*(a + b*\text{ArcCsch}[c*x]))/(5*x^5) - (2*d*e*(a + b*\text{ArcCsch}[c*x]))/(3*x^3) - (e^2*(a + b*\text{ArcCsch}[c*x]))/x$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

### Rule 270

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a + b\*x^n)^(p+1))/(a\*e\*(m+1)), x] + Dist[(a\*d\*(m+1) - b\*c\*(m+n\*(p+1)+1))/(a\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

### Rule 1265

Int[((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := With[{Qx = PolynomialQuotient[(a + b\*x^2 +

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x],
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
- e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

### Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx))}{x^6} dx &= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{5x^5} - \frac{2de (a + b \operatorname{csch}^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \operatorname{csch}^{-1}(cx))}{x} \\ &= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{5x^5} - \frac{2de (a + b \operatorname{csch}^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \operatorname{csch}^{-1}(cx))}{x} \\ &= \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{25x^4 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{5x^5} - \frac{2de (a + b \operatorname{csch}^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \operatorname{csch}^{-1}(cx))}{x} \\ &= \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{25x^4 \sqrt{-c^2 x^2}} - \frac{2bcd (6c^2 d - 25e) \sqrt{-1 - c^2 x^2}}{225x^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{5x^5} \\ &= \frac{bc (24c^4 d^2 - 100c^2 de + 225e^2) \sqrt{-1 - c^2 x^2}}{225 \sqrt{-c^2 x^2}} + \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{25x^4 \sqrt{-c^2 x^2}} - \frac{2bcd (a + b \operatorname{csch}^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \operatorname{csch}^{-1}(cx))}{x} \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 126, normalized size = 0.67

$$\frac{-15a(3d^2 + 10dex^2 + 15e^2x^4) + bcx\sqrt{\frac{1}{c^2x^2} + 1}(-50dex^2(2c^2x^2 - 1) + 3d^2(8c^4x^4 - 4c^2x^2 + 3) + 225e^2x^4) - 1}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6, x]
```

```
[Out] (-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(225
*e^2*x^4 - 50*d*e*x^2*(-1 + 2*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4))
- 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c*x])/(225*x^5)
```

**fricas [A]** time = 0.63, size = 165, normalized size = 0.87

$$\frac{225ae^2x^4 + 150adex^2 + 45ad^2 + 15(15be^2x^4 + 10bdex^2 + 3bd^2) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}+1}}{cx}\right) - ((24bc^5d^2 - 100bc^3d^2 - 100bc^3de + 150bc^3e^2) \sqrt{-1 - c^2x^2} - (24bc^5d^2 - 100bc^3d^2 - 100bc^3de + 150bc^3e^2) \operatorname{csch}^{-1}(cx))}{225x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

[Out]  $-1/225*(225*a*e^{2*x^4} + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^{2*x^4} + 10*b*d*e*x^2 + 3*b*d^2)*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)) - ((24*b*c^5*d^2 - 100*b*c^3*d*e + 225*b*c*e^2)*x^5 + 9*b*c*d^2*x - 2*(6*b*c^3*d^2 - 25*b*c*d*e)*x^3)*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)})/x^5$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^6, x)`

**maple** [A] time = 0.06, size = 191, normalized size = 1.01

$$c^5 \left[ \frac{a \left( -\frac{e^2}{cx} - \frac{2de}{3cx^3} - \frac{d^2}{5cx^5} \right)}{c^4} + \frac{b \left( -\frac{\operatorname{arccsch}(cx)e^2}{cx} - \frac{2\operatorname{arccsch}(cx)de}{3cx^3} - \frac{\operatorname{arccsch}(cx)d^2}{5cx^5} + \frac{(c^2x^2+1)(24c^8d^2x^4-100c^6dex^4-12c^6d^2x^2+225c^4e^2)}{225\sqrt{\frac{c^2x^2+1}{c^2x^2}}c^6x^6} \right)}{c^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x)`

[Out]  $c^5*(a/c^4*(-e^2/c/x-2/3/c*d*e/x^3-1/5*d^2/c/x^5)+b/c^4*(-\operatorname{arccsch}(c*x)*e^2/c/x-2/3*\operatorname{arccsch}(c*x)/c*d*e/x^3-1/5*\operatorname{arccsch}(c*x)*d^2/c/x^5+1/225*(c^2*x^2+1)*(24*c^8*d^2*x^4-100*c^6*d*e*x^4-12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2+9*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/c^6/x^6))$

**maxima** [A] time = 0.36, size = 175, normalized size = 0.93

$$\left( c\sqrt{\frac{1}{c^2x^2} + 1} - \frac{\operatorname{arcsch}(cx)}{x} \right) be^2 + \frac{1}{75} bd^2 \left( \frac{3c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} - 10c^6 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} + 15c^6 \sqrt{\frac{1}{c^2x^2} + 1}}{c} - \frac{15 \operatorname{arcsch}(cx)}{x^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")`

[Out]  $(c*\sqrt{1/(c^2*x^2) + 1} - \operatorname{arccsch}(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(1/(c^2*x^2) + 1)^{(5/2)} - 10*c^6*(1/(c^2*x^2) + 1)^{(3/2)} + 15*c^6*\sqrt{1/(c^2*x^2) + 1})/c - 15*\operatorname{arccsch}(c*x)/x^5) + 2/9*b*d*e*((c^4*(1/(c^2*x^2) + 1)^{(3/2)} - 3*c^4*\sqrt{1/(c^2*x^2) + 1})/c - 3*\operatorname{arccsch}(c*x)/x^3) - a*e^2/x - 2/3*a*d*e/x^3 - 1/5*a*d^2/x^5$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d)^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^6,x)`

[Out] `int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^6, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**6, x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**6, x)
```

$$3.93 \quad \int \frac{(d+ex^2)^2 (a+bcsch^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=249

$$\frac{d^2 (a + bcsch^{-1}(cx))}{7x^7} - \frac{2de (a + bcsch^{-1}(cx))}{5x^5} - \frac{e^2 (a + bcsch^{-1}(cx))}{3x^3} + \frac{bcd^2 \sqrt{-c^2x^2 - 1}}{49x^6 \sqrt{-c^2x^2}} - \frac{2bcd \sqrt{-c^2x^2 - 1} (15c^2d)}{1225x^4 \sqrt{-c^2x^2}}$$

[Out]  $-1/7*d^2*(a+b*arccsch(c*x))/x^7-2/5*d*e*(a+b*arccsch(c*x))/x^5-1/3*e^2*(a+b*arccsch(c*x))/x^3-2/11025*b*c^3*(360*c^4*d^2-1176*c^2*d*e+1225*e^2)*(-c^2*x^2-1)^{(1/2)}/(-c^2*x^2)^{(1/2)}+1/49*b*c*d^2*(-c^2*x^2-1)^{(1/2)}/x^6/(-c^2*x^2)^{(1/2)}-2/1225*b*c*d*(15*c^2*d-49*e)*(-c^2*x^2-1)^{(1/2)}/x^4/(-c^2*x^2)^{(1/2)}+1/11025*b*c*(360*c^4*d^2-1176*c^2*d*e+1225*e^2)*(-c^2*x^2-1)^{(1/2)}/x^2/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.20, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {270, 6302, 12, 1265, 453, 271, 264}

$$\frac{d^2 (a + bcsch^{-1}(cx))}{7x^7} - \frac{2de (a + bcsch^{-1}(cx))}{5x^5} - \frac{e^2 (a + bcsch^{-1}(cx))}{3x^3} - \frac{2bc^3 \sqrt{-c^2x^2 - 1} (360c^4d^2 - 1176c^2de + 1225e^2)}{11025 \sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x^8,x]

[Out]  $(-2*b*c^3*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*\text{Sqrt}[-1 - c^2*x^2])/((11025*\text{Sqrt}[-(c^2*x^2)]) + (b*c*d^2*\text{Sqrt}[-1 - c^2*x^2]))/(49*x^6*\text{Sqrt}[-(c^2*x^2)]) - (2*b*c*d*(15*c^2*d - 49*e)*\text{Sqrt}[-1 - c^2*x^2])/((1225*x^4*\text{Sqrt}[-(c^2*x^2)])) + (b*c*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*\text{Sqrt}[-1 - c^2*x^2])/((11025*x^2*\text{Sqrt}[-(c^2*x^2)])) - (d^2*(a + b*ArcCsch[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcCsch[c*x]))/(5*x^5) - (e^2*(a + b*ArcCsch[c*x]))/(3*x^3)$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

### Rule 270

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m+1)\*(a+b\*x^n)^(p+1))/(a\*(m+1)), x] - Dist[(b\*(m+n\*(p+1)+1))/(a\*(m+1)), Int[x^(m+n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m+1)/n+p+1], 0] && NeQ[m, -1]

### Rule 453



```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 1265

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]}, Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

### Rule 6302

```
Int[((a_) + ArcCsch[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + bcsch^{-1}(cx))}{x^8} dx &= -\frac{d^2 (a + bcsch^{-1}(cx))}{7x^7} - \frac{2de (a + bcsch^{-1}(cx))}{5x^5} - \frac{e^2 (a + bcsch^{-1}(cx))}{3x^3} \\ &= -\frac{d^2 (a + bcsch^{-1}(cx))}{7x^7} - \frac{2de (a + bcsch^{-1}(cx))}{5x^5} - \frac{e^2 (a + bcsch^{-1}(cx))}{3x^3} \\ &= \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{49x^6 \sqrt{-c^2 x^2}} - \frac{d^2 (a + bcsch^{-1}(cx))}{7x^7} - \frac{2de (a + bcsch^{-1}(cx))}{5x^5} - \frac{e^2 (a + bcsch^{-1}(cx))}{3x^3} \\ &= \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{49x^6 \sqrt{-c^2 x^2}} - \frac{2bcd (15c^2 d - 49e) \sqrt{-1 - c^2 x^2}}{1225x^4 \sqrt{-c^2 x^2}} - \frac{d^2 (a + bcsch^{-1}(cx))}{7x^7} \\ &= \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{49x^6 \sqrt{-c^2 x^2}} - \frac{2bcd (15c^2 d - 49e) \sqrt{-1 - c^2 x^2}}{1225x^4 \sqrt{-c^2 x^2}} + \frac{bc (360c^4 d^2 - 1176c^2 de + 1225e^2)}{11025} \\ &= -\frac{2bc^3 (360c^4 d^2 - 1176c^2 de + 1225e^2) \sqrt{-1 - c^2 x^2}}{11025 \sqrt{-c^2 x^2}} + \frac{bcd^2 \sqrt{-1 - c^2 x^2}}{49x^6 \sqrt{-c^2 x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 152, normalized size = 0.61

$$\frac{-105a (15d^2 + 42dex^2 + 35e^2x^4) + bcx \sqrt{\frac{1}{c^2 x^2} + 1} (1225e^2x^4 (1 - 2c^2x^2) + 294dex^2 (8c^4x^4 - 4c^2x^2 + 3) - 45d^2)}{11025x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x^8,x]

[Out]  $(-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*\text{Sqrt}[1 + 1/(c^2*x^2)])*x*(1225*e^2*x^4*(1 - 2*c^2*x^2) + 294*d*e*x^2*(3 - 4*c^2*x^2 + 8*c^4*x^4) - 45*d^2*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4)*\text{ArcCsch}[c*x]/(11025*x^7)$

**fricas** [A] time = 1.01, size = 197, normalized size = 0.79

$$3675 ae^2x^4 + 4410 adex^2 + 1575 ad^2 + 105 (35 be^2x^4 + 42 bdex^2 + 15 bd^2) \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx}\right) + (2(360 bc^7d^2 - 1176 b^2c^5de + 1225 b^3c^3e^2)x^7 - (360 b^2c^5d^2 - 1176 b^3c^3de + 1225 b^4c^1e^2)x^5 - 225 b^2c^4d^2x + 18(15 b^3c^3d^2 - 49 b^4c^2de)x^3) \sqrt{\frac{c^2x^2+1}{c^2x^2}}/x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")`

[Out]  $-1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4 + 42*b*d*e*x^2 + 15*b*d^2))*\log((c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*(360*b*c^7*d^2 - 1176*b*c^5*d*e + 1225*b*c^3*e^2)*x^7 - (360*b*c^5*d^2 - 1176*b*c^3*d*e + 1225*b*c*e^2)*x^5 - 225*b*c*d^2*x + 18*(15*b*c^3*d^2 - 49*b*c*d*e)*x^3)*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2))/x^7$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^8, x)`

**maple** [A] time = 0.06, size = 223, normalized size = 0.90

$$c^7 \left[ \frac{a \left( -\frac{e^2}{3c^3x^3} - \frac{d^2}{7c^3x^7} - \frac{2de}{5c^3x^5} \right)}{c^4} + \frac{b \left( -\frac{\operatorname{arccsch}(cx)e^2}{3c^3x^3} - \frac{\operatorname{arccsch}(cx)d^2}{7c^3x^7} - \frac{2\operatorname{arccsch}(cx)de}{5c^3x^5} - \frac{(c^2x^2+1)(720c^{10}d^2x^6-2352c^8dex^6-360c^8d^2x^4)}{c^4} \right)}{c^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x)`

[Out]  $c^7*(a/c^4*(-1/3*e^2/c^3/x^3-1/7*d^2/c^3/x^7-2/5/c^3*d*e/x^5)+b/c^4*(-1/3*arccsch(c*x)*e^2/c^3/x^3-1/7*arccsch(c*x)*d^2/c^3/x^7-2/5*arccsch(c*x)/c^3*d*e/x^5-1/11025*(c^2*x^2+1)*(720*c^10*d^2*x^6-2352*c^8*d*e*x^6-360*c^8*d^2*x^4+2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2-1225*c^4*e^2*x^4-882*c^4*d*e*x^2-225*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^8/x^8))$

**maxima** [A] time = 0.45, size = 232, normalized size = 0.93

$$\frac{1}{245} bd^2 \left( \frac{5c^8 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{7}{2}} - 21c^8 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{5}{2}} + 35c^8 \left( \frac{1}{c^2x^2} + 1 \right)^{\frac{3}{2}} - 35c^8 \sqrt{\frac{1}{c^2x^2} + 1}}{c} - \frac{35 \operatorname{arcsch}(cx)}{x^7} \right) + \frac{2}{75} bde \left( \frac{3c^6}{c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^8,x, algorithm="maxima")

[Out] 1/245\*b\*d^2\*((5\*c^8\*(1/(c^2\*x^2) + 1)^(7/2) - 21\*c^8\*(1/(c^2\*x^2) + 1)^(5/2) + 35\*c^8\*(1/(c^2\*x^2) + 1)^(3/2) - 35\*c^8\*sqrt(1/(c^2\*x^2) + 1))/c - 35\*arccsch(c\*x)/x^7) + 2/75\*b\*d\*e\*((3\*c^6\*(1/(c^2\*x^2) + 1)^(5/2) - 10\*c^6\*(1/(c^2\*x^2) + 1)^(3/2) + 15\*c^6\*sqrt(1/(c^2\*x^2) + 1))/c - 15\*arccsch(c\*x)/x^5) + 1/9\*b\*e^2\*((c^4\*(1/(c^2\*x^2) + 1)^(3/2) - 3\*c^4\*sqrt(1/(c^2\*x^2) + 1))/c - 3\*arccsch(c\*x)/x^3) - 1/3\*a\*e^2/x^3 - 2/5\*a\*d\*e/x^5 - 1/7\*a\*d^2/x^7

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^2 \left( a + b \operatorname{arsinh}\left(\frac{1}{cx}\right) \right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))))/x^8,x)

[Out] int(((d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))))/x^8, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*acsch(c\*x))/x\*\*8,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)\*\*2/x\*\*8, x)

### 3.94 $\int x^3 (d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=250

$$\frac{1}{4}d^2x^4(a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3}dex^6(a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{8}e^2x^8(a + b \operatorname{csch}^{-1}(cx)) + \frac{bex(-c^2x^2 - 1)^{5/2}(8c^2d - 9e) - be^2}{120c^7\sqrt{-c^2x^2}}$$

[Out]  $\frac{1}{4}d^2x^4(a + b \operatorname{arccsch}(cx)) + \frac{1}{3}dex^6(a + b \operatorname{arccsch}(cx)) + \frac{1}{8}e^2x^8(a + b \operatorname{arccsch}(cx)) - \frac{1}{72}b(6c^4d^2 - 16c^2de + 9e^2)x(-c^2x^2 - 1)^{3/2}/c^7/(-c^2x^2)^{1/2} + \frac{1}{120}b(8c^2d - 9e)ex(-c^2x^2 - 1)^{5/2}/c^7/(-c^2x^2)^{1/2} - \frac{1}{56}be^2x(-c^2x^2 - 1)^{7/2}/c^7/(-c^2x^2)^{1/2} - \frac{1}{24}b(6c^4d^2 - 8c^2de + 3e^2)x(-c^2x^2 - 1)^{1/2}/c^7/(-c^2x^2)^{1/2}$

**Rubi [A]** time = 0.23, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {266, 43, 6302, 12, 1251, 771}

$$\frac{1}{4}d^2x^4(a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3}dex^6(a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{8}e^2x^8(a + b \operatorname{csch}^{-1}(cx)) - \frac{bx(-c^2x^2 - 1)^{3/2}(6c^4d^2 - 16c^2de + 9e^2)}{72c^7\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]), x]

[Out]  $-(b(6c^4d^2 - 8c^2de + 3e^2)x\sqrt{-1 - c^2x^2})/(24c^7\sqrt{-(c^2x^2)}) - (b(6c^4d^2 - 16c^2de + 9e^2)x(-1 - c^2x^2)^{3/2})/(72c^7\sqrt{-(c^2x^2)}) + (b(8c^2d - 9e)ex(-1 - c^2x^2)^{5/2})/(120c^7\sqrt{-(c^2x^2)}) - (be^2x(-1 - c^2x^2)^{7/2})/(56c^7\sqrt{-(c^2x^2)}) + (d^2x^4(a + b \operatorname{ArcCsch}[c*x]))/4 + (dex^6(a + b \operatorname{ArcCsch}[c*x]))/3 + (e^2x^8(a + b \operatorname{ArcCsch}[c*x]))/8$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 771

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((f\_.) + (g\_.)\*(x\_))\*((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)\*(a + b\*x + c\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4\*a\*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

#### Rule 1251

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]
```

### Rule 6302

```
Int[((a_) + ArcCsch[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned} \int x^3 (d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{1}{4} d^2 x^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{1}{4} d^2 x^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{1}{4} d^2 x^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \operatorname{csch}^{-1}(cx)) \\ &= \frac{1}{4} d^2 x^4 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \operatorname{csch}^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \operatorname{csch}^{-1}(cx)) \\ &= -\frac{b(6c^4 d^2 - 8c^2 de + 3e^2)x\sqrt{-1 - c^2 x^2}}{24c^7 \sqrt{-c^2 x^2}} - \frac{b(6c^4 d^2 - 16c^2 de + 9e^2)x}{72c^7 \sqrt{-c^2 x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.39, size = 159, normalized size = 0.64

$$x \left( 105ax^3 (6d^2 + 8dex^2 + 3e^2x^4) + \frac{b\sqrt{\frac{1}{c^2x^2} + 1} (3c^6(70d^2x^2 + 56dex^4 + 15e^2x^6) - 2c^4(210d^2 + 112dex^2 + 27e^2x^4) + 8c^2e(56d + 9ex^2) - 144e^2)}{c^7} \right)$$

2520

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]), x]

[Out] (x\*(105\*a\*x^3\*(6\*d^2 + 8\*d\*e\*x^2 + 3\*e^2\*x^4) + (b\*Sqrt[1 + 1/(c^2\*x^2)]\*(-144\*e^2 + 8\*c^2\*e\*(56\*d + 9\*e\*x^2) - 2\*c^4\*(210\*d^2 + 112\*d\*e\*x^2 + 27\*e^2\*x^4) + 3\*c^6\*(70\*d^2\*x^2 + 56\*d\*e\*x^4 + 15\*e^2\*x^6))))/c^7 + 105\*b\*x^3\*(6\*d^2 + 8\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsch[c\*x])/2520

**fricas [A]** time = 1.25, size = 224, normalized size = 0.90

$$315 ac^7 e^2 x^8 + 840 ac^7 dex^6 + 630 ac^7 d^2 x^4 + 105 (3 bc^7 e^2 x^8 + 8 bc^7 dex^6 + 6 bc^7 d^2 x^4) \log \left( \frac{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1}{cx} \right) + (45 bc^7 e^2 x^8 + 140 bc^7 dex^6 + 90 bc^7 d^2 x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 1/2520\*(315\*a\*c^7\*e^2\*x^8 + 840\*a\*c^7\*d\*e\*x^6 + 630\*a\*c^7\*d^2\*x^4 + 105\*(3\*b\*c^7\*e^2\*x^8 + 8\*b\*c^7\*d\*e\*x^6 + 6\*b\*c^7\*d^2\*x^4)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (45\*b\*c^6\*e^2\*x^7 + 6\*(28\*b\*c^6\*d\*e - 9\*b\*c^4\*e^2)\*x^5 + 2\*(105\*b\*c^6\*d^2 - 112\*b\*c^4\*d\*e + 36\*b\*c^2\*e^2)\*x^3 - 4\*(105\*b\*c^4\*d^2 - 112\*b\*c^2\*d\*e + 36\*b\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/c^7

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsch(c\*x) + a)\*x^3, x)

**maple** [A] time = 0.05, size = 214, normalized size = 0.86

$$\frac{a\left(\frac{1}{8}e^2c^8x^8 + \frac{1}{3}c^8dex^6 + \frac{1}{4}c^8d^2x^4\right)}{c^4} + \frac{b\left(\frac{\operatorname{arccsch}(cx)e^{2c^8x^8}}{8} + \frac{\operatorname{arccsch}(cx)c^8dex^6}{3} + \frac{\operatorname{arccsch}(cx)c^8x^4d^2}{4} + \frac{(c^2x^2+1)(45c^6e^2x^6+168c^6dex^4+210c^6d^2x^2-54c^4e^2x^4-224c^4dex^2-420c^4d^2x^2+72c^2e^2x^2+448c^2d^2e-144e^2)}{2520\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^4} \operatorname{arcsch}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)),x)

[Out] 1/c^4\*(a/c^4\*(1/8\*e^2\*c^8\*x^8+1/3\*c^8\*d\*e\*x^6+1/4\*c^8\*d^2\*x^4)+b/c^4\*(1/8\*a\*arccsch(c\*x)\*e^2\*c^8\*x^8+1/3\*arccsch(c\*x)\*c^8\*d\*e\*x^6+1/4\*arccsch(c\*x)\*c^8\*x^4\*d^2+1/2520\*(c^2\*x^2+1)\*(45\*c^6\*e^2\*x^6+168\*c^6\*d\*e\*x^4+210\*c^6\*d^2\*x^2-54\*c^4\*e^2\*x^4-224\*c^4\*d\*e\*x^2-420\*c^4\*d^2+72\*c^2\*e^2\*x^2+448\*c^2\*d\*e-144\*e^2)/((c^2\*x^2+1)/c^2/x^2)^(1/2)/c/x))

**maxima** [A] time = 0.37, size = 244, normalized size = 0.98

$$\frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{12}\left(3x^4 \operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2} + 1}}{c^3}\right)bd^2 + \frac{1}{45}\left(15x^6 \operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2} + 1}}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/8\*a\*e^2\*x^8 + 1/3\*a\*d\*e\*x^6 + 1/4\*a\*d^2\*x^4 + 1/12\*(3\*x^4\*arccsch(c\*x) + (c^2\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) - 3\*x\*sqrt(1/(c^2\*x^2) + 1))/c^3)\*b\*d^2 + 1/45\*(15\*x^6\*arccsch(c\*x) + (3\*c^4\*x^5\*(1/(c^2\*x^2) + 1)^(5/2) - 10\*c^2\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) + 15\*x\*sqrt(1/(c^2\*x^2) + 1))/c^5)\*b\*d\*e + 1/280\*(35\*x^8\*arccsch(c\*x) + (5\*c^6\*x^7\*(1/(c^2\*x^2) + 1)^(7/2) - 21\*c^4\*x^5\*(1/(c^2\*x^2) + 1)^(5/2) + 35\*c^2\*x^3\*(1/(c^2\*x^2) + 1)^(3/2) - 35\*x\*sqrt(1/(c^2\*x^2) + 1))/c^7)\*b\*e^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (ex^2 + d)^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{acsch}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

### 3.95 $\int x (d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=203

$$\frac{(d + ex^2)^3 (a + b \operatorname{csch}^{-1}(cx))}{6e} - \frac{bcd^3 x \tan^{-1}\left(\sqrt{-c^2 x^2 - 1}\right)}{6e\sqrt{-c^2 x^2}} - \frac{bex(-c^2 x^2 - 1)^{3/2} (3c^2 d - 2e)}{18c^5 \sqrt{-c^2 x^2}} + \frac{be^2 x (-c^2 x^2 - 1)^{5/2}}{30c^5 \sqrt{-c^2 x^2}} + b$$

[Out]  $1/6*(e*x^2+d)^3*(a+b*\operatorname{arccsch}(c*x))/e-1/18*b*(3*c^2*d-2*e)*e*x*(-c^2*x^2-1)^{(3/2)}/c^5/(-c^2*x^2)^{(1/2)}+1/30*b*e^2*x*(-c^2*x^2-1)^{(5/2)}/c^5/(-c^2*x^2)^{(1/2)}-1/6*b*c*d^3*x*\operatorname{arctan}((-c^2*x^2-1)^{(1/2)})/e/(-c^2*x^2)^{(1/2)}+1/6*b*(3*c^4*d^2-3*c^2*d*e+e^2)*x*(-c^2*x^2-1)^{(1/2)}/c^5/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.16, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {6300, 446, 88, 63, 205}

$$\frac{(d + ex^2)^3 (a + b \operatorname{csch}^{-1}(cx))}{6e} + \frac{bx\sqrt{-c^2 x^2 - 1} (3c^4 d^2 - 3c^2 d e + e^2)}{6c^5 \sqrt{-c^2 x^2}} - \frac{bcd^3 x \tan^{-1}\left(\sqrt{-c^2 x^2 - 1}\right)}{6e\sqrt{-c^2 x^2}} - \frac{bex(-c^2 x^2 - 1)^{3/2}}{18c^5 \sqrt{-c^2 x^2}}$$

Antiderivative was successfully verified.

[In] `Int[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]`

[Out]  $(b*(3*c^4*d^2 - 3*c^2*d*e + e^2)*x*\operatorname{Sqrt}[-1 - c^2*x^2])/(6*c^5*\operatorname{Sqrt}[-(c^2*x^2 - 2)]) - (b*(3*c^2*d - 2*e)*e*x*(-1 - c^2*x^2)^{(3/2)})/(18*c^5*\operatorname{Sqrt}[-(c^2*x^2 - 2)]) + (b*e^2*x*(-1 - c^2*x^2)^{(5/2)})/(30*c^5*\operatorname{Sqrt}[-(c^2*x^2 - 2)]) + ((d + e*x^2)^3*(a + b*\operatorname{ArcCsch}[c*x]))/(6*e) - (b*c*d^3*x*\operatorname{ArcTan}[\operatorname{Sqrt}[-1 - c^2*x^2]])/(6*e*\operatorname{Sqrt}[-(c^2*x^2 - 2)])$

#### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

#### Rule 88

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))`

#### Rule 205

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]`

#### Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

#### Rule 6300



```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*(x_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

### Rubi steps

$$\begin{aligned}
\int x(d+ex^2)^2(a+bcsch^{-1}(cx))dx &= \frac{(d+ex^2)^3(a+bcsch^{-1}(cx))}{6e} - \frac{(bcx)\int\frac{(d+ex^2)^3}{x\sqrt{-1-c^2x^2}}dx}{6e\sqrt{-c^2x^2}} \\
&= \frac{(d+ex^2)^3(a+bcsch^{-1}(cx))}{6e} - \frac{(bcx)\text{Subst}\left(\int\frac{(d+ex^2)^3}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{12e\sqrt{-c^2x^2}} \\
&= \frac{(d+ex^2)^3(a+bcsch^{-1}(cx))}{6e} - \frac{(bcx)\text{Subst}\left(\int\left(\frac{e(3c^4d^2-3c^2de+e^2)}{c^4\sqrt{-1-c^2x}}+\frac{d}{x\sqrt{-1-c^2x}}\right)dx,x,x^2\right)}{12e\sqrt{-c^2x^2}} \\
&= \frac{b(3c^4d^2-3c^2de+e^2)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b(3c^2d-2e)ex(-1-c^2x^2)^{3/2}}{18c^5\sqrt{-c^2x^2}} \\
&= \frac{b(3c^4d^2-3c^2de+e^2)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b(3c^2d-2e)ex(-1-c^2x^2)^{3/2}}{18c^5\sqrt{-c^2x^2}} \\
&= \frac{b(3c^4d^2-3c^2de+e^2)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b(3c^2d-2e)ex(-1-c^2x^2)^{3/2}}{18c^5\sqrt{-c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 123, normalized size = 0.61

$$\frac{1}{90}x\left(15ax(3d^2+3dex^2+e^2x^4)+\frac{b\sqrt{\frac{1}{c^2x^2}+1}(3c^4(15d^2+5dex^2+e^2x^4)-2c^2e(15d+2ex^2)+8e^2)}{c^5}+15bcx\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]), x]
```

```
[Out] (x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(8*e^2 - 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4))))/c^5 + 15*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsch[c*x])/90
```

**fricas [A]** time = 1.16, size = 189, normalized size = 0.93

$$\frac{15ac^5e^2x^6 + 45ac^5dex^4 + 45ac^5d^2x^2 + 15(bc^5e^2x^6 + 3bc^5dex^4 + 3bc^5d^2x^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + (3bc^4e^2x^5 + \dots)}{90c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^5*e^2*x^6 + 45*a*c^5*d*e*x^4 + 45*a*c^5*d^2*x^2 + 15*(b*c^5*e^2*x^6 + 3*b*c^5*d*e*x^4 + 3*b*c^5*d^2*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2
```

$(x^2 + 1)/(cx) + (3bc^4e^2x^5 + (15bc^4de - 4bc^2e^2)x^3 + (45bc^4d^2 - 30bc^2de + 8be^2)x)\sqrt{(c^2x^2 + 1)/(c^2x^2)})/c^5$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsch(c\*x) + a)\*x, x)

**maple** [A] time = 0.05, size = 182, normalized size = 0.90

$$\frac{a\left(\frac{1}{6}c^6e^2x^6 + \frac{1}{2}c^6dex^4 + \frac{1}{2}c^6d^2x^2\right)}{c^4} + \frac{b\left(\frac{\operatorname{arccsch}(cx)e^2c^6x^6}{6} + \frac{\operatorname{arccsch}(cx)c^6dex^4}{2} + \frac{\operatorname{arccsch}(cx)c^6x^2d^2}{2} + \frac{(c^2x^2+1)(3c^4e^2x^4+15c^4dex^2+45d^2c^4-4c^2e^2x^2-30c^2de+8e^2)}{90\sqrt{\frac{c^2x^2+1}{c^2x^2}}cx}\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)),x)

[Out]  $1/c^2*(a/c^4*(1/6*c^6*e^2*x^6+1/2*c^6*d*e*x^4+1/2*c^6*d^2*x^2)+b/c^4*(1/6*a \operatorname{rccsch}(c*x)*e^2*c^6*x^6+1/2*\operatorname{arccsch}(c*x)*c^6*d*e*x^4+1/2*\operatorname{arccsch}(c*x)*c^6*x^2*d^2+1/90*(c^2*x^2+1)*(3*c^4*e^2*x^4+15*c^4*d*e*x^2+45*c^4*d^2-4*c^2*e^2*x^2-30*c^2*d*e+8*e^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x)$

**maxima** [A] time = 0.37, size = 183, normalized size = 0.90

$$\frac{1}{6}ae^2x^6 + \frac{1}{2}adex^4 + \frac{1}{2}ad^2x^2 + \frac{1}{2}\left(x^2 \operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2} + 1}}{c}\right)bd^2 + \frac{1}{6}\left(3x^4 \operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2} + 1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2}}}{c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out]  $1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*\operatorname{arccsch}(c*x) + x*\sqrt{1/(c^2*x^2) + 1}/c)*b*d^2 + 1/6*(3*x^4*\operatorname{arccsch}(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x*\sqrt{1/(c^2*x^2) + 1})/c^3)*b*d*e + 1/90*(15*x^6*\operatorname{arccsch}(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*\sqrt{1/(c^2*x^2) + 1})/c^5)*b*e^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x (ex^2 + d)^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x\*(d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{acsch}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

$$3.96 \quad \int \frac{(d+ex^2)^2 (a+bcsch^{-1}(cx))}{x} dx$$

Optimal. Leaf size=178

$$-d^2 \log\left(\frac{1}{x}\right) (a + bcsch^{-1}(cx)) + dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a + bcsch^{-1}(cx)) + \frac{be^2 x^3 \sqrt{\frac{1}{c^2 x^2} + 1}}{12c} + \frac{bex \sqrt{\frac{1}{c^2 x^2} + 1}}{6c}$$

[Out]  $1/2*b*d^2*arccsch(c*x)^2+d*e*x^2*(a+b*arccsch(c*x))+1/4*e^2*x^4*(a+b*arccsch(c*x))-b*d^2*arccsch(c*x)*\ln(1-(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)+b*d^2*arccsch(c*x)*\ln(1/x)-d^2*(a+b*arccsch(c*x))*\ln(1/x)-1/2*b*d^2*polylog(2,(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)+1/6*b*(6*c^2*d-e)*e*x*(1+1/c^2/x^2)^{(1/2)}/c^3+1/12*b*e^2*x^3*(1+1/c^2/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.42, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {6304, 266, 43, 5789, 6742, 453, 264, 2325, 5659, 3716, 2190, 2279, 2391}

$$-\frac{1}{2}bd^2\text{PolyLog}\left(2, e^{2csch^{-1}(cx)}\right) - d^2 \log\left(\frac{1}{x}\right) (a + bcsch^{-1}(cx)) + dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a + bcsch^{-1}(cx)) +$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x,x]

[Out]  $(b*(6*c^2*d - e)*e*\text{Sqrt}[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*\text{Sqrt}[1 + 1/(c^2*x^2)]*x^3)/(12*c) + (b*d^2*ArcCsch[c*x]^2)/2 + d*e*x^2*(a + b*ArcCsch[c*x]) + (e^2*x^4*(a + b*ArcCsch[c*x]))/4 - b*d^2*ArcCsch[c*x]*\text{Log}[1 - E^{(2*ArcCsch[c*x])}] + b*d^2*ArcCsch[c*x]*\text{Log}[x^{(-1)}] - d^2*(a + b*ArcCsch[c*x])*\text{Log}[x^{(-1)}] - (b*d^2*PolyLog[2, E^{(2*ArcCsch[c*x])}])/2$

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 453

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*e\*(m + 1)), x] + Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (

LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_))/  
((a\_) + (b\_)\*(F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)  
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*(c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol]  
:= Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))  
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2325

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)])\*(b\_)]/Sqrt[(d\_) + (e\_)\*(x\_)^2], x\_Symb  
ol] := Simp[(ArcSinh[(Rt[e, 2]\*x)/Sqrt[d]]\*(a + b\*Log[c\*x^n]))/Rt[e, 2], x]  
- Dist[(b\*n)/Rt[e, 2], Int[ArcSinh[(Rt[e, 2]\*x)/Sqrt[d]]/x, x], x] /; Free  
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2  
, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3716

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + Pi\*(k\_) + (Complex[0, fz\_])\*(f\_  
\_)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2  
\*I, Int[((c + d\*x)^m\*E^(2\*(-(I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-(I\*  
e) + f\*fz\*x))/E^(2\*I\*k\*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ  
erQ[4\*k] && IGtQ[m, 0]

### Rule 5659

Int[((a\_) + ArcSinh[(c\_)\*(x\_)])\*(b\_)]^(n\_)/(x\_), x\_Symbol] := Subst[Int[  
(a + b\*x)^n/Tanh[x], x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,  
0]

### Rule 5789

Int[((a\_) + ArcSinh[(c\_)\*(x\_)])\*(b\_))\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x  
\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dis  
t[a + b\*ArcSinh[c\*x], u, x] - Dist[b\*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^  
2\*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2\*d] &&  
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

### Rule 6304

Int[((a\_) + ArcSch[(c\_)\*(x\_)])\*(b\_)]^(n\_)\*(x\_)^(m\_)\*((d\_) + (e\_)\*(x\_  
\_)^2)^(p\_), x\_Symbol] := -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n)/  
x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0  
] && IntegersQ[m, p]

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d + ex^2)^2 (a + bcsch^{-1}(cx))}{x} dx = -\text{Subst} \left( \int \frac{(e + dx^2)^2 (a + b \sinh^{-1}(\frac{x}{c}))}{x^5} dx, x, \frac{1}{x} \right)$$

$$= dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx)) - d^2 (a + bcsch^{-1}(cx)) \log(x)$$

$$= dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx)) - d^2 (a + bcsch^{-1}(cx)) \log(x)$$

$$= dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx)) - d^2 (a + bcsch^{-1}(cx)) \log(x)$$

$$= \frac{be^2\sqrt{1 + \frac{1}{c^2x^2}}x^3}{12c} + dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx)) + bd^2 \log(x)$$

$$= \frac{b(6c^2d - e)e\sqrt{1 + \frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1 + \frac{1}{c^2x^2}}x^3}{12c} + dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx))$$

$$= \frac{b(6c^2d - e)e\sqrt{1 + \frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1 + \frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2csch^{-1}(cx)^2 + dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx))$$

$$= \frac{b(6c^2d - e)e\sqrt{1 + \frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1 + \frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2csch^{-1}(cx)^2 + dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx))$$

$$= \frac{b(6c^2d - e)e\sqrt{1 + \frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1 + \frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2csch^{-1}(cx)^2 + dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx))$$

$$= \frac{b(6c^2d - e)e\sqrt{1 + \frac{1}{c^2x^2}}x}{6c^3} + \frac{be^2\sqrt{1 + \frac{1}{c^2x^2}}x^3}{12c} + \frac{1}{2}bd^2csch^{-1}(cx)^2 + dex^2 (a + bcsch^{-1}(cx)) + \frac{1}{4}e^2x^4 (a + bcsch^{-1}(cx))$$

**Mathematica [A]** time = 0.42, size = 148, normalized size = 0.83

$$ad^2 \log(x) + adex^2 + \frac{1}{4}ae^2x^4 + \frac{bdex \left( \sqrt{\frac{1}{c^2x^2} + 1} + cxcsch^{-1}(cx) \right)}{c} + \frac{be^2x \left( 3c^3x^3csch^{-1}(cx) + \sqrt{\frac{1}{c^2x^2} + 1} (c^2x^2 - 2) \right)}{12c^3} + \dots$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(((d + e*x^2)^2*(a + b*ArcCsch[c*x])))/x,x]
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + (b*d*e*x*(Sqrt[1 + 1/(c^2*x^2)] + c*x*ArcCsch[c*x]))/c + (b*e^2*x*(Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2) + 3*c^3*x^3*ArcCsch[c*x]))/(12*c^3) + a*d^2*Log[x] + (b*d^2*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[c*x])])) + PolyLog[2, E^(-2*ArcCsch[c*x])]))/2
```

**fricas** [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2) \operatorname{arcsch}(cx)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x,x, algorithm="fricas")

[Out] integral((a\*e^2\*x^4 + 2\*a\*d\*e\*x^2 + a\*d^2 + (b\*e^2\*x^4 + 2\*b\*d\*e\*x^2 + b\*d^2)\*arccsch(c\*x))/x, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsch(c\*x) + a)/x, x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (a + b \operatorname{arccsch}(cx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x,x)

[Out] int((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}ae^2x^4 + 4bc^2d^2 \int \frac{x \log(x)}{4(\sqrt{c^2x^2 + 1}c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1)} dx + adex^2 - bd^2 \log(c) \log(x) - \frac{1}{4} (2 \log(c^2x^2 + 1) + \log(\sqrt{c^2x^2 + 1} + 1)) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x,x, algorithm="maxima")

[Out] 1/4\*a\*e^2\*x^4 + 4\*b\*c^2\*d^2\*integrate(1/4\*x\*log(x)/(sqrt(c^2\*x^2 + 1)\*c^2\*x^2 + c^2\*x^2 + sqrt(c^2\*x^2 + 1) + 1), x) + a\*d\*e\*x^2 - b\*d^2\*log(c)\*log(x) - 1/4\*(2\*log(c^2\*x^2 + 1)\*log(x) + dilog(-c^2\*x^2))\*b\*d^2 + a\*d^2\*log(x) + 1/2\*b\*d\*e\*(2\*sqrt(c^2\*x^2 + 1) - log(c^2\*x^2 + 1))/c^2 - 1/24\*(3\*c^2\*x^2 - 2\*(c^2\*x^2 + 1)^(3/2) + 6\*sqrt(c^2\*x^2 + 1) - 3\*log(c^2\*x^2 + 1) + 3)\*b\*e^2/c^4 - 1/8\*(2\*b\*c^2\*e^2\*x^4\*log(c) + 4\*b\*c^2\*d^2\*log(x)^2 + (8\*c^2\*d\*e\*log(c) - e^2)\*b\*x^2 + 2\*(b\*c^2\*e^2\*x^4 + 4\*b\*c^2\*d\*e\*x^2)\*log(x) - 2\*(b\*c^2\*e^2\*x^4 + 4\*b\*c^2\*d\*e\*x^2 + 4\*b\*c^2\*d^2\*log(x))\*log(sqrt(c^2\*x^2 + 1) + 1))/c^2 + 1/8\*(4\*c^2\*d\*e - e^2)\*b\*log(c^2\*x^2 + 1)/c^4

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d)^2 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x,x)`

[Out] `int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x,x)`

[Out] `Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x, x)`



$$3.97 \quad \int \frac{(d+ex^2)^2 (a+b\operatorname{csch}^{-1}(cx))}{x^3} dx$$

**Optimal.** Leaf size=178

$$-\frac{d^2 (a + b\operatorname{csch}^{-1}(cx))}{2x^2} - 2de \log\left(\frac{1}{x}\right) (a + b\operatorname{csch}^{-1}(cx)) + \frac{1}{2}e^2x^2 (a + b\operatorname{csch}^{-1}(cx)) + \frac{bcd^2 \sqrt{\frac{1}{c^2x^2} + 1}}{4x} - \frac{1}{4}bc^2d^2\operatorname{csch}^{-1}(cx)$$

[Out]  $-1/4*b*c^2*d^2*\operatorname{arccsch}(c*x)+b*d*e*\operatorname{arccsch}(c*x)^2-1/2*d^2*(a+b*\operatorname{arccsch}(c*x))/x^2+1/2*e^2*x^2*(a+b*\operatorname{arccsch}(c*x))-2*b*d*e*\operatorname{arccsch}(c*x)*\ln(1-(1/c/x+(1+1/c^2/x^2)^{1/2}))^2+2*b*d*e*\operatorname{arccsch}(c*x)*\ln(1/x)-2*d*e*(a+b*\operatorname{arccsch}(c*x))*\ln(1/x)-b*d*e*\operatorname{polylog}(2,(1/c/x+(1+1/c^2/x^2)^{1/2}))^2+1/4*b*c*d^2*(1+1/c^2/x^2)^{1/2}/x+1/2*b*e^2*x*(1+1/c^2/x^2)^{1/2}/c$

**Rubi [A]** time = 0.42, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {6304, 266, 43, 5789, 12, 6742, 264, 321, 215, 2325, 5659, 3716, 2190, 2279, 2391}

$$-bde\operatorname{PolyLog}\left(2, e^{2\operatorname{csch}^{-1}(cx)}\right) - \frac{d^2 (a + b\operatorname{csch}^{-1}(cx))}{2x^2} - 2de \log\left(\frac{1}{x}\right) (a + b\operatorname{csch}^{-1}(cx)) + \frac{1}{2}e^2x^2 (a + b\operatorname{csch}^{-1}(cx)) +$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e*x^2)^2*(a + b*\operatorname{ArcCsch}[c*x])/x^3, x]$

[Out]  $(b*c*d^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(4*x) + (b*e^2*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x)/(2*c) - (b*c^2*d^2*\operatorname{ArcCsch}[c*x])/4 + b*d*e*\operatorname{ArcCsch}[c*x]^2 - (d^2*(a + b*\operatorname{ArcCsch}[c*x]))/(2*x^2) + (e^2*x^2*(a + b*\operatorname{ArcCsch}[c*x]))/2 - 2*b*d*e*\operatorname{ArcCsch}[c*x]*\operatorname{Log}[1 - E^{(2*\operatorname{ArcCsch}[c*x])}] + 2*b*d*e*\operatorname{ArcCsch}[c*x]*\operatorname{Log}[x^{-1}] - 2*d*e*(a + b*\operatorname{ArcCsch}[c*x])*\operatorname{Log}[x^{-1}] - b*d*e*\operatorname{PolyLog}[2, E^{(2*\operatorname{ArcCsch}[c*x])}]$

### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

### Rule 43

$\operatorname{Int}[(a_*)(x_*)^{(m_*)}((c_*) + (d_*)(x_*))^{(n_*)}, x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (!\operatorname{IntegerQ}[n] || (\operatorname{EqQ}[c, 0] \&\& \operatorname{LeQ}[7*m + 4*n + 4, 0]) || \operatorname{LtQ}[9*m + 5*(n + 1), 0] || \operatorname{GtQ}[m + n + 2, 0])$

### Rule 215

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_*) + (b_*)(x_*)^2], x\_Symbol] := \operatorname{Simp}[\operatorname{ArcSinh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Sqrt}[a]]/\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{PosQ}[b]$

### Rule 264

$\operatorname{Int}[(c_*)(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] := \operatorname{Simp}[(c*x)^{m+1}*(a + b*x^n)^{p+1}/(a*c*(m+1)), x] /; \operatorname{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \operatorname{EqQ}[(m+1)/n + p + 1, 0] \&\& \operatorname{NeQ}[m, -1]$

### Rule 266

$\operatorname{Int}[(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x\_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /; \operatorname{FreeQ}[\{a, b$

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.))\*((c\_.) + (d\_.)\*(x\_)^(m\_.))/((a\_) + (b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*(F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_))]^(n\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2325

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/Sqrt[(d\_) + (e\_.)\*(x\_)^2], x\_Symbol] := Simp[(ArcSinh[Rt[e, 2]\*x]/Sqrt[d]]\*(a + b\*Log[c\*x^n])/Rt[e, 2], x] - Dist[(b\*n)/Rt[e, 2], Int[ArcSinh[Rt[e, 2]\*x]/Sqrt[d]/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_)^(m\_.))\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 5659

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tanh[x], x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

### Rule 5789

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))\*((f\_.)\*(x\_)^(m\_.))\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSinh[c\*x], u, x] - Dist[b\*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^2\*x^2], x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2\*d] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

### Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

### Rubi steps

$$\int \frac{(d + ex^2)^2 (a + b \operatorname{csch}^{-1}(cx))}{x^3} dx = -\operatorname{Subst}\left(\int \frac{(e + dx^2)^2 (a + b \sinh^{-1}(\frac{x}{c}))}{x^3} dx, x, \frac{1}{x}\right)$$

$$= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{2x^2} + \frac{1}{2}e^2 x^2 (a + b \operatorname{csch}^{-1}(cx)) - 2de (a + b \operatorname{csch}^{-1}(cx))$$

$$= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{2x^2} + \frac{1}{2}e^2 x^2 (a + b \operatorname{csch}^{-1}(cx)) - 2de (a + b \operatorname{csch}^{-1}(cx))$$

$$= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{2x^2} + \frac{1}{2}e^2 x^2 (a + b \operatorname{csch}^{-1}(cx)) - 2de (a + b \operatorname{csch}^{-1}(cx))$$

$$= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{2x^2} + \frac{1}{2}e^2 x^2 (a + b \operatorname{csch}^{-1}(cx)) - 2de (a + b \operatorname{csch}^{-1}(cx))$$

$$= \frac{bcd^2 \sqrt{1 + \frac{1}{c^2 x^2}}}{4x} + \frac{be^2 \sqrt{1 + \frac{1}{c^2 x^2}} x}{2c} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{2x^2} + \frac{1}{2}e^2 x^2 (a + b \operatorname{csch}^{-1}(cx))$$

$$= \frac{bcd^2 \sqrt{1 + \frac{1}{c^2 x^2}}}{4x} + \frac{be^2 \sqrt{1 + \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{4}bc^2 d^2 \operatorname{csch}^{-1}(cx) - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{2x^2}$$

$$= \frac{bcd^2 \sqrt{1 + \frac{1}{c^2 x^2}}}{4x} + \frac{be^2 \sqrt{1 + \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{4}bc^2 d^2 \operatorname{csch}^{-1}(cx) + bde \operatorname{csch}^{-1}(cx)^2$$

$$= \frac{bcd^2 \sqrt{1 + \frac{1}{c^2 x^2}}}{4x} + \frac{be^2 \sqrt{1 + \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{4}bc^2 d^2 \operatorname{csch}^{-1}(cx) + bde \operatorname{csch}^{-1}(cx)^2$$

$$= \frac{bcd^2 \sqrt{1 + \frac{1}{c^2 x^2}}}{4x} + \frac{be^2 \sqrt{1 + \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{4}bc^2 d^2 \operatorname{csch}^{-1}(cx) + bde \operatorname{csch}^{-1}(cx)^2$$

$$= \frac{bcd^2 \sqrt{1 + \frac{1}{c^2 x^2}}}{4x} + \frac{be^2 \sqrt{1 + \frac{1}{c^2 x^2}} x}{2c} - \frac{1}{4}bc^2 d^2 \operatorname{csch}^{-1}(cx) + bde \operatorname{csch}^{-1}(cx)^2$$

**Mathematica** [A] time = 0.91, size = 187, normalized size = 1.05

$$\frac{1}{4} \left( -\frac{2ad^2}{x^2} + 8ade \log(x) + 2ae^2x^2 - \frac{bd^2 \left( -c^2x^2 + c^2x^2 \sqrt{c^2x^2 + 1} \tanh^{-1} \left( \sqrt{c^2x^2 + 1} \right) - 1 \right)}{cx^3 \sqrt{\frac{1}{c^2x^2} + 1}} + \frac{2be^2x \left( \sqrt{\frac{1}{c^2x^2} + 1} + c \right)}{c} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]))/x^3,x]

[Out] ((-2\*a\*d^2)/x^2 + 2\*a\*e^2\*x^2 - (2\*b\*d^2\*ArcCsch[c\*x])/x^2 + (2\*b\*e^2\*x\*(Sqrt[1 + 1/(c^2\*x^2)] + c\*x\*ArcCsch[c\*x])/c - (b\*d^2\*(-1 - c^2\*x^2 + c^2\*x^2\*Sqrt[1 + c^2\*x^2]\*ArcTanh[Sqrt[1 + c^2\*x^2]])))/(c\*Sqrt[1 + 1/(c^2\*x^2)]\*x^3) - 4\*b\*d\*e\*ArcCsch[c\*x]\*(ArcCsch[c\*x] + 2\*Log[1 - E^(-2\*ArcCsch[c\*x])]) + 8\*a\*d\*e\*Log[x] + 4\*b\*d\*e\*PolyLog[2, E^(-2\*ArcCsch[c\*x])])/4

**fricas** [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2) \operatorname{arcsch}(cx)}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="fricas")

[Out] integral((a\*e^2\*x^4 + 2\*a\*d\*e\*x^2 + a\*d^2 + (b\*e^2\*x^4 + 2\*b\*d\*e\*x^2 + b\*d^2)\*arccsch(c\*x))/x^3, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsch(c\*x) + a)/x^3, x)

**maple** [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^2 (a + b \operatorname{arccsch}(cx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^3,x)

[Out] int((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$4bc^2de \int \frac{x \log(x)}{2 \left( \sqrt{c^2x^2 + 1} c^2x^2 + c^2x^2 + \sqrt{c^2x^2 + 1} + 1 \right)} dx - \frac{1}{2} be^2x^2 \log(c) - \frac{1}{2} be^2x^2 \log(x) + \frac{1}{2} ae^2x^2 - 2bde \log(c) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^2\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="maxima")

[Out] 4\*b\*c^2\*d\*e\*integrate(1/2\*x\*log(x)/(sqrt(c^2\*x^2 + 1)\*c^2\*x^2 + c^2\*x^2 + sqrt(c^2\*x^2 + 1) + 1), x) - 1/2\*b\*e^2\*x^2\*log(c) - 1/2\*b\*e^2\*x^2\*log(x) + 1/2\*a\*e^2\*x^2 - 2\*b\*d\*e\*log(c)\*log(x) - b\*d\*e\*log(x)^2 - 1/2\*(2\*log(c^2\*x^2 + 1)\*log(x) + dilog(-c^2\*x^2))\*b\*d\*e + 1/8\*b\*d^2\*((2\*c^4\*x\*sqrt(1/(c^2\*x^2 + 1))/(c^2\*x^2\*(1/(c^2\*x^2) + 1) - 1) - c^3\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) + 1) + c^3\*log(c\*x\*sqrt(1/(c^2\*x^2) + 1) - 1))/c - 4\*arccsch(c\*x)/x^2) + 2\*a\*d\*e\*log(x) + 1/4\*b\*e^2\*(2\*sqrt(c^2\*x^2 + 1) - log(c^2\*x^2 + 1))/c^2 + 1/4\*b\*e^2\*log(c^2\*x^2 + 1)/c^2 + 1/2\*(b\*e^2\*x^2 + 4\*b\*d\*e\*log(x))\*log(sqrt(c^2\*x^2 + 1) + 1) - 1/2\*a\*d^2/x^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(ex^2 + d)^2 \left( a + b \operatorname{arsinh}\left(\frac{1}{cx}\right) \right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))))/x^3,x)

[Out] int(((d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*2\*(a+b\*acsch(c\*x))/x\*\*3,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)\*\*2/x\*\*3, x)

$$3.98 \quad \int \frac{x^2(a + b \operatorname{csch}^{-1}(cx))}{d + ex^2} dx$$

**Optimal.** Leaf size=512

$$\frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2e^{3/2}} - \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}} + 1\right)}{2e^{3/2}} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{2e^{3/2}}$$

[Out]  $x*(a+b*\operatorname{arccsch}(c*x))/e+b*\operatorname{arctanh}((1+1/c^2/x^2)^{(1/2)})/c/e+1/2*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(3/2)}-1/2*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(3/2)}+1/2*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(3/2)}-1/2*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(3/2)}-1/2*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(3/2)}+1/2*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(3/2)}-1/2*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(3/2)}+1/2*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(3/2)}$

**Rubi [A]** time = 1.20, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6304, 5791, 5661, 266, 63, 208, 5706, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2e^{3/2}} - \frac{b\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2e^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^2*(a + b*\operatorname{ArcCsch}[c*x]))/(d + e*x^2), x]$

[Out]  $(x*(a + b*\operatorname{ArcCsch}[c*x]))/e + (b*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + 1/(c^2*x^2)]])/c/e + (\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(2*e^{(3/2)}) - (\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(2*e^{(3/2)}) + (\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(2*e^{(3/2)}) - (\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(2*e^{(3/2)}) - (b*\operatorname{Sqrt}[-d]* \operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e]))])/(2*e^{(3/2)}) + (b*\operatorname{Sqrt}[-d]* \operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(2*e^{(3/2)}) - (b*\operatorname{Sqrt}[-d]* \operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e]))])/(2*e^{(3/2)}) + (b*\operatorname{Sqrt}[-d]* \operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(2*e^{(3/2)})$

### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_)^m)*((c_. + (d_.)*(x_)^n)], x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 208

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b]$

### Rule 266

$\text{Int}[(x)^{m \cdot} \cdot (a + (b \cdot x)^n)^{p \cdot}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[m + 1]/n - 1) \cdot (a + b \cdot x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \ \&\& \ \text{IntegerQ}[\text{Simplify}[m + 1]/n]$

### Rule 2190

$\text{Int}[(F)^{(g \cdot) \cdot (e \cdot) + (f \cdot) \cdot (x \cdot))}^{n \cdot} \cdot ((c \cdot) + (d \cdot) \cdot (x \cdot))^{m \cdot} / ((a \cdot) + (b \cdot) \cdot (F)^{(g \cdot) \cdot (e \cdot) + (f \cdot) \cdot (x \cdot))}^{n \cdot}), x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F^{g \cdot (e + f \cdot x)})^n)/a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^{m - 1} \cdot \text{Log}[1 + (b \cdot (F^{g \cdot (e + f \cdot x)})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[(a + (b \cdot x) \cdot (F)^{(e \cdot) \cdot (c \cdot) + (d \cdot) \cdot (x \cdot))}^{n \cdot})], x\_Symbol] \rightarrow \text{Dist}[1/(d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x]/x, x], x, (F^{e \cdot (c + d \cdot x)})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c \cdot) \cdot ((d \cdot) + (e \cdot) \cdot (x \cdot)^{n \cdot})] / (x \cdot), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c \cdot d, 1]$

### Rule 5561

$\text{Int}[(\text{Cosh}[(c \cdot) + (d \cdot) \cdot (x \cdot)] \cdot ((e \cdot) + (f \cdot) \cdot (x \cdot))^{m \cdot}) / ((a \cdot) + (b \cdot) \cdot \text{Sinh}[(c \cdot) + (d \cdot) \cdot (x \cdot)]), x\_Symbol] \rightarrow -\text{Simp}[(e + f \cdot x)^{m + 1} / (b \cdot f \cdot (m + 1)), x] + (\text{Int}[(e + f \cdot x)^m \cdot E^{(c + d \cdot x)} / (a - \text{Rt}[a^2 + b^2, 2] + b \cdot E^{(c + d \cdot x)}), x] + \text{Int}[(e + f \cdot x)^m \cdot E^{(c + d \cdot x)} / (a + \text{Rt}[a^2 + b^2, 2] + b \cdot E^{(c + d \cdot x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

### Rule 5661

$\text{Int}[(a \cdot) + \text{ArcSinh}[(c \cdot) \cdot (x \cdot)] \cdot (b \cdot)]^{n \cdot} \cdot ((d \cdot) \cdot (x \cdot))^{m \cdot}, x\_Symbol] \rightarrow \text{Simp}[(d \cdot x)^{m + 1} \cdot (a + b \cdot \text{ArcSinh}[c \cdot x])^n / (d \cdot (m + 1)), x] - \text{Dist}[(b \cdot c \cdot n) / (d \cdot (m + 1)), \text{Int}[(d \cdot x)^{m + 1} \cdot (a + b \cdot \text{ArcSinh}[c \cdot x])^{n - 1} / \text{Sqrt}[1 + c^2 \cdot x^2], x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$

### Rule 5706

$\text{Int}[(a \cdot) + \text{ArcSinh}[(c \cdot) \cdot (x \cdot)] \cdot (b \cdot)]^{n \cdot} \cdot ((d \cdot) + (e \cdot) \cdot (x \cdot)^2)^{p \cdot}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSinh}[c \cdot x])^n \cdot (d + e \cdot x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x\} \ \&\& \ \text{NeQ}[e, c^2 \cdot d] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (p > 0 \ || \ \text{IGtQ}[n, 0])$

### Rule 5791

$\text{Int}[(a \cdot) + \text{ArcSinh}[(c \cdot) \cdot (x \cdot)] \cdot (b \cdot)]^{n \cdot} \cdot ((f \cdot) \cdot (x \cdot))^{m \cdot} \cdot ((d \cdot) + (e \cdot) \cdot (x \cdot)^2)^{p \cdot}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSinh}[c \cdot x])^n \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \ \text{NeQ}[e, c^2 \cdot d] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$

### Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x]]
/; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

### Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol]
:> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0]
] && IntegersQ[m, p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{d + ex^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x^2 (e + dx^2)} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{ex^2} - \frac{d (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x^2} dx, x, \frac{1}{x} \right)}{e} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e} - \frac{b \operatorname{Subst} \left( \int \frac{1}{x \sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{ce} + \frac{d \operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e} (\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e} (\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e^{3/2}} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e^{3/2}} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{d \operatorname{Subst} \left( \int \frac{(a+bx) \cosh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sinh(x)} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2e^{3/2}} + \frac{d \operatorname{Subst} \left( \int \frac{(a+bx) \cosh(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \sinh(x)} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2e^{3/2}} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{b \tanh^{-1} \left( \sqrt{1 + \frac{1}{c^2 x^2}} \right)}{ce} + \frac{d \operatorname{Subst} \left( \int \frac{e^{x(a+bx)}}{\frac{\sqrt{e}}{c} - \frac{\sqrt{-c^2 d + e}}{c} - \sqrt{-d} e^x} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2e^{3/2}} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{b \tanh^{-1} \left( \sqrt{1 + \frac{1}{c^2 x^2}} \right)}{ce} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right)}{2e^{3/2}} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{b \tanh^{-1} \left( \sqrt{1 + \frac{1}{c^2 x^2}} \right)}{ce} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right)}{2e^{3/2}} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{b \tanh^{-1} \left( \sqrt{1 + \frac{1}{c^2 x^2}} \right)}{ce} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2 d + e}} \right)}{2e^{3/2}}
\end{aligned}$$

**Mathematica** [C] time = 1.67, size = 1221, normalized size = 2.38

result too large to display

Warning: Unable to verify antiderivative.



[In] Integrate[(x^2\*(a + b\*ArcSch[c\*x]))/(d + e\*x^2),x]

[Out] (4\*a\*c\*Sqrt[e]\*x + 4\*b\*c\*Sqrt[e]\*x\*ArcSch[c\*x] - 4\*a\*c\*Sqrt[d]\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]] - (8\*I)\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] - Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcSch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - (8\*I)\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcSch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] + b\*c\*Sqrt[d]\*Pi\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] - (2\*I)\*b\*c\*Sqrt[d]\*ArcSch[c\*x]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] + 4\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] - b\*c\*Sqrt[d]\*Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] + (2\*I)\*b\*c\*Sqrt[d]\*ArcSch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] - 4\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] - b\*c\*Sqrt[d]\*Pi\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] + (2\*I)\*b\*c\*Sqrt[d]\*ArcSch[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] + 4\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] + b\*c\*Sqrt[d]\*Pi\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] - (2\*I)\*b\*c\*Sqrt[d]\*ArcSch[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] - 4\*b\*c\*Sqrt[d]\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] + b\*c\*Sqrt[d]\*Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x] - b\*c\*Sqrt[d]\*Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x] - 4\*b\*Sqrt[e]\*Log[Tanh[ArcSch[c\*x]/2]] + (2\*I)\*b\*c\*Sqrt[d]\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] - (2\*I)\*b\*c\*Sqrt[d]\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] - (2\*I)\*b\*c\*Sqrt[d]\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])] + (2\*I)\*b\*c\*Sqrt[d]\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcSch[c\*x])/(c\*Sqrt[d])])]/(4\*c\*e^(3/2))

**fricas** [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arcsch}(cx) + ax^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsch(c\*x) + a\*x^2)/(e\*x^2 + d), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2/(e\*x^2 + d), x)

**maple** [F] time = 3.40, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccsch}(cx))}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d),x)

[Out] `int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{d \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e} - \frac{x}{e} \right) + b \int \frac{x^2 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")`

[Out] `-a*(d*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e) - x/e) + b*integrate(x^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)`

[Out] `int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acsch}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d),x)`

[Out] `Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2), x)`

$$3.99 \quad \int \frac{x(a+b\operatorname{csch}^{-1}(cx))}{d+ex^2} dx$$

**Optimal.** Leaf size=467

$$\frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e} + \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e} + \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}$$

[Out]  $-(a+b\operatorname{arccsch}(c*x))^2/b/e-(a+b\operatorname{arccsch}(c*x))*\ln(1-1/(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)/e+1/2*(a+b\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)}))*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)})/e+1/2*(a+b\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)}))*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)})/e+1/2*(a+b\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)}))*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)})/e+1/2*(a+b\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)}))*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)})/e+1/2*b*\operatorname{polylog}(2,1/(1/c/x+(1+1/c^2/x^2)^{(1/2)})^2)/e+1/2*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)}))*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)})/e+1/2*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)}))*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)})/e+1/2*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)}))*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)})/e+1/2*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)}))*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)})/e$

**Rubi [A]** time = 1.11, antiderivative size = 449, normalized size of antiderivative = 0.96, number of steps used = 26, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {6304, 5791, 5659, 3716, 2190, 2279, 2391, 5799, 5561}

$$\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e}$$

Warning: Unable to verify antiderivative.

[In] Int[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2), x]

[Out]  $((a+b\operatorname{ArcCsch}[c*x])*\operatorname{Log}[1-(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])]/(2*e)+((a+b\operatorname{ArcCsch}[c*x])*\operatorname{Log}[1+(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])]/(2*e)+((a+b\operatorname{ArcCsch}[c*x])*\operatorname{Log}[1-(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])]/(2*e)+((a+b\operatorname{ArcCsch}[c*x])*\operatorname{Log}[1+(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])]/(2*e)-((a+b\operatorname{ArcCsch}[c*x])*\operatorname{Log}[1-E^{(2*\operatorname{ArcCsch}[c*x])}])/e+(b*\operatorname{PolyLog}[2,-((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])]/(2*e)+(b*\operatorname{PolyLog}[2,(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])]/(2*e)+(b*\operatorname{PolyLog}[2,-((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])]/(2*e)+(b*\operatorname{PolyLog}[2,(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])]/(2*e)-((b*\operatorname{PolyLog}[2,E^{(2*\operatorname{ArcCsch}[c*x])}])/e$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_)+(f\_)\*(x\_)))^(n\_)\*((c\_)+(d\_)\*(x\_))^(m\_))/((a\_)+(b\_)\*((F\_)^(g\_)\*((e\_)+(f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[(((c+d\*x)^m\*Log[1+(b\*(F^(g\*(e+f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c+d\*x)^(m-1)\*Log[1+(b\*(F^(g\*(e+f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2279**

Int[Log[(a\_)+(b\_)\*((F\_)^(g\_)\*((c\_)+(d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a+b\*x]/x, x], x, (F^(e\*(c+d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x)))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)])*((e_.) + (f_.)*(x_)^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.)^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.)^(n_.)*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.)^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)^(n_.)*(x_)^(m_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \operatorname{csch}^{-1}(cx))}{d + ex^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x(e + dx^2)} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{ex} - \frac{dx(a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e} + \frac{d \operatorname{Subst} \left( \int \frac{x(a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e} \\
&= -\frac{\operatorname{Subst} \left( \int (a + bx) \operatorname{coth}(x) dx, x, \operatorname{csch}^{-1}(cx) \right)}{e} + \frac{d \operatorname{Subst} \left( \int \left( -\frac{\sqrt{-d}(a + b \sinh^{-1} \left( \frac{x}{c} \right))}{2d(\sqrt{e} - \sqrt{-d}x)} + \right) dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2be} + \frac{2 \operatorname{Subst} \left( \int \frac{e^{2x(a+bx)}}{1-e^{2x}} dx, x, \operatorname{csch}^{-1}(cx) \right)}{e} - \frac{\sqrt{-d} \operatorname{Subst} \left( \int \frac{a+bx}{\sqrt{e}-\sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2be} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log(1 - e^{2 \operatorname{csch}^{-1}(cx)})}{e} + \frac{b \operatorname{Subst} \left( \int \log(1 - \frac{e^{2x(a+bx)}}{1-e^{2x}}) dx, x, \operatorname{csch}^{-1}(cx) \right)}{e} \\
&= -\frac{(a + b \operatorname{csch}^{-1}(cx)) \log(1 - e^{2 \operatorname{csch}^{-1}(cx)})}{e} + \frac{b \operatorname{Subst} \left( \int \frac{\log(1-x)}{x} dx, x, e^{2 \operatorname{csch}^{-1}(cx)} \right)}{2e} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2e} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2e} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2e} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2e} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2e} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2e}
\end{aligned}$$

**Mathematica [C]** time = 0.35, size = 1103, normalized size = 2.36

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2), x]

[Out] (b\*Pi^2 - (4\*I)\*b\*Pi\*ArcCsch[c\*x] - 8\*b\*ArcCsch[c\*x]^2 + 16\*b\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] - Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 16\*b\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*b\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])] + (2\*I)\*b\*Pi\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*ArcCsch[c\*x]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (8\*I)\*b\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (2\*I)\*b\*Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*ArcCsch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])])

$$\begin{aligned} & \left. \right) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d]) \left. \right) + (8*I) * b * \text{ArcSin}[\text{Sqrt}[1 - \text{Sqrt}[e] / (c * \text{Sqrt}[d])] / \text{Sqrt}[2]] * \text{Log}[1 + (I * (-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) + (2*I) * b * \text{Pi} * \text{Log}[1 - (I * (\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) + 4 * b * \text{ArcCsch}[c*x] * \text{Log}[1 - (I * (\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) - (8*I) * b * \text{ArcSin}[\text{Sqrt}[1 - \text{Sqrt}[e] / (c * \text{Sqrt}[d])] / \text{Sqrt}[2]] * \text{Log}[1 - (I * (\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) + (2*I) * b * \text{Pi} * \text{Log}[1 + (I * (\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) + 4 * b * \text{ArcCsch}[c*x] * \text{Log}[1 + (I * (\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) - (8*I) * b * \text{ArcSin}[\text{Sqrt}[1 + \text{Sqrt}[e] / (c * \text{Sqrt}[d])] / \text{Sqrt}[2]] * \text{Log}[1 + (I * (\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) - (2*I) * b * \text{Pi} * \text{Log}[\text{Sqrt}[e] - (I * \text{Sqrt}[d]) / x] - (2*I) * b * \text{Pi} * \text{Log}[\text{Sqrt}[e] + (I * \text{Sqrt}[d]) / x] + 4 * a * \text{Log}[d + e * x^2] + 4 * b * \text{PolyLog}[2, E^{(-2 * \text{ArcCsch}[c*x])}] + 4 * b * \text{PolyLog}[2, ((-I) * (-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) + 4 * b * \text{PolyLog}[2, (I * (-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) + 4 * b * \text{PolyLog}[2, ((-I) * (\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])) + 4 * b * \text{PolyLog}[2, (I * (\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e]) * E^{\text{ArcCsch}[c*x]} / (c * \text{Sqrt}[d])))] / (8 * e) \end{aligned}$$

**fricas** [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx \operatorname{arcsch}(cx) + ax}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*x\*arccsch(c\*x) + a\*x)/(e\*x^2 + d), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x/(e\*x^2 + d), x)

**maple** [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{arccsch}(cx))}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d),x)

[Out] int(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{x \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{ex^2 + d} dx + \frac{a \log(ex^2 + d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d),x, algorithm="maxima")

[Out] b\*integrate(x\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e\*x^2 + d), x) + 1/2\*a\*log(e\*x^2 + d)/e

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2),x)

[Out] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x (a + b \operatorname{acsch}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d),x)

[Out] Integral(x\*(a + b\*acsch(c\*x))/(d + e\*x\*\*2), x)

$$3.100 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{d + ex^2} dx$$

**Optimal.** Leaf size=477

$$\frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2d}} + 1\right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

[Out]  $\frac{1}{2}*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}-1/2*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}+1/2*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}-1/2*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}-1/2*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}+1/2*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}-1/2*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}+1/2*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))/(-d)^{(1/2)}/e^{(1/2)}$

**Rubi [A]** time = 0.85, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {6294, 5706, 5799, 5561, 2190, 2279, 2391}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2d}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2d}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2d} + \sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2d} + \sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/(d + e\*x^2), x]

[Out]  $((a + b*\operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) - ((a + b*\operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) + ((a + b*\operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) - ((a + b*\operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) - (b*\operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e]))])/(2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) + (b*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) - (b*\operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e]))])/(2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]) + (b*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(2*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])$

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

**Rule 2279**

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]



Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)])*((e_.) + (f_.)*(x_)^(m_.)))/((a_) + (b_.)*Sin h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p > 0 || IGtQ[n, 0])
```

Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6294

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{d + ex^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}\left(\frac{x}{c}\right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2\sqrt{e}} - \frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2\sqrt{e}} \\
&= -\frac{\operatorname{Subst} \left( \int \frac{(a+bx) \cosh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sinh(x)} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2\sqrt{e}} - \frac{\operatorname{Subst} \left( \int \frac{(a+bx) \cosh(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \sinh(x)} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2\sqrt{e}} \\
&= -\frac{\operatorname{Subst} \left( \int \frac{e^{x(a+bx)}}{\frac{\sqrt{e}}{c} - \frac{\sqrt{-c^2d+e}}{c} - \sqrt{-d} e^x} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2\sqrt{e}} - \frac{\operatorname{Subst} \left( \int \frac{e^{x(a+bx)}}{\frac{\sqrt{e}}{c} + \frac{\sqrt{-c^2d+e}}{c} - \sqrt{-d} e^x} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2\sqrt{e}} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} + \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} + \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 + \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2\sqrt{-d} \sqrt{e}} +
\end{aligned}$$

**Mathematica [C]** time = 0.51, size = 1055, normalized size = 2.21

$$4a \tan^{-1} \left( \frac{\sqrt{e}x}{\sqrt{d}} \right) + 8ib \sin^{-1} \left( \frac{\sqrt{\frac{\sqrt{e}}{c\sqrt{d}} + 1}}{\sqrt{2}} \right) \tan^{-1} \left( \frac{(c\sqrt{d} - \sqrt{e}) \cot \left( \frac{1}{4} (2i \operatorname{csch}^{-1}(cx) + \pi) \right)}{\sqrt{e - c^2d}} \right) + 8ib \sin^{-1} \left( \frac{\sqrt{1 - \frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}} \right) \tan^{-1} \left( \frac{(\sqrt{d}c + \sqrt{e})}{\sqrt{e - c^2d}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x^2), x]

[Out] (4\*a\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]] + (8\*I)\*b\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] - Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] + (8\*I)\*b\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - b\*Pi\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (2\*I)\*b\*ArcCsch[c\*x]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] - 4\*b\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + b\*Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] - (2\*I)\*b\*ArcCsch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*S

$\text{qrt}[d]] + b\pi \cdot \text{Log}[1 - (I \cdot (\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] - (2 \cdot I) \cdot b \cdot \text{ArcCsch}[c \cdot x] \cdot \text{Log}[1 - (I \cdot (\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] - 4 \cdot b \cdot \text{ArcSin}[\text{Sqrt}[1 - \text{Sqrt}[e] / (c \cdot \text{Sqrt}[d])] / \text{Sqrt}[2]] \cdot \text{Log}[1 - (I \cdot (\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] - b \cdot \pi \cdot \text{Log}[1 + (I \cdot (\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] + (2 \cdot I) \cdot b \cdot \text{ArcCsch}[c \cdot x] \cdot \text{Log}[1 + (I \cdot (\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] + 4 \cdot b \cdot \text{ArcSin}[\text{Sqrt}[1 + \text{Sqrt}[e] / (c \cdot \text{Sqrt}[d])] / \text{Sqrt}[2]] \cdot \text{Log}[1 + (I \cdot (\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] - b \cdot \pi \cdot \text{Log}[\text{Sqrt}[e] - (I \cdot \text{Sqrt}[d]) / x] + b \cdot \pi \cdot \text{Log}[\text{Sqrt}[e] + (I \cdot \text{Sqrt}[d]) / x] - (2 \cdot I) \cdot b \cdot \text{PolyLog}[2, ((-I) \cdot (-\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] + (2 \cdot I) \cdot b \cdot \text{PolyLog}[2, (I \cdot (-\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] + (2 \cdot I) \cdot b \cdot \text{PolyLog}[2, ((-I) \cdot (\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])] - (2 \cdot I) \cdot b \cdot \text{PolyLog}[2, (I \cdot (\text{Sqrt}[e] + \text{Sqrt}[-(c^2 \cdot d) + e]) \cdot E^{\text{ArcCsch}[c \cdot x]}) / (c \cdot \text{Sqrt}[d])]) / (4 \cdot \text{Sqrt}[d] \cdot \text{Sqrt}[e])$

**fricas** [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e\*x^2 + d), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x^2 + d), x)

**maple** [F] time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x^2+d),x)

[Out] int((a+b\*arccsch(c\*x))/(e\*x^2+d),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{ex^2 + d} dx + \frac{a \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d),x, algorithm="maxima")

[Out] b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e\*x^2 + d), x) + a\*arctan(e\*x/sqrt(d\*e))/sqrt(d\*e)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*asinh(1/(c*x)))/(d + e*x^2), x)`

[Out] `int((a + b*asinh(1/(c*x)))/(d + e*x^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))/(e*x**2+d), x)`

[Out] `Integral((a + b*acsch(c*x))/(d + e*x**2), x)`

$$3.101 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)} dx$$

**Optimal.** Leaf size=425

$$\frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2d} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}} + 1\right)}{2d} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2d}$$

```
[Out] 1/2*(a+b*arccsch(c*x))^2/b/d-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d
```

**Rubi [A]** time = 0.86, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6304, 5791, 5799, 5561, 2190, 2279, 2391}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2d} - \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2d} - \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2d} - \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)), x]
```

```
[Out] (a + b*ArcCsch[c*x])^2/(2*b*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, -(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, -(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d)
```

**Rule 2190**

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

**Rule 2279**

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

**Rule 2391**

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_)^(m_.)))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

#### Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^n_)*((f_.)*(x_)^(m_.))*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
(f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

#### Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^n_)/((d_.) + (e_.)*(x_)), x_Symbo
l] := Subst[Int[(a + b*x)^n*Cosh[x]/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^n_)*(x_)^(m_.))*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)} dx &= -\operatorname{Subst} \left( \int \frac{x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( -\frac{\sqrt{-d} \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} - \sqrt{-d}x)} + \frac{\sqrt{-d} \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2\sqrt{-d}} + \frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2\sqrt{-d}} \\
&= -\frac{\operatorname{Subst} \left( \int \frac{(a+bx) \cosh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sinh(x)} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2\sqrt{-d}} + \frac{\operatorname{Subst} \left( \int \frac{(a+bx) \cosh(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \sinh(x)} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2\sqrt{-d}} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2bd} - \frac{\operatorname{Subst} \left( \int \frac{e^{x(a+bx)}}{\frac{\sqrt{e}}{c} - \sqrt{-c^2d+e} - \sqrt{-d}ex} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2\sqrt{-d}} - \frac{\operatorname{Subst} \left( \int \frac{e^{x(a+bx)}}{\frac{\sqrt{e}}{c} + \sqrt{-c^2d+e} + \sqrt{-d}ex} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2\sqrt{-d}} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2bd} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2d} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2d} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2bd} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2d} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2d} \\
&= \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2bd} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2d} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2d}
\end{aligned}$$

**Mathematica [C]** time = 0.98, size = 387, normalized size = 0.91

$$-2a \log(d + ex^2) + 4a \log(x) + b \left( \operatorname{Li}_2 \left( \frac{(dc^2 - 2e + 2\sqrt{e(e - c^2d)})e^{-2\operatorname{csch}^{-1}(cx)}}{c^2d} \right) + \operatorname{Li}_2 \left( \frac{(c^2d - 2(e + \sqrt{e(e - c^2d)})e^{-2\operatorname{csch}^{-1}(cx)})}{c^2d} \right) - 2 \left( \operatorname{Li}_2 \left( \frac{(dc^2 - 2e + 2\sqrt{e(e - c^2d)})e^{-2\operatorname{csch}^{-1}(cx)}}{c^2d} \right) + \operatorname{Li}_2 \left( \frac{(c^2d - 2(e + \sqrt{e(e - c^2d)})e^{-2\operatorname{csch}^{-1}(cx)})}{c^2d} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)), x]

[Out] (4\*a\*Log[x] - 2\*a\*Log[d + e\*x^2] + b\*(-2\*(ArcCsch[c\*x]^2 + I\*ArcSin[Sqrt[e/(c^2\*d)]]\*(2\*ArcTanh[Sqrt[e\*(-(c^2\*d) + e)]/(c\*e\*Sqrt[1 + 1/(c^2\*x^2)]]\*x)] - Log[(2\*e - 2\*Sqrt[e\*(-(c^2\*d) + e)] + c^2\*d\*(-1 + E^(2\*ArcCsch[c\*x]))])/(c^2\*d\*E^(2\*ArcCsch[c\*x]))] + Log[(2\*(e + Sqrt[e\*(-(c^2\*d) + e)]) + c^2\*d\*(-1 + E^(2\*ArcCsch[c\*x]))])/(c^2\*d\*E^(2\*ArcCsch[c\*x]))] + ArcCsch[c\*x]\*(Log[(2\*e - 2\*Sqrt[e\*(-(c^2\*d) + e)] + c^2\*d\*(-1 + E^(2\*ArcCsch[c\*x]))])/(c^2\*d\*E^(2\*ArcCsch[c\*x]))] + Log[(2\*(e + Sqrt[e\*(-(c^2\*d) + e)]) + c^2\*d\*(-1 + E^(2\*ArcCsch[c\*x]))])/(c^2\*d\*E^(2\*ArcCsch[c\*x]))] + PolyLog[2, (c^2\*d - 2\*e + 2\*Sqrt[e\*(-(c^2\*d) + e)]/(c^2\*d\*E^(2\*ArcCsch[c\*x]))] + PolyLog[2, (c^2\*d - 2\*(e + Sqrt[e\*(-(c^2\*d) + e)]))]/(c^2\*d\*E^(2\*ArcCsch[c\*x]))]))/(4\*d)

**fricas [F]** time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{b \operatorname{arcsch}(cx) + a}{ex^3 + dx}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e\*x^3 + d\*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)\*x), x)

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsch}(cx)}{x(e x^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d),x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} a \left( \frac{\log(ex^2 + d)}{d} - \frac{2 \log(x)}{d} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{ex^3 + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d),x, algorithm="maxima")

[Out] -1/2\*a\*(log(e\*x^2 + d)/d - 2\*log(x)/d) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e\*x^3 + d\*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x(e x^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x(d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(e\*x\*\*2+d),x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*(d + e\*x\*\*2)), x)



$$3.102 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2(d + ex^2)} dx$$

**Optimal.** Leaf size=518

$$\frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2(-d)^{3/2}} - \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}} + 1\right)}{2(-d)^{3/2}} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{2(-d)^{3/2}}$$

[Out]  $-a/d/x - b \operatorname{arccsch}(c*x)/d/x + 1/2*(a + b \operatorname{arccsch}(c*x)) * \ln(1 - c*(1/c/x + (1 + 1/c^2/x^2)^{1/2}))^{1/2} * (-d)^{1/2} / (e^{1/2} - (-c^2*d + e)^{1/2}) * e^{1/2} / (-d)^{3/2} - 1/2*(a + b \operatorname{arccsch}(c*x)) * \ln(1 + c*(1/c/x + (1 + 1/c^2/x^2)^{1/2}))^{1/2} * (-d)^{1/2} / (e^{1/2} - (-c^2*d + e)^{1/2}) * e^{1/2} / (-d)^{3/2} + 1/2*(a + b \operatorname{arccsch}(c*x)) * \ln(1 - c*(1/c/x + (1 + 1/c^2/x^2)^{1/2}))^{1/2} * (-d)^{1/2} / (e^{1/2} + (-c^2*d + e)^{1/2}) * e^{1/2} / (-d)^{3/2} - 1/2*(a + b \operatorname{arccsch}(c*x)) * \ln(1 + c*(1/c/x + (1 + 1/c^2/x^2)^{1/2}))^{1/2} * (-d)^{1/2} / (e^{1/2} + (-c^2*d + e)^{1/2}) * e^{1/2} / (-d)^{3/2} - 1/2*b * \operatorname{polylog}(2, -c*(1/c/x + (1 + 1/c^2/x^2)^{1/2}))^{1/2} * (-d)^{1/2} / (e^{1/2} - (-c^2*d + e)^{1/2}) * e^{1/2} / (-d)^{3/2} + 1/2*b * \operatorname{polylog}(2, c*(1/c/x + (1 + 1/c^2/x^2)^{1/2}))^{1/2} * (-d)^{1/2} / (e^{1/2} - (-c^2*d + e)^{1/2}) * e^{1/2} / (-d)^{3/2} - 1/2*b * \operatorname{polylog}(2, -c*(1/c/x + (1 + 1/c^2/x^2)^{1/2}))^{1/2} * (-d)^{1/2} / (e^{1/2} + (-c^2*d + e)^{1/2}) * e^{1/2} / (-d)^{3/2} + 1/2*b * \operatorname{polylog}(2, c*(1/c/x + (1 + 1/c^2/x^2)^{1/2}))^{1/2} * (-d)^{1/2} / (e^{1/2} + (-c^2*d + e)^{1/2}) * e^{1/2} / (-d)^{3/2} + b*c*(1 + 1/c^2/x^2)^{1/2}/d$

**Rubi [A]** time = 1.06, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {6304, 5791, 5653, 261, 5706, 5799, 5561, 2190, 2279, 2391}

$$-\frac{b\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2(-d)^{3/2}} - \frac{b\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2(-d)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x^2)), x]

[Out]  $(b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/d - a/(d*x) - (b*\operatorname{ArcCsch}[c*x])/(d*x) + (\operatorname{Sqrt}[e] * (a + b*\operatorname{ArcCsch}[c*x]) * \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])]) / (2*(-d)^{3/2}) - (\operatorname{Sqrt}[e] * (a + b*\operatorname{ArcCsch}[c*x]) * \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])]) / (2*(-d)^{3/2}) + (\operatorname{Sqrt}[e] * (a + b*\operatorname{ArcCsch}[c*x]) * \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])]) / (2*(-d)^{3/2}) - (\operatorname{Sqrt}[e] * (a + b*\operatorname{ArcCsch}[c*x]) * \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])]) / (2*(-d)^{3/2}) - (b*\operatorname{Sqrt}[e] * \operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])]) / (2*(-d)^{3/2}) + (b*\operatorname{Sqrt}[e] * \operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])]) / (2*(-d)^{3/2}) - (b*\operatorname{Sqrt}[e] * \operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])]) / (2*(-d)^{3/2}) + (b*\operatorname{Sqrt}[e] * \operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])]) / (2*(-d)^{3/2}))$

**Rule 261**

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

**Rule 2190**

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp

$$\left[ \frac{((c + d*x)^m \log[1 + (b*(F^{(g*(e + f*x)))^n])/a])}{(b*f*g*n \log[F])}, x \right] - \text{Dist} \left[ \frac{(d*m)}{(b*f*g*n \log[F])}, \text{Int}[(c + d*x)^{(m-1)} \log[1 + (b*(F^{(g*(e + f*x)))^n])/a], x], x \right] /; \text{FreeQ}[\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$$

#### Rule 2279

$$\text{Int}[\log[(a_) + (b_)*(F_)^{(e_)*((c_) + (d_)*(x_))}]^{(n_)}], x\_Symbol] \rightarrow \text{Dist} \left[ \frac{1}{(d*e*n \log[F])}, \text{Subst}[\text{Int}[\log[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x \right] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$$

#### Rule 2391

$$\text{Int}[\log[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_)], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$$

#### Rule 5561

$$\text{Int}[(\cosh[(c_) + (d_)*(x_)]*(e_) + (f_)*(x_))^{(m_)}]/((a_) + (b_)*\sinh[(c_) + (d_)*(x_)]), x\_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{(m+1)}/(b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m * E^{(c + d*x)}/(a - \text{Rt}[a^2 + b^2, 2] + b * E^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m * E^{(c + d*x)}/(a + \text{Rt}[a^2 + b^2, 2] + b * E^{(c + d*x)}), x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$$

#### Rule 5653

$$\text{Int}[(a_) + \text{ArcSinh}[(c_)*(x_)]*(b_)]^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcSinh}[c*x])^n, x] - \text{Dist}[b*c*n, \text{Int}[(x*(a + b*\text{ArcSinh}[c*x])^{(n-1)})/\sqrt{1 + c^2*x^2}], x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{GtQ}[n, 0]$$

#### Rule 5706

$$\text{Int}[(a_) + \text{ArcSinh}[(c_)*(x_)]*(b_)]^{(n_)*((d_) + (e_)*(x_)^2)^{(p_)}], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSinh}[c*x])^n, (d + e*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{NeQ}[e, c^2*d] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (p > 0 \ || \ \text{IGtQ}[n, 0])$$

#### Rule 5791

$$\text{Int}[(a_) + \text{ArcSinh}[(c_)*(x_)]*(b_)]^{(n_)*((f_)*(x_))^{(m_)*((d_) + (e_)*(x_)^2)^{(p_)}], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSinh}[c*x])^n, (f*x)^m * (d + e*x^2)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[e, c^2*d] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$$

#### Rule 5799

$$\text{Int}[(a_) + \text{ArcSinh}[(c_)*(x_)]*(b_)]^{(n_)} / ((d_) + (e_)*(x_)), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n * \cosh[x]/(c*d + e*\sinh[x]), x], x, \text{ArcSinh}[c*x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{IGtQ}[n, 0]$$

#### Rule 6304

$$\text{Int}[(a_) + \text{ArcCsch}[(c_)*(x_)]*(b_)]^{(n_)*((d_) + (e_)*(x_)^2)^{(p_)}], x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p * (a + b*\text{ArcSinh}[x/c])^n / x^{(m+2*(p+1))}, x], x, 1/x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegersQ}[m, p]$$

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2(d + ex^2)} dx &= -\operatorname{Subst} \left( \int \frac{x^2 \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{d} - \frac{e \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{a}{dx} - \frac{b \operatorname{Subst} \left( \int \sinh^{-1} \left( \frac{x}{c} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{a}{dx} - \frac{b \operatorname{csch}^{-1}(cx)}{dx} + \frac{b \operatorname{Subst} \left( \int \frac{x}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{cd} + \frac{\sqrt{e} \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2d} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e} \operatorname{Subst} \left( \int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d}\sinh(x)} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2d} + \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e} \operatorname{Subst} \left( \int \frac{e^{x(a+bx)}}{\frac{\sqrt{e}}{c} - \frac{\sqrt{-c^2d+e}}{c} - \sqrt{-d}e^x} dx, x, \operatorname{csch}^{-1}(cx) \right)}{2d} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2(-d)^{3/2}} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2(-d)^{3/2}} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b \operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{-c^2d+e}} \right)}{2(-d)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 1.69, size = 1211, normalized size = 2.34

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x^2)), x]

[Out]  $-(a/(d*x)) - (a*\sqrt{e}*\operatorname{ArcTan}[(\sqrt{e}*x)/\sqrt{d}])/d^{3/2} + b*((c*\sqrt{1 + 1/(c^2*x^2)} - \operatorname{ArcCsch}[c*x])/x)/d - ((I/16)*\sqrt{e}*(\pi^2 - (4*I)*\pi*\operatorname{ArcCsch}[c*x] - 8*\operatorname{ArcCsch}[c*x]^2 + 32*\operatorname{ArcSin}[\sqrt{1 + \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2})*\operatorname{ArcTan}[(c*\sqrt{d} - \sqrt{e})*\operatorname{Cot}[(\pi + (2*I)*\operatorname{ArcCsch}[c*x])/4])/(\sqrt{-(c^2*d + e)} - 8*\operatorname{ArcCsch}[c*x]*\operatorname{Log}[1 - E^{-(2*\operatorname{ArcCsch}[c*x])}] + (4*I)*\pi*\operatorname{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2*d + e)})*E^{\operatorname{ArcCsch}[c*x]})/(c*\sqrt{d})] + 8*\operatorname{ArcCsch}[c*x]*\operatorname{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2*d + e)})*E^{\operatorname{ArcCsch}[c*x]})/(c*\sqrt{d})] + (16*I)*\operatorname{ArcSin}[\sqrt{1 + \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2})*\operatorname{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2*d + e)})*E^{\operatorname{ArcCsch}[c*x]})/(c*\sqrt{d})] + (4*I)*\pi*\operatorname{Log}[1$

+ (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])) + 8\*ArcCsch[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] - (16\*I)\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] - (4\*I)\*Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] + 8\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))])/d^(3/2) + ((I/16)\*Sqrt[e]\*(Pi^2 - (4\*I)\*Pi\*ArcCsch[c\*x] - 8\*ArcCsch[c\*x]^2 - 32\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])] + (4\*I)\*Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] + 8\*ArcCsch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] + (16\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] + (4\*I)\*Pi\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] + 8\*ArcCsch[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] - (16\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] - (4\*I)\*Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))] + 8\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))])/d^(3/2))

**fricas** [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d),x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e\*x^4 + d\*x^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)\*x^2), x)

**maple** [F] time = 2.76, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 (ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d),x)

[Out] int((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{e \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}d} + \frac{1}{dx} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{ex^4 + dx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d),x, algorithm="maxima")

[Out] -a\*(e\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d) + 1/(d\*x)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e\*x^4 + d\*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2 (ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x^2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^2 (d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*2/(e\*x\*\*2+d),x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*\*2\*(d + e\*x\*\*2)), x)

$$3.103 \quad \int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=591

$$\frac{d(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{e^3} - \frac{d(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}} + 1\right)}{e^3} - \frac{d(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}} + 1\right)}{e^3} - \frac{d(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}} + 1\right)}{e^3}$$

[Out]  $\frac{1}{2} d (a + b \operatorname{arccsch}(c x)) / e^2 / (e + d/x^2) + \frac{1}{2} x^2 (a + b \operatorname{arccsch}(c x)) / e^2 + 2 d (a + b \operatorname{arccsch}(c x))^2 / b e^3 + 2 d (a + b \operatorname{arccsch}(c x)) \ln(1 - 1 / (1/c/x + (1 + 1/c^2/x^2)^{1/2}))^2 / e^3 - d (a + b \operatorname{arccsch}(c x)) \ln(1 - c (1/c/x + (1 + 1/c^2/x^2)^{1/2})) (-d)^{1/2} / (e^{1/2} - (-c^2 d + e)^{1/2}) / e^3 - d (a + b \operatorname{arccsch}(c x)) \ln(1 + c (1/c/x + (1 + 1/c^2/x^2)^{1/2})) (-d)^{1/2} / (e^{1/2} - (-c^2 d + e)^{1/2}) / e^3 - d (a + b \operatorname{arccsch}(c x)) \ln(1 - c (1/c/x + (1 + 1/c^2/x^2)^{1/2})) (-d)^{1/2} / (e^{1/2} + (-c^2 d + e)^{1/2}) / e^3 - d (a + b \operatorname{arccsch}(c x)) \ln(1 + c (1/c/x + (1 + 1/c^2/x^2)^{1/2})) (-d)^{1/2} / (e^{1/2} + (-c^2 d + e)^{1/2}) / e^3 - b d \operatorname{polylog}(2, 1 / (1/c/x + (1 + 1/c^2/x^2)^{1/2}))^2 / e^3 - b d \operatorname{polylog}(2, -c (1/c/x + (1 + 1/c^2/x^2)^{1/2})) (-d)^{1/2} / (e^{1/2} - (-c^2 d + e)^{1/2}) / e^3 - b d \operatorname{polylog}(2, c (1/c/x + (1 + 1/c^2/x^2)^{1/2})) (-d)^{1/2} / (e^{1/2} - (-c^2 d + e)^{1/2}) / e^3 - b d \operatorname{polylog}(2, -c (1/c/x + (1 + 1/c^2/x^2)^{1/2})) (-d)^{1/2} / (e^{1/2} + (-c^2 d + e)^{1/2}) / e^3 - b d \operatorname{polylog}(2, c (1/c/x + (1 + 1/c^2/x^2)^{1/2})) (-d)^{1/2} / (e^{1/2} + (-c^2 d + e)^{1/2}) / e^3 - 1/2 b d \operatorname{arctan}((c^2 d - e)^{1/2} / c/x/e^{1/2} / (1 + 1/c^2/x^2)^{1/2}) / e^{5/2} / (c^2 d - e)^{1/2} + 1/2 b x (1 + 1/c^2/x^2)^{1/2} / c/e^2$

**Rubi [A]** time = 1.28, antiderivative size = 571, normalized size of antiderivative = 0.97, number of steps used = 31, number of rules used = 14, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6304, 5791, 5661, 264, 5659, 3716, 2190, 2279, 2391, 5787, 377, 205, 5799, 5561}

$$\frac{bd \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{e^3}$$

Warning: Unable to verify antiderivative.

[In]  $\operatorname{Int}[(x^5 (a + b \operatorname{ArcCsch}[c x])) / (d + e x^2)^2, x]$

[Out]  $(b \sqrt{1 + 1/(c^2 x^2)}) x / (2 c e^2) + (d (a + b \operatorname{ArcCsch}[c x])) / (2 e^2 (e + d/x^2)) + (x^2 (a + b \operatorname{ArcCsch}[c x])) / (2 e^2) - (b d \operatorname{ArcTan}[\sqrt{c^2 d - e} / (c \sqrt{e} \sqrt{1 + 1/(c^2 x^2)}) x]) / (2 \sqrt{c^2 d - e} e^{5/2}) - (d (a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 - (c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}) / (\sqrt{e} - \sqrt{-(c^2 d + e)})]) / e^3 - (d (a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 + (c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}) / (\sqrt{e} - \sqrt{-(c^2 d + e)})]) / e^3 - (d (a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 - (c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}) / (\sqrt{e} + \sqrt{-(c^2 d + e)})]) / e^3 - (d (a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 + (c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}) / (\sqrt{e} + \sqrt{-(c^2 d + e)})]) / e^3 + (2 d (a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 - E^{(2 \operatorname{ArcCsch}[c x])}]) / e^3 - (b d \operatorname{PolyLog}[2, -((c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}) / (\sqrt{e} - \sqrt{-(c^2 d + e)})])]) / e^3 - (b d \operatorname{PolyLog}[2, (c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}) / (\sqrt{e} - \sqrt{-(c^2 d + e)})]) / e^3 - (b d \operatorname{PolyLog}[2, -((c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}) / (\sqrt{e} + \sqrt{-(c^2 d + e)})])]) / e^3 - (b d \operatorname{PolyLog}[2, (c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}) / (\sqrt{e} + \sqrt{-(c^2 d + e)})]) / e^3 + (b d \operatorname{PolyLog}[2, E^{(2 \operatorname{ArcCsch}[c x])}]) / e^3$

Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 264

Int[((c\_)\*(x\_)^(m\_))\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

#### Rule 377

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 2190

Int[(((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)/((a\_) + (b\_)\*((F\_)^(g\_)\*((e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m-1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^(e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 3716

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + Pi\*(k\_) + (Complex[0, fz\_])\*(f\_)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m+1))/(d\*(m+1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))]/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 5561

Int[(Cosh[(c\_) + (d\_)\*(x\_)])\*((e\_) + (f\_)\*(x\_))^(m\_)/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m+1)/(b\*f\*(m+1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

#### Rule 5659

Int[((a\_) + ArcSinh[(c\_)\*(x\_)])\*(b\_)^(n\_)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tanh[x], x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

#### Rule 5661

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^ (n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 +
c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

#### Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

#### Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
(f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

#### Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^ (n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x^3 (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e^2 x^3} - \frac{2d (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e^3 x} + \frac{d^2 x (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)^2} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{(2d) \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^3} - \frac{(2d^2) \operatorname{Subst} \left( \int \frac{x (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} - \dots \\
&= \frac{d (a + b \operatorname{csch}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{2e^2} + \frac{(2d) \operatorname{Subst} \left( \int (a + bx) \operatorname{coth}(x) dx \right)}{e^3} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{2e^2} - \frac{d (a + b \operatorname{csch}^{-1}(cx))}{be^3} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{2e^2} - \frac{d (a + b \operatorname{csch}^{-1}(cx))}{be^3} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{2e^2} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}}} \right)}{2\sqrt{c^2 d - e} e^5} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{2e^2} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}}} \right)}{2\sqrt{c^2 d - e} e^5} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{2e^2} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}}} \right)}{2\sqrt{c^2 d - e} e^5} \\
&= \frac{b \sqrt{1 + \frac{1}{c^2 x^2}} x}{2ce^2} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{2e^2} - \frac{bd \tan^{-1} \left( \frac{\sqrt{c^2 d}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2}}} \right)}{2\sqrt{c^2 d - e} e^5}
\end{aligned}$$

**Mathematica [C]** time = 6.36, size = 1447, normalized size = 2.45

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2,x]

[Out] 
$$-1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*\text{Log}[d + e*x^2] + b*(d*\text{Pi}^2 - (2*e*\text{Sqrt}[1 + 1/(c^2*x^2)]*x)/c - (4*I)*d*\text{Pi}*ArcCsch[c*x] - 2*e*x^2*ArcCsch[c*x] + (d^{3/2}*ArcCsch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^{3/2}*ArcCsch[c*x])/(Sqrt[d] + I*Sqrt[e]*x) - 8*d*ArcCsch[c*x]^2 - 2*d*ArcSinh[1/(c*x)] + 16*d*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(\text{Pi} + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*d*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(\text{Pi} + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*d*ArcCsch[c*x]*\text{Log}[1 - E^{-(2*ArcCsch[c*x])}] + (2*I)*d*\text{Pi}*\text{Log}[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*\text{Log}[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + (8*I)*d*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*\text{Log}[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + (2*I)*d*\text{Pi}*\text{Log}[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*\text{Log}[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + (8*I)*d*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*\text{Log}[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + (2*I)*d*\text{Pi}*\text{Log}[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*\text{Log}[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] - (8*I)*d*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*\text{Log}[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + (2*I)*d*\text{Pi}*\text{Log}[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*\text{Log}[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] - (8*I)*d*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*\text{Log}[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] - (2*I)*d*\text{Pi}*\text{Log}[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*d*\text{Pi}*\text{Log}[Sqrt[e] + (I*Sqrt[d])/x] + (d*Sqrt[e]*\text{Log}[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e])*Sqrt[1 + 1/(c^2*x^2)])*x)/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x)))/Sqrt[-(c^2*d) + e] + (d*Sqrt[e]*\text{Log}[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e])*Sqrt[1 + 1/(c^2*x^2)])*x)/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x)))/Sqrt[-(c^2*d) + e] + 4*d*PolyLog[2, E^{-(2*ArcCsch[c*x])}] + 4*d*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + 4*d*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + 4*d*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])] + 4*d*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^{ArcCsch[c*x]})/(c*Sqrt[d])))/e^3$$

**fricas** [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^5 \operatorname{arcsch}(cx) + ax^5}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^5\*arccsch(c\*x) + a\*x^5)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^5/(e\*x^2 + d)^2, x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

[Out] int(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{d^2}{e^4x^2 + de^3} - \frac{x^2}{e^2} + \frac{2d \log(ex^2 + d)}{e^3}\right) + b \int \frac{x^5 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^2x^4 + 2dex^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2\*a\*(d^2/(e^4\*x^2 + d\*e^3) - x^2/e^2 + 2\*d\*log(e\*x^2 + d)/e^3) + b\*integrate(x^5\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.104 \quad \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx$$

**Optimal.** Leaf size=553

$$\frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2e^2} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}} + 1\right)}{2e^2} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2e^2}$$

[Out]  $\frac{1}{2} * (-a - b * \operatorname{arccsch}(c * x)) / e / (e + d / x^2) - (a + b * \operatorname{arccsch}(c * x))^2 / b / e^2 - (a + b * \operatorname{arccsch}(c * x)) * \ln(1 - 1 / ((1 / c / x + (1 + 1 / c^2 / x^2)^{1/2})^2) / e^2 + 1 / 2 * (a + b * \operatorname{arccsch}(c * x)) * \ln(1 - c * (1 / c / x + (1 + 1 / c^2 / x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} - (-c^2 * d + e)^{1/2})) / e^2 + 1 / 2 * (a + b * \operatorname{arccsch}(c * x)) * \ln(1 + c * (1 / c / x + (1 + 1 / c^2 / x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} - (-c^2 * d + e)^{1/2})) / e^2 + 1 / 2 * (a + b * \operatorname{arccsch}(c * x)) * \ln(1 - c * (1 / c / x + (1 + 1 / c^2 / x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} + (-c^2 * d + e)^{1/2})) / e^2 + 1 / 2 * (a + b * \operatorname{arccsch}(c * x)) * \ln(1 + c * (1 / c / x + (1 + 1 / c^2 / x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} + (-c^2 * d + e)^{1/2})) / e^2 + 1 / 2 * b * \operatorname{polylog}(2, 1 / ((1 / c / x + (1 + 1 / c^2 / x^2)^{1/2})^2) / e^2 + 1 / 2 * b * \operatorname{polylog}(2, -c * (1 / c / x + (1 + 1 / c^2 / x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} - (-c^2 * d + e)^{1/2})) / e^2 + 1 / 2 * b * \operatorname{polylog}(2, c * (1 / c / x + (1 + 1 / c^2 / x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} - (-c^2 * d + e)^{1/2})) / e^2 + 1 / 2 * b * \operatorname{polylog}(2, -c * (1 / c / x + (1 + 1 / c^2 / x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} + (-c^2 * d + e)^{1/2})) / e^2 + 1 / 2 * b * \operatorname{polylog}(2, c * (1 / c / x + (1 + 1 / c^2 / x^2)^{1/2}) * (-d)^{1/2} / (e^{1/2} + (-c^2 * d + e)^{1/2})) / e^2 + 1 / 2 * b * \operatorname{arctan}((c^2 * d - e)^{1/2} / c / x / e^{1/2} / (1 + 1 / c^2 / x^2)^{1/2}) / e^{3/2} / (c^2 * d - e)^{1/2}$

**Rubi [A]** time = 1.25, antiderivative size = 535, normalized size of antiderivative = 0.97, number of steps used = 29, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6304, 5791, 5659, 3716, 2190, 2279, 2391, 5787, 377, 205, 5799, 5561}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2e^2}$$

Warning: Unable to verify antiderivative.

[In] Int[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2, x]

[Out]  $-(a + b * \operatorname{ArcCsch}[c * x]) / (2 * e * (e + d / x^2)) + (b * \operatorname{ArcTan}[\operatorname{Sqrt}[c^2 * d - e] / (c * \operatorname{Sqrt}[e] * \operatorname{Sqrt}[1 + 1 / (c^2 * x^2)] * x)]) / (2 * \operatorname{Sqrt}[c^2 * d - e] * e^{3/2}) + ((a + b * \operatorname{ArcCsch}[c * x]) * \operatorname{Log}[1 - (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsch}[c * x]}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * e^2) + ((a + b * \operatorname{ArcCsch}[c * x]) * \operatorname{Log}[1 + (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsch}[c * x]}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * e^2) + ((a + b * \operatorname{ArcCsch}[c * x]) * \operatorname{Log}[1 - (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsch}[c * x]}) / (\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * e^2) + ((a + b * \operatorname{ArcCsch}[c * x]) * \operatorname{Log}[1 + (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsch}[c * x]}) / (\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * e^2) - ((a + b * \operatorname{ArcCsch}[c * x]) * \operatorname{Log}[1 - E^{(2 * \operatorname{ArcCsch}[c * x])}]) / e^2 + (b * \operatorname{PolyLog}[2, -((c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsch}[c * x]}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2 * d) + e])])]) / (2 * e^2) + (b * \operatorname{PolyLog}[2, (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsch}[c * x]}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * e^2) + (b * \operatorname{PolyLog}[2, -((c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsch}[c * x]}) / (\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2 * d) + e])])]) / (2 * e^2) + (b * \operatorname{PolyLog}[2, (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsch}[c * x]}) / (\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * e^2) - (b * \operatorname{PolyLog}[2, E^{(2 * \operatorname{ArcCsch}[c * x])}]) / (2 * e^2)$

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)])\*((e\_.) + (f\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

Rule 5659

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/(x\_), x\_Symbol] := Subst[Int[(a + b\*x)^n/Tanh[x], x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 5787

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))\*(x\_)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSinh[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c)/(2\*e\*(p + 1)), Int[(d + e\*x^2)^(p + 1)/Sqrt[1 + c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2\*d] && NeQ[p, -1]

Rule 5791

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcSinh[c\*x])^n, (f\*x)^m\*(d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^2]

2\*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

Rule 5799

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)]\*(b\_.))^n\_./((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Subst[Int[((a + b\*x)^n\*Cosh[x])/(c\*d + e\*Sinh[x]), x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 6304

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)]\*(b\_.))^n\_.\*(x\_)^m\_.\*((d\_.) + (e\_.)\*(x\_)^2)^p\_., x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e^2 x} - \frac{dx (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)^2} - \frac{dx (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)} \right) \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left( \int \frac{x (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e + dx^2} \right) dx, x, \frac{1}{x} \right)}{e^2} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left( \int (a + bx) \operatorname{coth}(x) dx, x, \operatorname{csch}^{-1}(cx) \right)}{e^2} + \frac{d \operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e + dx^2} \right) dx, x, \frac{1}{x} \right)}{e^2} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2be^2} + \frac{2 \operatorname{Subst} \left( \int \frac{e^{2x(a+bx)}}{1-e^{2x}} dx, x, \operatorname{csch}^{-1}(cx) \right)}{e^2} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2be^2} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2\sqrt{c^2 d - e} e^{3/2}} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - e^{2 \operatorname{csch}^{-1}(cx)} \right)}{e^2} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2\sqrt{c^2 d - e} e^{3/2}} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - e^{2 \operatorname{csch}^{-1}(cx)} \right)}{e^2} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2\sqrt{c^2 d - e} e^{3/2}} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{d}} \right)}{2e^2} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2\sqrt{c^2 d - e} e^{3/2}} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{d}} \right)}{2e^2} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e \left( e + \frac{d}{x^2} \right)} + \frac{b \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c \sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2\sqrt{c^2 d - e} e^{3/2}} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log \left( 1 - \frac{c \sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{d}} \right)}{2e^2}
\end{aligned}$$

**Mathematica [C]** time = 2.41, size = 1410, normalized size = 2.55

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2,x]

[Out] (b\*Pi^2 + (4\*a\*d)/(d + e\*x^2) - (4\*I)\*b\*Pi\*ArcCsch[c\*x] + (2\*b\*Sqrt[d]\*ArcCsch[c\*x]))/(Sqrt[d] - I\*Sqrt[e]\*x) + (2\*b\*Sqrt[d]\*ArcCsch[c\*x))/(Sqrt[d] + I\*Sqrt[e]\*x) - 8\*b\*ArcCsch[c\*x]^2 - 4\*b\*ArcSinh[1/(c\*x)] + 16\*b\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] - Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 16\*b\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*b\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])] + (2\*I)\*b\*Pi\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*ArcCsch[c\*x]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (8\*I)\*b\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (2\*I)\*b\*Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*ArcCsch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (8\*I)\*b\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (2\*I)\*b\*Pi\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*ArcCsch[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] - (8\*I)\*b\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (2\*I)\*b\*Pi\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] - (2\*I)\*b\*Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x] - (2\*I)\*b\*Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x] + (2\*b\*Sqrt[e]\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] + I\*Sqrt[-(c^2\*d) + e])\*Sqrt[1 + 1/(c^2\*x^2)])\*x))/(Sqrt[-(c^2\*d) + e]\*(I\*Sqrt[d] + Sqrt[e]\*x)))/Sqrt[-(c^2\*d) + e] + (2\*b\*Sqrt[e]\*Log[(-2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) + e])\*Sqrt[1 + 1/(c^2\*x^2)])\*x))/(Sqrt[-(c^2\*d) + e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) + e] + 4\*a\*Log[d + e\*x^2] + 4\*b\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 4\*b\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 4\*b\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])])]/(8\*e^2)

**fricas** [F] time = 1.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3 \operatorname{arcsch}(cx) + ax^3}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^3\*arccsch(c\*x) + a\*x^3)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^3/(e\*x^2 + d)^2, x)



**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

[Out] int(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{d}{e^3 x^2 + d e^2} + \frac{\log(ex^2 + d)}{e^2} \right) + b \int \frac{x^3 \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{e^2 x^4 + 2 d e x^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(d/(e^3\*x^2 + d\*e^2) + log(e\*x^2 + d)/e^2) + b\*integrate(x^3\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.105 \quad \int \frac{x(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=139

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex^2)} + \frac{bcx \tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{2de\sqrt{-c^2x^2}} + \frac{bcx \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{e}\sqrt{-c^2x^2}\sqrt{c^2d-e}}$$

[Out] 1/2\*(-a-b\*arccsch(c\*x))/e/(e\*x^2+d)+1/2\*b\*c\*x\*arctan((-c^2\*x^2-1)^(1/2))/d/e/(-c^2\*x^2)^(1/2)+1/2\*b\*c\*x\*arctanh(e^(1/2)\*(-c^2\*x^2-1)^(1/2)/(c^2\*d-e)^(1/2))/d/(c^2\*d-e)^(1/2)/e^(1/2)/(-c^2\*x^2)^(1/2)

**Rubi [A]** time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {6300, 446, 86, 63, 205, 208}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex^2)} + \frac{bcx \tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{2de\sqrt{-c^2x^2}} + \frac{bcx \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{e}\sqrt{-c^2x^2}\sqrt{c^2d-e}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2,x]

[Out] -(a + b\*ArcCsch[c\*x])/(2\*e\*(d + e\*x^2)) + (b\*c\*x\*ArcTan[Sqrt[-1 - c^2\*x^2]])/(2\*d\*e\*Sqrt[-(c^2\*x^2)]) + (b\*c\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 - c^2\*x^2])/Sqrt[c^2\*d - e]])/(2\*d\*Sqrt[c^2\*d - e]\*Sqrt[e]\*Sqrt[-(c^2\*x^2)])

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 86

Int[((e\_.) + (f\_.)\*(x\_))^(p\_)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x] - Dist[d/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[

$b*c - a*d, 0]$  && IntegerQ[Simplify[(m + 1)/n]]

### Rule 6300

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))\*(x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsch[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[-(c^2\*x^2)]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[-1 - c^2\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{x(a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex^2)} + \frac{(bcx) \int \frac{1}{x\sqrt{-1-c^2x^2}(d+ex^2)} dx}{2e\sqrt{-c^2x^2}} \\ &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex^2)} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}(d+ex)} dx, x, x^2\right)}{4e\sqrt{-c^2x^2}} \\ &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex^2)} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}(d+ex)} dx, x, x^2\right)}{4d\sqrt{-c^2x^2}} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}} dx, x, x^2\right)}{4de\sqrt{-c^2x^2}} \\ &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex^2)} + \frac{(bx) \operatorname{Subst}\left(\int \frac{1}{d - \frac{e}{c^2} - \frac{ex^2}{c^2}} dx, x, \sqrt{-1 - c^2x^2}\right)}{2cd\sqrt{-c^2x^2}} - \frac{(bx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}} dx, x, \sqrt{-1 - c^2x^2}\right)}{2de\sqrt{-c^2x^2}} \\ &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{2e(d + ex^2)} + \frac{bcx \tan^{-1}\left(\sqrt{-1 - c^2x^2}\right)}{2de\sqrt{-c^2x^2}} + \frac{bcx \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{c^2d-e}\sqrt{e}\sqrt{-c^2x^2}} \end{aligned}$$

**Mathematica [C]** time = 0.79, size = 271, normalized size = 1.95

$$\frac{\frac{2a}{d+ex^2} + \frac{b\sqrt{e} \log\left(\frac{4(cd\sqrt{ex}(c\sqrt{d+ix}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d})+ide)}{b\sqrt{e-c^2d}(\sqrt{d-ix}\sqrt{ex})}\right)}{d\sqrt{e-c^2d}} + \frac{b\sqrt{e} \log\left(\frac{4i(de+cd\sqrt{ex}\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d+ic\sqrt{d}}\right))}{b\sqrt{e-c^2d}(\sqrt{d+ix}\sqrt{ex})}\right)}{d\sqrt{e-c^2d}} + \frac{2b\operatorname{csch}^{-1}(cx)}{d+ex^2} - \frac{2b\operatorname{sinh}^{-1}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{d}}{4e}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2, x]

[Out]  $-1/4*((2*a)/(d + e*x^2) + (2*b*ArcCsch[c*x]))/(d + e*x^2) - (2*b*ArcSinh[1/(c*x)]/d + (b*Sqrt[e]*Log[(-4*(I*d*e + c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e])*Sqrt[1 + 1/(c^2*x^2)]))x])/(b*Sqrt[-(c^2*d) + e]*(Sqrt[d] - I*Sqrt[e]*x)))/(d*Sqrt[-(c^2*d) + e]) + (b*Sqrt[e]*Log[((4*I)*(d*e + c*d*Sqrt[e])*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e])*Sqrt[1 + 1/(c^2*x^2)]))x])/(b*Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x)))/(d*Sqrt[-(c^2*d) + e]))/e$

**fricas [B]** time = 0.94, size = 615, normalized size = 4.42

$$\frac{2ac^2d^2 - 2ade + \sqrt{-c^2de + e^2}(bex^2 + bd) \log\left(\frac{c^2ex^2 - c^2d - 2\sqrt{-c^2de + e^2}cx\sqrt{\frac{c^2x^2+1}{c^2x^2} + 2e}}{ex^2 + d}\right) - 2(bc^2d^2 - bde + (bc^2de - 2ade + 2a^2))\sqrt{-c^2de + e^2}}{4(c^2d + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] [-1/4*(2*a*c^2*d^2 - 2*a*d*e + sqrt(-c^2*d*e + e^2)*(b*e*x^2 + b*d)*log((c^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*e)/(e*x^2 + d)) - 2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*c^2*d^2 - b*d*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2), -1/2*(a*c^2*d^2 - a*d*e + sqrt(c^2*d*e - e^2)*(b*e*x^2 + b*d)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*d - e)) - (b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^2*d^2 - b*d*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^2, x)
```

**maple** [B] time = 0.07, size = 360, normalized size = 2.59

$$-\frac{c^2 a}{2e(c^2 x^2 e + c^2 d)} - \frac{c^2 b \operatorname{arccsch}(cx)}{2e(c^2 x^2 e + c^2 d)} + \frac{b\sqrt{c^2 x^2 + 1} \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right)}{2ce\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} xd} - \frac{b\sqrt{c^2 x^2 + 1} \ln\left(\frac{2\left(\sqrt{c^2 x^2 + 1} \sqrt{\frac{-c^2 d - e}{e}} e + \sqrt{-c^2 d - e}\right)}{-cxe + \sqrt{-c^2 d - e}}\right)}{4ce\sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} xd\sqrt{\frac{-c^2 d - e}{e}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
[Out] -1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arccsch(c*x)+1/2/c*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d*arctanh(1/(c^2*x^2+1)^(1/2))-1/4/c*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-(c^2*d*e)^(1/2)*c*x+e)/(-c*x*e+(-(c^2*d*e)^(1/2))))-1/4/c*b/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-(c^2*d*e)^(1/2)*c*x-e)/(c*x*e+(-(c^2*d*e)^(1/2))))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4} \left( 4c^2 \int \frac{x}{2(c^2 e^2 x^4 + (c^2 d e + e^2)x^2 + d e + (c^2 e^2 x^4 + (c^2 d e + e^2)x^2 + d e)\sqrt{c^2 x^2 + 1})} dx - \frac{2c^2 d^2 \log(c) - 2(c^2 d e + e^2)}{2(c^2 e^2 x^4 + (c^2 d e + e^2)x^2 + d e + (c^2 e^2 x^4 + (c^2 d e + e^2)x^2 + d e)\sqrt{c^2 x^2 + 1})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
[Out] -1/4*(4*c^2*integrate(1/2*x/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e + (c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(c^2*x^2 + 1)), x) - (2*c^2*d^2*1
```

$\log(c) - 2*(c^2*d*e - e^2)*x^2*\log(x) - 2*d*e*\log(c) + (c^2*d*e*x^2 + c^2*d^2)*\log(c^2*x^2 + 1) - 2*(c^2*d^2 - d*e)*\log(\sqrt{c^2*x^2 + 1} + 1)/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2) + \log(e*x^2 + d)/(c^2*d^2 - d*e)*b - 1/2*a/(e^2*x^2 + d*e)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.106 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^2} dx$$

Optimal. Leaf size=515

$$\frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2d^2} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}} + 1\right)}{2d^2} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2d^2}$$

[Out]  $-1/2 * e * (a + b * \operatorname{arccsch}(c * x)) / d^2 / (e + d / x^2) + 1/2 * (a + b * \operatorname{arccsch}(c * x))^2 / b / d^2 - 1/2 * (a + b * \operatorname{arccsch}(c * x)) * \ln(1 - c * (1/c/x + (1 + 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (-c^2 * d + e)^{1/2}) / d^2 - 1/2 * (a + b * \operatorname{arccsch}(c * x)) * \ln(1 + c * (1/c/x + (1 + 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (-c^2 * d + e)^{1/2}) / d^2 - 1/2 * (a + b * \operatorname{arccsch}(c * x)) * \ln(1 - c * (1/c/x + (1 + 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (-c^2 * d + e)^{1/2}) / d^2 - 1/2 * (a + b * \operatorname{arccsch}(c * x)) * \ln(1 + c * (1/c/x + (1 + 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (-c^2 * d + e)^{1/2}) / d^2 - 1/2 * b * \operatorname{polylog}(2, -c * (1/c/x + (1 + 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (-c^2 * d + e)^{1/2}) / d^2 - 1/2 * b * \operatorname{polylog}(2, c * (1/c/x + (1 + 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} - (-c^2 * d + e)^{1/2}) / d^2 - 1/2 * b * \operatorname{polylog}(2, -c * (1/c/x + (1 + 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (-c^2 * d + e)^{1/2}) / d^2 - 1/2 * b * \operatorname{polylog}(2, c * (1/c/x + (1 + 1/c^2/x^2)^{1/2})) * (-d)^{1/2} / (e^{1/2} + (-c^2 * d + e)^{1/2}) / d^2 + 1/2 * b * \operatorname{arctan}((c^2 * d - e)^{1/2} / c / x / e^{1/2} / (1 + 1/c^2/x^2)^{1/2}) * e^{1/2} / d^2 / (c^2 * d - e)^{1/2}$

**Rubi [A]** time = 1.15, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {6304, 5791, 5787, 377, 205, 5799, 5561, 2190, 2279, 2391}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2d^2} - \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2 d}}}\right)}{2d^2} - \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2d^2} - \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2d^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b * \operatorname{ArcCsCh}[c * x]) / (x * (d + e * x^2)^2), x]$

[Out]  $-(e * (a + b * \operatorname{ArcCsCh}[c * x])) / (2 * d^2 * (e + d / x^2)) + (a + b * \operatorname{ArcCsCh}[c * x])^2 / (2 * b * d^2) + (b * \operatorname{Sqrt}[e] * \operatorname{ArcTan}[\operatorname{Sqrt}[c^2 * d - e] / (c * \operatorname{Sqrt}[e] * \operatorname{Sqrt}[1 + 1 / (c^2 * x^2)] * x)]) / (2 * d^2 * \operatorname{Sqrt}[c^2 * d - e]) - ((a + b * \operatorname{ArcCsCh}[c * x]) * \operatorname{Log}[1 - (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsCh}[c * x]}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * d^2) - ((a + b * \operatorname{ArcCsCh}[c * x]) * \operatorname{Log}[1 + (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsCh}[c * x]}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * d^2) - ((a + b * \operatorname{ArcCsCh}[c * x]) * \operatorname{Log}[1 - (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsCh}[c * x]}) / (\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * d^2) - ((a + b * \operatorname{ArcCsCh}[c * x]) * \operatorname{Log}[1 + (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsCh}[c * x]}) / (\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * d^2) - (b * \operatorname{PolyLog}[2, -((c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsCh}[c * x]}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * d^2) - (b * \operatorname{PolyLog}[2, (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsCh}[c * x]}) / (\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * d^2) - (b * \operatorname{PolyLog}[2, -((c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsCh}[c * x]}) / (\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * d^2) - (b * \operatorname{PolyLog}[2, (c * \operatorname{Sqrt}[-d] * E^{\operatorname{ArcCsCh}[c * x]}) / (\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2 * d) + e])]) / (2 * d^2)$

Rule 205

$\operatorname{Int}[(a + (b * x^2)^{-1}), x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[a/b, 2] * \operatorname{ArcTan}[x / \operatorname{Rt}[a/b, 2]]) / a, x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 2190

Int((((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)\*((c\_) + (d\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*((F\_)^((g\_)\*((e\_) + (f\_)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 5561

Int[(Cosh[(c\_) + (d\_)\*(x\_)]\*((e\_) + (f\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

#### Rule 5787

Int[((a\_) + ArcSinh[(c\_)\*(x\_)]\*(b\_))\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcSinh[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c)/(2\*e\*(p + 1)), Int[(d + e\*x^2)^(p + 1)/Sqrt[1 + c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2\*d] && NeQ[p, -1]

#### Rule 5791

Int[((a\_) + ArcSinh[(c\_)\*(x\_)]\*(b\_))^(n\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*ArcSinh[c\*x])^n, (f\*x)^m\*(d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^2\*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 5799

Int[((a\_) + ArcSinh[(c\_)\*(x\_)]\*(b\_))^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Subst[Int[((a + b\*x)^n\*Cosh[x])/(c\*d + e\*Sinh[x]), x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 6304

Int[((a\_) + ArcCsch[(c\_)\*(x\_)]\*(b\_))^(n\_)\*(x\_)^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m, p]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{x^3 \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( -\frac{ex \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)^2} + \frac{x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left( \int \frac{x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left( \int \left( -\frac{\sqrt{-d} \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} - \sqrt{-d}x)} + \frac{\sqrt{-d} \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d} + \dots \\
&= -\frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} - \frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} \\
&= -\frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c\sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2d^2 \sqrt{c^2 d - e}} + \frac{\operatorname{Subst} \left( \int \frac{(a + bx) \cosh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \sinh(x)} dx, x, \operatorname{csch}^{-1} \left( \frac{x}{c} \right) \right)}{2(-d)^{3/2}} \\
&= -\frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)^2}{2bd^2} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c\sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2d^2 \sqrt{c^2 d - e}} + \frac{\operatorname{Subst} \left( \int \frac{\frac{1}{\sqrt{e}}}{\frac{1}{\sqrt{e}} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} \\
&= -\frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)^2}{2bd^2} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c\sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2d^2 \sqrt{c^2 d - e}} - \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)}{2(-d)^{3/2}} \\
&= -\frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)^2}{2bd^2} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c\sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2d^2 \sqrt{c^2 d - e}} - \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)}{2(-d)^{3/2}} \\
&= -\frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{2d^2 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)^2}{2bd^2} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{\sqrt{c^2 d - e}}{c\sqrt{e} \sqrt{1 + \frac{1}{c^2 x^2} x}} \right)}{2d^2 \sqrt{c^2 d - e}} - \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)}{2(-d)^{3/2}}
\end{aligned}$$

**Mathematica** [F] time = 44.15, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^2} dx$$

Verification is Not applicable to the result.



[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^2), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^2), x]

**fricas** [F] time = 1.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{e^2x^5 + 2dex^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e^2\*x^5 + 2\*d\*e\*x^3 + d^2\*x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)^2\*x), x)

**maple** [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x(e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^2,x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{1}{dex^2 + d^2} - \frac{\log(ex^2 + d)}{d^2} + \frac{2 \log(x)}{d^2} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^2x^5 + 2dex^3 + d^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(1/(d\*e\*x^2 + d^2) - log(e\*x^2 + d)/d^2 + 2\*log(x)/d^2) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e^2\*x^5 + 2\*d\*e\*x^3 + d^2\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x(e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)^2), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.107 \quad \int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx$$

**Optimal.** Leaf size=756

$$\frac{3\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{4e^{5/2}} - \frac{3\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}} + 1\right)}{4e^{5/2}} + \frac{3\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{4e^{5/2}}$$

[Out]  $x*(a+b*\operatorname{arccsch}(c*x))/e^2+b*\operatorname{arctanh}((1+1/c^2/x^2)^{(1/2)})/c/e^2+3/4*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)}-3/4*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)}+3/4*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)}-3/4*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)}-3/4*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)}+3/4*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)}-3/4*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)}+3/4*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))*(-d)^{(1/2)}/e^{(5/2)}+1/4*b*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)})*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d-e)^{(1/2)}/(1+1/c^2/x^2)^{(1/2)})*d^{(1/2)}/e^2/(c^2*d-e)^{(1/2)}+1/4*b*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)})*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d-e)^{(1/2)}/(1+1/c^2/x^2)^{(1/2)})*d^{(1/2)}/e^2/(c^2*d-e)^{(1/2)}-1/4*d*(a+b*\operatorname{arccsch}(c*x))/e^2/(-d/x+(-d)^{(1/2)})*e^{(1/2)}+1/4*d*(a+b*\operatorname{arccsch}(c*x))/e^2/(d/x+(-d)^{(1/2)})*e^{(1/2)}$

**Rubi [A]** time = 2.47, antiderivative size = 756, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 15, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {6304, 5791, 5661, 266, 63, 208, 5706, 5801, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$-\frac{3b\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{4e^{5/2}} + \frac{3b\sqrt{-d} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{4e^{5/2}} - \frac{3b\sqrt{-d} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{4e^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^4*(a + b*\operatorname{ArcCsch}[c*x]))/(d + e*x^2)^2, x]$

[Out]  $-(d*(a + b*\operatorname{ArcCsch}[c*x]))/(4*e^2*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)) + (d*(a + b*\operatorname{ArcCsch}[c*x]))/(4*e^2*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)) + (x*(a + b*\operatorname{ArcCsch}[c*x]))/e^2 + (b*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 + 1/(c^2*x^2)]])/c/e^2 + (b*\operatorname{Sqrt}[d]*\operatorname{ArcTanh}[(c^2*d - (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x)/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])))/(4*\operatorname{Sqrt}[c^2*d - e]*e^2) + (b*\operatorname{Sqrt}[d]*\operatorname{ArcTanh}[(c^2*d + (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x)/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])))/(4*\operatorname{Sqrt}[c^2*d - e]*e^2) + (3*\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(4*e^{(5/2)}) - (3*\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(4*e^{(5/2)}) + (3*\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(4*e^{(5/2)}) - (3*\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(4*e^{(5/2)}) - (3*b*\operatorname{Sqrt}[-d]*\operatorname{PolyLog}[2, -(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(4*e^{(5/2)}) + (3*b*\operatorname{Sqrt}[-d]*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(4*e^{(5/2)}) - (3*b*\operatorname{Sqrt}[-d]*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(4*e^{(5/2)}) + (3*b*\operatorname{Sqrt}[-d]*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(4*e^{(5/2)})$

```
rt[-d]*E^ArcCsch[c*x]/(Sqrt[e - Sqrt[-(c^2*d) + e]])/(4*e^(5/2)) - (3*b*
Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
+ e]))]/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])
/(Sqrt[e] + Sqrt[-(c^2*d) + e]))]/(4*e^(5/2))
```

### Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_)^(m_.)))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

, x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

#### Rule 5661

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d\*x)^(m + 1)\*(a + b\*ArcSinh[c\*x])^n)/(d\*(m + 1)), x] - Dist[(b\*c\*n)/(d\*(m + 1)), Int[((d\*x)^(m + 1)\*(a + b\*ArcSinh[c\*x])^(n - 1))/Sqrt[1 + c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 5706

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*ArcSinh[c\*x])^n, (d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2\*d] && IntegerQ[p] && (p > 0 || IGtQ[n, 0])

#### Rule 5791

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*ArcSinh[c\*x])^n, (f\*x)^m\*(d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^2\*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 5799

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Subst[Int[((a + b\*x)^n\*Cosh[x])/(c\*d + e\*Sinh[x]), x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 5801

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcSinh[c\*x])^n)/(e\*(m + 1)), x] - Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcSinh[c\*x])^(n - 1))/Sqrt[1 + c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 6304

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m, p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x^2 (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e^2 x^2} - \frac{d (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)^2} - \frac{d (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)} \right) dx, x \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} - \frac{b \operatorname{Subst} \left( \int \frac{1}{x \sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{ce^2} + \frac{d \operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{e^2} \\
&= \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e^{5/2}} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{2e^{5/2}} \\
&= -\frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{e^2} \\
&= -\frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{\frac{e-d}{e+d}} \right)}{ce^2} \\
&= -\frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{\frac{e-d}{e+d}} \right)}{ce^2} \\
&= -\frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{\frac{e-d}{e+d}} \right)}{ce^2} \\
&= -\frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{\frac{e-d}{e+d}} \right)}{ce^2} \\
&= -\frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{\frac{e-d}{e+d}} \right)}{ce^2} \\
&= -\frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{4e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{b \tanh^{-1} \left( \sqrt{\frac{e-d}{e+d}} \right)}{ce^2}
\end{aligned}$$

**Mathematica [C]** time = 6.17, size = 1583, normalized size = 2.09

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2,x]

[Out] (a\*x)/e^2 + (a\*d\*x)/(2\*e^2\*(d + e\*x^2)) - (3\*a\*Sqrt[d]\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(2\*e^(5/2)) + b\*(-1/4\*(d\*(-ArcCsch[c\*x]/(I\*Sqrt[d]\*Sqrt[e] + e\*x)) - (I\*(ArcSinh[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] + I\*Sqrt[-(c^2\*d) + e])\*Sqrt[1 + 1/(c^2\*x^2)])\*x])/(Sqrt[-(c^2\*d) + e]\*(I\*Sqrt[d] + Sqrt[e]\*x)))/Sqrt[-(c^2\*d) + e]))/Sqrt[d]))/e^2 - (d\*(-ArcCsch[c\*x]/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x)) + (I\*(ArcSinh[1/(c\*x)]/Sqrt[e] - Log[-2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) + e])\*Sqrt[1 + 1/(c^2\*x^2)])\*x])/(Sqrt[-(c^2\*d) + e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) + e]))/Sqrt[d]))/(4\*e^2) - (((3\*I)/32)\*Sqrt[d]\*(Pi^2 - (4\*I)\*Pi\*ArcCsch[c\*x] - 8\*ArcCsch[c\*x]^2 + 32\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] - Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])]) + (4\*I)\*Pi\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) + 8\*ArcCsch[c\*x]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) + (16\*I)\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) + (4\*I)\*Pi\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) + 8\*ArcCsch[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) - (16\*I)\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) - (4\*I)\*Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 8\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))])/e^(5/2) + (((3\*I)/32)\*Sqrt[d]\*(Pi^2 - (4\*I)\*Pi\*ArcCsch[c\*x] - 8\*ArcCsch[c\*x]^2 - 32\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])]) + (4\*I)\*Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) + 8\*ArcCsch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) + (16\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) + (4\*I)\*Pi\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) + 8\*ArcCsch[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) - (16\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]) - (4\*I)\*Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 8\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d]))])/e^(5/2) + ((ArcCsch[c\*x]\*Coth[ArcCsch[c\*x]/2])/2 - Log[Tanh[ArcCsch[c\*x]/2]] - (ArcCsch[c\*x]\*Tanh[ArcCsch[c\*x]/2]))/(c\*e^2))

**fricas [F]** time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4 \operatorname{arcsch}(cx) + ax^4}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^4\*arccsch(c\*x) + a\*x^4)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^4}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^4/(e\*x^2 + d)^2, x)

**maple** [F] time = 15.13, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

[Out] int(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{dx}{e^3 x^2 + d e^2} - \frac{3 d \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de} e^2} + \frac{2x}{e^2} \right) + b \int \frac{x^4 \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{e^2 x^4 + 2 dex^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(d\*x/(e^3\*x^2 + d\*e^2) - 3\*d\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*e^2) + 2\*x/e^2) + b\*integrate(x^4\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int((x^4\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out



$$3.108 \quad \int \frac{x^2(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^2} dx$$

**Optimal.** Leaf size=719

$$\frac{(a+b\operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}} + 1\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}} + 1\right)}{4\sqrt{-d}e^{3/2}}$$

[Out]  $\frac{1}{4}(a+b\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/e^{3/2}/(-d)^{1/2}-1/4*(a+b\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/e^{3/2}/(-d)^{1/2}+1/4*(a+b\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/e^{3/2}/(-d)^{1/2}-1/4*(a+b\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/e^{3/2}/(-d)^{1/2}-1/4*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/e^{3/2}/(-d)^{1/2}+1/4*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/e^{3/2}/(-d)^{1/2}-1/4*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/e^{3/2}/(-d)^{1/2}+1/4*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/e^{3/2}/(-d)^{1/2}-1/4*b*\operatorname{arctanh}((c^2*d-(-d)^{1/2}*e^{1/2}/x)/c/d^{1/2}/(c^2*d-e)^{1/2}/(1+1/c^2/x^2)^{1/2}))/e/d^{1/2}/(c^2*d-e)^{1/2}-1/4*b*\operatorname{arctanh}((c^2*d+(-d)^{1/2}*e^{1/2}/x)/c/d^{1/2}/(c^2*d-e)^{1/2}/(1+1/c^2/x^2)^{1/2}))/e/d^{1/2}/(c^2*d-e)^{1/2}+1/4*(a+b\operatorname{arccsch}(c*x))/e/(-d/x+(-d)^{1/2}*e^{1/2}))+1/4*(-a-b\operatorname{arccsch}(c*x))/e/(d/x+(-d)^{1/2}*e^{1/2}))$

**Rubi [A]** time = 1.24, antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {6304, 5706, 5801, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2, x]

[Out]  $\frac{(a+b\operatorname{ArcCsch}[c*x])/(4*e*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]-d/x))-(a+b\operatorname{ArcCsch}[c*x])/(4*e*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]+d/x))-(b*\operatorname{ArcTanh}[(c^2*d-(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x]/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d-e]*\operatorname{Sqrt}[1+1/(c^2*x^2)])))/(4*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d-e]*e)-(b*\operatorname{ArcTanh}[(c^2*d+(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x]/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d-e]*\operatorname{Sqrt}[1+1/(c^2*x^2)])))/(4*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d-e]*e)+((a+b\operatorname{ArcCsch}[c*x])*Log[1-(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})-((a+b\operatorname{ArcCsch}[c*x])*Log[1+(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})+((a+b\operatorname{ArcCsch}[c*x])*Log[1-(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})-((a+b\operatorname{ArcCsch}[c*x])*Log[1+(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})-(b*\operatorname{PolyLog}[2,-((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})+(b*\operatorname{PolyLog}[2,(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})-(b*\operatorname{PolyLog}[2,-((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})-(b*\operatorname{PolyLog}[2,(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})+(b*\operatorname{PolyLog}[2,-((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})+(b*\operatorname{PolyLog}[2,(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]-\operatorname{Sqrt}[-(c^2*d)+e])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})+(b*\operatorname{PolyLog}[2,-((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])])])/(4*\operatorname{Sqrt}[-d]*e^{3/2})+(b*\operatorname{PolyLog}[2,(c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e]+\operatorname{Sqrt}[-(c^2*d)+e])])/(4*\operatorname{Sqrt}[-d]*e^{3/2}))$

$g[2, (c*\text{Sqrt}[-d]*E^{\text{ArcCsch}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])]/(4*\text{Sqrt}[-d]*e^{(3/2)})$

### Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 725

$\text{Int}[1/(((d_ + (e_)*(x_))*\text{Sqrt}[(a_ + (c_)*(x_)^2])), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] /; \text{FreeQ}\{a, c, d, e\}, x]$

### Rule 2190

$\text{Int}((((F_)^{((g_)*((e_ + (f_)*(x_))))})^{(n_)*((c_ + (d_)*(x_))^{(m_))})}/((a_ + (b_)*((F_)^{((g_)*((e_ + (f_)*(x_))))})^{(n_)})), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \ \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[(a_ + (b_)*((F_)^{((e_)*((c_ + (d_)*(x_))))})^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_ + (e_)*(x_)^{(n_)}))/x], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x \ \&\& \ \text{EqQ}[c*d, 1]$

### Rule 5561

$\text{Int}[(\text{Cosh}[(c_ + (d_)*(x_)]*(e_ + (f_)*(x_))^{(m_)}]/((a_ + (b_)*\text{Sinh}[(c_ + (d_)*(x_)])), x\_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{(m+1)}/(b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m*E^{(c + d*x)}/(a - \text{Rt}[a^2 + b^2, 2] + b*E^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*E^{(c + d*x)}/(a + \text{Rt}[a^2 + b^2, 2] + b*E^{(c + d*x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

### Rule 5706

$\text{Int}(((a_ + \text{ArcSinh}[(c_)*(x_)]*(b_))^{(n_)*((d_ + (e_)*(x_)^2)^{(p_)}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSinh}[c*x])^n, (d + e*x^2)^p], x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{NeQ}[e, c^2*d] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (p > 0 \ || \ \text{IGtQ}[n, 0])$

### Rule 5799

$\text{Int}(((a_ + \text{ArcSinh}[(c_)*(x_)]*(b_))^{(n_)} / ((d_ + (e_)*(x_))), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n*\text{Cosh}[x]/(c*d + e*\text{Sinh}[x]), x], x, \text{ArcSinh}[c*x]] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{IGtQ}[n, 0]$

### Rule 5801

$\text{Int}(((a_ + \text{ArcSinh}[(c_)*(x_)]*(b_))^{(n_)*((d_ + (e_)*(x_))^{(m_)}), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m+1)}*(a + b*\text{ArcSinh}[c*x])^n]/(e*(m+1)), x] - \text{Dist}[(b*c*n)/(e*(m+1)), \text{Int}[(d + e*x)^{(m+1)}*(a + b*\text{ArcSinh}[c*x])^n]$

- 1))/Sqrt[1 + c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 6304

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)]\*(b\_.))^n\_\*(x\_)^m\_\*((d\_.) + (e\_.)\*(x\_)^2)^p\_, x\_Symbol] :> -Subst[Int[(e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n]/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{d(a + b \sinh^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d(a + b \sinh^{-1} \left( \frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} + dx)^2} - \frac{d(a + b \sinh^{-1} \left( \frac{x}{c} \right))}{2e(-de - d^2x^2)} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{4e} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{4e} + \frac{d \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{-de - d^2x^2} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \operatorname{Subst} \left( \int \frac{1}{(\sqrt{-d}\sqrt{e} - dx) \sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4ce} + \frac{b \operatorname{Subst} \left( \int \frac{1}{(\sqrt{-d}\sqrt{e} + dx) \sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{4ce} \\
&= \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{4e^{3/2}} - \frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{4e^{3/2}} \\
&= \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} \\
&= \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} \\
&= \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} \\
&= \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} \\
&= \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}} - \frac{b \tanh^{-1} \left( \frac{c^2d - \sqrt{-d}\sqrt{e}}{c\sqrt{d}\sqrt{c^2d - e} \sqrt{1 + \frac{1}{c^2x^2}}} \right)}{4\sqrt{d}\sqrt{c^2d - e}}
\end{aligned}$$

**Mathematica [C]** time = 2.74, size = 1442, normalized size = 2.01

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2,x]

[Out] 
$$\begin{aligned} &((-4*a*\sqrt{e}*x)/(d + e*x^2) + (4*a*\text{ArcTan}[\sqrt{e}*x/\sqrt{d}])/\sqrt{d} + \\ &b*((2*\text{ArcCsch}[c*x])/(I*\sqrt{d} - \sqrt{e}*x) - (2*\text{ArcCsch}[c*x])/(I*\sqrt{d} \\ &+ \sqrt{e}*x) + ((8*I)*\text{ArcSin}[\sqrt{1 + \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2})*\text{ArcTan} \\ &(((c*\sqrt{d} - \sqrt{e})*\text{Cot}[(\text{Pi} + (2*I)*\text{ArcCsch}[c*x])/4])/ \sqrt{-(c^2*d) + e} \\ &)]/\sqrt{d} + ((8*I)*\text{ArcSin}[\sqrt{1 - \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2})*\text{ArcTan}(((c \\ &*\sqrt{d} + \sqrt{e})*\text{Cot}[(\text{Pi} + (2*I)*\text{ArcCsch}[c*x])/4])/ \sqrt{-(c^2*d) + e} \\ &)]/\sqrt{d} - (\text{Pi}*\text{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]})/( \\ &c*\sqrt{d})])/ \sqrt{d} + ((2*I)*\text{ArcCsch}[c*x]*\text{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2 \\ &*\sqrt{d} + \sqrt{e})*E^{\text{ArcCsch}[c*x]})/(c*\sqrt{d})}))/ \sqrt{d} - (4*\text{ArcSin}[\sqrt{1 + \sqrt{e} \\ &/c*\sqrt{d}}]/\sqrt{2})*\text{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]}) \\ &/c*\sqrt{d}])/ \sqrt{d} + (\text{Pi}*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]}) \\ &/c*\sqrt{d}])/ \sqrt{d} - ((2*I)*\text{ArcCsch}[c*x]*\text{Log}[1 + (I \\ &*(-\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]})/(c*\sqrt{d})]])/ \sqrt{d} + (4 \\ &*\text{ArcSin}[\sqrt{1 - \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2})*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]}) \\ &/c*\sqrt{d}])/ \sqrt{d} + (\text{Pi}*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]}) \\ &/c*\sqrt{d}])/ \sqrt{d} - ((2*I)*\text{ArcCsch}[c*x]*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]}) \\ &/c*\sqrt{d}])]/ \sqrt{d} - (4*\text{ArcSin}[\sqrt{1 - \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2})*\text{Log}[1 - \\ &(I*(\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]})/(c*\sqrt{d})]])/ \sqrt{d} - ( \\ &\text{Pi}*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]})/(c*\sqrt{d})]])/ \\ &\sqrt{d} + ((2*I)*\text{ArcCsch}[c*x]*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]}) \\ &/c*\sqrt{d}])]/ \sqrt{d} + (4*\text{ArcSin}[\sqrt{1 + \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2})*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]}) \\ &/c*\sqrt{d}])]/ \sqrt{d} - (\text{Pi}*\text{Log}[\sqrt{e} - (I*\sqrt{d})/x])/ \sqrt{d} + (\text{Pi}*\text{Log}[\sqrt{e} \\ &+ (I*\sqrt{d})/x])/ \sqrt{d} + ((2*I)*\sqrt{e}*\text{Log}[(2*\sqrt{d}*\sqrt{e}*(I*\sqrt{e} \\ &+ c*(c*\sqrt{d} + I*\sqrt{-(c^2*d) + e})*\sqrt{1 + 1/(c^2*x^2)})*x])/(\sqrt{-(c^2*d) + e}*(I*\sqrt{d} + \sqrt{e}*x)))/(\sqrt{d}*\sqrt{-(c^2*d) + e}) - (( \\ &2*I)*\sqrt{e}*\text{Log}[(-2*\sqrt{d}*\sqrt{e}*(\sqrt{e} + c*(I*c*\sqrt{d} + \sqrt{-(c^2 \\ &*\sqrt{d} + \sqrt{e})*\sqrt{1 + 1/(c^2*x^2)})*x))/(\sqrt{-(c^2*d) + e}*(\sqrt{d} + I*\sqrt{e} \\ &]*x)))/(\sqrt{d}*\sqrt{-(c^2*d) + e}) - ((2*I)*\text{PolyLog}[2, ((-I)*(-\sqrt{e} + \\ &\sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]})/(c*\sqrt{d})]])/ \sqrt{d} + ((2*I)*\text{PolyLog}[ \\ &2, (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]})/(c*\sqrt{d})]])/ \sqrt{d} \\ &+ ((2*I)*\text{PolyLog}[2, ((-I)*(\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]})/( \\ &c*\sqrt{d})]])/ \sqrt{d} - ((2*I)*\text{PolyLog}[2, (I*(\sqrt{e} + \sqrt{-(c^2*d) + e})*E^{\text{ArcCsch}[c*x]})/( \\ &c*\sqrt{d})]])/ \sqrt{d}))/ (8*e^{(3/2)}) \end{aligned}$$

**fricas** [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arcsch}(cx) + ax^2}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsch(c\*x) + a\*x^2)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2/(e\*x^2 + d)^2, x)

**maple** [F] time = 3.51, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

[Out] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a \left( \frac{x}{e^2x^2 + de} - \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e} \right) + b \int \frac{x^2 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^2x^4 + 2dex^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2\*a\*(x/(e^2\*x^2 + d\*e) - arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*e)) + b\*integrate(x^2\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.109 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex^2)^2} dx$$

Optimal. Leaf size=713

$$\frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}} + 1\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

[Out]  $\frac{1}{4}b \operatorname{arctanh}\left(\frac{c^2d - (-d)^{1/2}e^{1/2}/x}{c/d^{1/2}/(c^2d - e)^{1/2}/(1 + 1/c^2/x^2)^{1/2}}\right)/d^{3/2}/(c^2d - e)^{1/2} + \frac{1}{4}b \operatorname{arctanh}\left(\frac{c^2d + (-d)^{1/2}e^{1/2}/x}{c/d^{1/2}/(c^2d - e)^{1/2}/(1 + 1/c^2/x^2)^{1/2}}\right)/d^{3/2}/(c^2d - e)^{1/2} - \frac{1}{4}(a + b \operatorname{arccsch}(cx)) \ln\left(\frac{1 - c(1/c/x + (1 + 1/c^2/x^2)^{1/2})(-d)^{1/2}/(e^{1/2} - (-c^2d + e)^{1/2})}{(-d)^{3/2}/e^{1/2} + 1}\right) + \frac{1}{4}(a + b \operatorname{arccsch}(cx)) \ln\left(\frac{1 + c(1/c/x + (1 + 1/c^2/x^2)^{1/2})(-d)^{1/2}/(e^{1/2} - (-c^2d + e)^{1/2})}{(-d)^{3/2}/e^{1/2} - 1}\right) + \frac{1}{4}(a + b \operatorname{arccsch}(cx)) \ln\left(\frac{1 - c(1/c/x + (1 + 1/c^2/x^2)^{1/2})(-d)^{1/2}/(e^{1/2} + (-c^2d + e)^{1/2})}{(-d)^{3/2}/e^{1/2} + 1}\right) + \frac{1}{4}(a + b \operatorname{arccsch}(cx)) \ln\left(\frac{1 + c(1/c/x + (1 + 1/c^2/x^2)^{1/2})(-d)^{1/2}/(e^{1/2} + (-c^2d + e)^{1/2})}{(-d)^{3/2}/e^{1/2} - 1}\right) + \frac{1}{4}b \operatorname{polylog}\left(2, \frac{-c(1/c/x + (1 + 1/c^2/x^2)^{1/2})(-d)^{1/2}/(e^{1/2} - (-c^2d + e)^{1/2})}{(-d)^{3/2}/e^{1/2} - 1}\right) - \frac{1}{4}b \operatorname{polylog}\left(2, \frac{c(1/c/x + (1 + 1/c^2/x^2)^{1/2})(-d)^{1/2}/(e^{1/2} - (-c^2d + e)^{1/2})}{(-d)^{3/2}/e^{1/2} - 1}\right) + \frac{1}{4}b \operatorname{polylog}\left(2, \frac{-c(1/c/x + (1 + 1/c^2/x^2)^{1/2})(-d)^{1/2}/(e^{1/2} + (-c^2d + e)^{1/2})}{(-d)^{3/2}/e^{1/2} + 1}\right) - \frac{1}{4}b \operatorname{polylog}\left(2, \frac{c(1/c/x + (1 + 1/c^2/x^2)^{1/2})(-d)^{1/2}/(e^{1/2} + (-c^2d + e)^{1/2})}{(-d)^{3/2}/e^{1/2} + 1}\right) + \frac{1}{4}(-a - b \operatorname{arccsch}(cx))/d/(-d/x + (-d)^{1/2}e^{1/2}) + \frac{1}{4}(a + b \operatorname{arccsch}(cx))/d/(d/x + (-d)^{1/2}e^{1/2})$

**Rubi [A]** time = 2.19, antiderivative size = 713, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {6294, 5791, 5706, 5801, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2d} + \sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2d} + \sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b \operatorname{ArcCsch}[c*x])/(d + e*x^2)^2, x]$

[Out]  $-\frac{(a + b \operatorname{ArcCsch}[c*x])}{(4*d*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x))} + \frac{(a + b \operatorname{ArcCsch}[c*x])}{(4*d*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x))} + \frac{(b \operatorname{ArcTanh}[(c^2*d - (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x])/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]))}{(4*d^{3/2}* \operatorname{Sqrt}[c^2*d - e])} + \frac{(b \operatorname{ArcTanh}[(c^2*d + (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x])/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]))}{(4*d^{3/2}* \operatorname{Sqrt}[c^2*d - e])} - \frac{((a + b \operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])}{(4*(-d)^{3/2}* \operatorname{Sqrt}[e])} + \frac{((a + b \operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])}{(4*(-d)^{3/2}* \operatorname{Sqrt}[e])} - \frac{((a + b \operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])}{(4*(-d)^{3/2}* \operatorname{Sqrt}[e])} + \frac{((a + b \operatorname{ArcCsch}[c*x])* \operatorname{Log}[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])}{(4*(-d)^{3/2}* \operatorname{Sqrt}[e])} + \frac{(b \operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])]}{(4*(-d)^{3/2}* \operatorname{Sqrt}[e])} - \frac{(b \operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])]}{(4*(-d)^{3/2}* \operatorname{Sqrt}[e])} + \frac{(b \operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])]}{(4*(-d)^{3/2}* \operatorname{Sqrt}[e])} - \frac{(b \operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])]}{(4*(-d)^{3/2}* \operatorname{Sqrt}[e])}$

$-(b \cdot \text{PolyLog}[2, (c \cdot \sqrt{-d}) \cdot E^{\text{ArcCsch}[c \cdot x]}]) / (\sqrt{e} + \sqrt{-(c^2 \cdot d) + e}) / (4 \cdot (-d)^{3/2} \cdot \sqrt{e})$

#### Rule 206

$\text{Int}[(a + (b \cdot x^2)^{-1}), x\_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

#### Rule 725

$\text{Int}[1/((d + (e \cdot x) \cdot \sqrt{(a + (c \cdot x)^2})]), x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c \cdot d^2 + a \cdot e^2 - x^2), x], x, (a \cdot e - c \cdot d \cdot x) / \sqrt{a + c \cdot x^2}] /; \text{FreeQ}\{a, c, d, e\}, x]$

#### Rule 2190

$\text{Int}[(F^{(g \cdot (e + f \cdot x))})^{(n \cdot (c + d \cdot x)^{m})} / ((a + b \cdot (F^{(g \cdot (e + f \cdot x))})^{(n)})], x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x))})^n) / a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x))})^n) / a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \ \text{IGtQ}[m, 0]$

#### Rule 2279

$\text{Int}[\text{Log}[(a + b \cdot (F^{(e \cdot (c + d \cdot x))})^{(n)})], x\_Symbol] \rightarrow \text{Dist}[1/(d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \text{GtQ}[a, 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c \cdot (d + (e \cdot x)^{n}))] / (x), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)] / n, x] /; \text{FreeQ}\{c, d, e, n\}, x \ \&\& \ \text{EqQ}[c \cdot d, 1]$

#### Rule 5561

$\text{Int}[(\text{Cosh}[(c + d \cdot x)] \cdot (e + f \cdot x)^{m}) / ((a + b \cdot \text{Sinh}[(c + d \cdot x)]), x\_Symbol] \rightarrow -\text{Simp}[(e + f \cdot x)^{m+1} / (b \cdot f \cdot (m+1)), x] + (\text{Int}[(e + f \cdot x)^m \cdot E^{(c + d \cdot x)} / (a - \text{Rt}[a^2 + b^2, 2] + b \cdot E^{(c + d \cdot x)}), x] + \text{Int}[(e + f \cdot x)^m \cdot E^{(c + d \cdot x)} / (a + \text{Rt}[a^2 + b^2, 2] + b \cdot E^{(c + d \cdot x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

#### Rule 5706

$\text{Int}[(a + \text{ArcSinh}[(c \cdot x)] \cdot (b \cdot x)^{n}) \cdot ((d + (e \cdot x)^2)^{p}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSinh}[c \cdot x])^n, (d + e \cdot x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{NeQ}[e, c^2 \cdot d] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (p > 0 \ || \ \text{IGtQ}[n, 0])$

#### Rule 5791

$\text{Int}[(a + \text{ArcSinh}[(c \cdot x)] \cdot (b \cdot x)^{n}) \cdot ((f \cdot x)^{m}) \cdot ((d + (e \cdot x)^2)^{p}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSinh}[c \cdot x])^n, (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[e, c^2 \cdot d] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$

#### Rule 5799

$\text{Int}[(a + \text{ArcSinh}[(c \cdot x)] \cdot (b \cdot x)^{n}) / ((d + (e \cdot x)^2), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b \cdot x)^n \cdot \text{Cosh}[x] / (c \cdot d + e \cdot \text{Sinh}[x]), x], x, \text{ArcSinh}[c \cdot x]$



]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 5801

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcSinh[c\*x])^n)/(e\*(m + 1)), x] - Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcSinh[c\*x])^(n - 1))/Sqrt[1 + c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 6294

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Subst[Int[(e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n]/x^(2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]

#### Rubi steps



**Mathematica [C]** time = 3.65, size = 1437, normalized size = 2.02

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x^2)^2, x]

[Out] ((a\*x)/(d^2 + d\*e\*x^2) + (a\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(d^(3/2)\*Sqrt[e]) + (b\*((2\*Sqrt[d]\*ArcCsch[c\*x])/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x) + (2\*Sqrt[d]\*ArcCsch[c\*x])/(I\*Sqrt[d]\*Sqrt[e] + e\*x) + ((8\*I)\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] - Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]]/Sqrt[e] + ((8\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]]/Sqrt[e] - (Pi\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] + ((2\*I)\*ArcCsch[c\*x]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] - (4\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] + (Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] - ((2\*I)\*ArcCsch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] + (4\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] + (Pi\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] - ((2\*I)\*ArcCsch[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] - (4\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] - (Pi\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] + ((2\*I)\*ArcCsch[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] + (4\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] - (Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x])/Sqrt[e] + (Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x])/Sqrt[e] - ((2\*I)\*Log[(2\*Sqrt[d]\*Sqrt[e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] + I\*Sqrt[-(c^2\*d) + e])\*Sqrt[1 + 1/(c^2\*x^2)])\*x)/(Sqrt[-(c^2\*d) + e]\*(I\*Sqrt[d] + Sqrt[e]\*x)))/Sqrt[-(c^2\*d) + e] + ((2\*I)\*Log[(-2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) + e])\*Sqrt[1 + 1/(c^2\*x^2)])\*x)/(Sqrt[-(c^2\*d) + e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))/Sqrt[-(c^2\*d) + e] - ((2\*I)\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] + ((2\*I)\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] + ((2\*I)\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e] - ((2\*I)\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e]))\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]/Sqrt[e]))/(4\*d^(3/2))/2

**fricas [F]** time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{e^2 x^4 + 2 d e x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^2, x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x^2 + d)^2, x)

**maple** [F] time = 4.65, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

[Out] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{x}{dex^2 + d^2} + \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}d} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^2x^4 + 2dex^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2\*a\*(x/(d\*e\*x^2 + d^2) + arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(d + e\*x^2)^2,x)

[Out] int((a + b\*asinh(1/(c\*x)))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.110 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex^2)^2} dx$$

**Optimal.** Leaf size=758

$$\frac{3\sqrt{e} (a + b\operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}} + \frac{3\sqrt{e} (a + b\operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}} + 1\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e} (a + b\operatorname{csch}^{-1}(cx))}{4(-d)^{5/2}}$$

[Out]  $-a/d^2/x - b*\operatorname{arccsch}(c*x)/d^2/x - 1/4*b*e*\operatorname{arctanh}((c^2*d - (-d)^{(1/2)}*e^{(1/2)})/x)/c/d^{(1/2)}/(c^2*d - e)^{(1/2)}/(1 + 1/c^2/x^2)^{(1/2)}/d^{(5/2)}/(c^2*d - e)^{(1/2)} - 1/4*b*e*\operatorname{arctanh}((c^2*d + (-d)^{(1/2)}*e^{(1/2)})/x)/c/d^{(1/2)}/(c^2*d - e)^{(1/2)}/(1 + 1/c^2/x^2)^{(1/2)}/d^{(5/2)}/(c^2*d - e)^{(1/2)} - 3/4*(a + b*\operatorname{arccsch}(c*x))*\ln(1 - c*(1/c/x + (1 + 1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} - (-c^2*d + e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 3/4*(a + b*\operatorname{arccsch}(c*x))*\ln(1 + c*(1/c/x + (1 + 1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} - (-c^2*d + e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} - 3/4*(a + b*\operatorname{arccsch}(c*x))*\ln(1 - c*(1/c/x + (1 + 1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} + (-c^2*d + e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 3/4*(a + b*\operatorname{arccsch}(c*x))*\ln(1 + c*(1/c/x + (1 + 1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} + (-c^2*d + e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 3/4*b*\operatorname{polylog}(2, -c*(1/c/x + (1 + 1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} - (-c^2*d + e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} - 3/4*b*\operatorname{polylog}(2, c*(1/c/x + (1 + 1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} - (-c^2*d + e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 3/4*b*\operatorname{polylog}(2, -c*(1/c/x + (1 + 1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} + (-c^2*d + e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} - 3/4*b*\operatorname{polylog}(2, c*(1/c/x + (1 + 1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)} + (-c^2*d + e)^{(1/2)}))*e^{(1/2)}/(-d)^{(5/2)} + 1/4*e*(a + b*\operatorname{arccsch}(c*x))/d^2/(-d/x + (-d)^{(1/2)}*e^{(1/2)}) - 1/4*e*(a + b*\operatorname{arccsch}(c*x))/d^2/(d/x + (-d)^{(1/2)}*e^{(1/2)}) + b*c*(1 + 1/c^2/x^2)^{(1/2)}/d^2$

**Rubi [A]** time = 2.25, antiderivative size = 758, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {6304, 5791, 5653, 261, 5706, 5801, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$\frac{3b\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}} + \frac{3b\sqrt{e} \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{5/2}} - \frac{3b\sqrt{e} \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x^2)^2), x]

[Out]  $(b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*\operatorname{ArcCsch}[c*x])/(d^2*x) + (e*(a + b*\operatorname{ArcCsch}[c*x]))/(4*d^2*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)) - (e*(a + b*\operatorname{ArcCsch}[c*x]))/(4*d^2*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)) - (b*e*\operatorname{ArcTanh}[(c^2*d - (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x)/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])])/(4*d^{(5/2)}*\operatorname{Sqrt}[c^2*d - e]) - (b*e*\operatorname{ArcTanh}[(c^2*d + (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x)/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])])/(4*d^{(5/2)}*\operatorname{Sqrt}[c^2*d - e]) - (3*\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(4*(-d)^{(5/2)}) + (3*\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(4*(-d)^{(5/2)}) - (3*\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(4*(-d)^{(5/2)}) + (3*\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcCsch}[c*x])*Log[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(4*(-d)^{(5/2)}) + (3*b*\operatorname{Sqrt}[e]*\operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])]/(4*(-d)^{(5/2)}) - (3*b*\operatorname{Sqrt}[e]*\operatorname{PolyLog}[2, (c*S$

$$\frac{\sqrt{-d} * E^{\text{ArcCsch}[c*x]} / (\sqrt{e} - \sqrt{-(c^2*d) + e})}{(4*(-d)^{5/2})} + (3*b*\sqrt{e}*\text{PolyLog}[2, -(c*\sqrt{-d} * E^{\text{ArcCsch}[c*x]} / (\sqrt{e} + \sqrt{-(c^2*d) + e}))]) / (4*(-d)^{5/2}) - (3*b*\sqrt{e}*\text{PolyLog}[2, (c*\sqrt{-d} * E^{\text{ArcCsch}[c*x]} / (\sqrt{e} + \sqrt{-(c^2*d) + e}))]) / (4*(-d)^{5/2})$$
Rule 206

$$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$$
Rule 261

$$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow \text{Simp}[(a + b*x^n)^{(p+1)} / (b*n*(p+1)), x] /; \text{FreeQ}\{a, b, m, n, p\}, x \ \&\& \ \text{EqQ}[m, n-1] \ \&\& \ \text{NeQ}[p, -1]$$
Rule 725

$$\text{Int}[1/((d_) + (e_)*(x_))*\sqrt{(a_) + (c_)*(x_)^2}], x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\sqrt{a + c*x^2}] /; \text{FreeQ}\{a, c, d, e\}, x]$$
Rule 2190

$$\text{Int}[(F_)^{((g_)*((e_) + (f_)*(x_)))^{(n_)}*((c_) + (d_)*(x_))^{(m_)}} / ((a_) + (b_)*((F_)^{((g_)*((e_) + (f_)*(x_)))^{(n_)}})), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]] / (b*f*g*n * \text{Log}[F]), x] - \text{Dist}[(d*m) / (b*f*g*n * \text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \ \text{IGtQ}[m, 0]$$
Rule 2279

$$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))^{(n_)}})], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n * \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \ \text{GtQ}[a, 0]$$
Rule 2391

$$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x \ \&\& \ \text{EqQ}[c*d, 1]$$
Rule 5561

$$\text{Int}[(\text{Cosh}[(c_)] + (d_)*(x_))*((e_) + (f_)*(x_))^{(m_)} / ((a_) + (b_)*\text{Sinh}[(c_)] + (d_)*(x_)), x\_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{(m+1)} / (b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m * E^{(c + d*x)} / (a - \text{Rt}[a^2 + b^2, 2] + b * E^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m * E^{(c + d*x)} / (a + \text{Rt}[a^2 + b^2, 2] + b * E^{(c + d*x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$$
Rule 5653

$$\text{Int}[(a_) + \text{ArcSinh}[(c_)*(x_)]*(b_))^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcSinh}[c*x])^n, x] - \text{Dist}[b*c*n, \text{Int}[(x*(a + b*\text{ArcSinh}[c*x])^{(n-1)})/\sqrt{1 + c^2*x^2}], x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{GtQ}[n, 0]$$
Rule 5706

$$\text{Int}[(a_) + \text{ArcSinh}[(c_)*(x_)]*(b_))^{(n_)}*((d_) + (e_)*(x_)^2)^{(p_)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcSinh}[c*x])^n, (d + e*x^2)^p], x],$$

$x]$  /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2\*d] && IntegerQ[p] && (p > 0 || IGtQ[n, 0])

#### Rule 5791

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^ (n\_.)\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*ArcSinh[c\*x])^n, (f\*x)^m\*(d + e\*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^2\*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

#### Rule 5799

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^ (n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Subst[Int[(a + b\*x)^n\*Cosh[x]]/(c\*d + e\*Sinh[x]), x], x, ArcSinh[c\*x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

#### Rule 5801

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)])\*(b\_.))^ (n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*ArcSinh[c\*x])^n)/(e\*(m + 1)), x] - Dist[(b\*c\*n)/(e\*(m + 1)), Int[((d + e\*x)^(m + 1)\*(a + b\*ArcSinh[c\*x])^(n - 1))/Sqrt[1 + c^2\*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

#### Rule 6304

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.))^ (n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 (d + ex^2)^2} dx &= -\operatorname{Subst} \left( \int \frac{x^4 \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{d^2} + \frac{e^2 \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^2} - \frac{2e \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \operatorname{Subst} \left( \int \frac{1}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{a}{d^2 x} - \frac{b \operatorname{Subst} \left( \int \sinh^{-1} \left( \frac{x}{c} \right) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{a}{d^2 x} - \frac{b \operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{b \operatorname{Subst} \left( \int \frac{x}{\sqrt{1 + \frac{x^2}{c^2}}} dx, x, \frac{1}{x} \right)}{cd^2} + \frac{\sqrt{e} \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} \operatorname{Subst} \left( \int \frac{1}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \frac{\sqrt{e} \operatorname{Subst} \left( \int \frac{1}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \frac{\operatorname{be} \operatorname{tan}^{-1} \left( \frac{\sqrt{-d} \sqrt{e} - \frac{d}{x}}{\sqrt{e}} \right)}{d^2} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \frac{\operatorname{be} \operatorname{tan}^{-1} \left( \frac{\sqrt{-d} \sqrt{e} - \frac{d}{x}}{\sqrt{e}} \right)}{d^2} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \frac{\operatorname{be} \operatorname{tan}^{-1} \left( \frac{\sqrt{-d} \sqrt{e} - \frac{d}{x}}{\sqrt{e}} \right)}{d^2} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \frac{\operatorname{be} \operatorname{tan}^{-1} \left( \frac{\sqrt{-d} \sqrt{e} - \frac{d}{x}}{\sqrt{e}} \right)}{d^2} \\
&= \frac{bc\sqrt{1 + \frac{1}{c^2 x^2}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{csch}^{-1}(cx)}{d^2 x} + \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e(a + b \operatorname{csch}^{-1}(cx))}{4d^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \frac{\operatorname{be} \operatorname{tan}^{-1} \left( \frac{\sqrt{-d} \sqrt{e} - \frac{d}{x}}{\sqrt{e}} \right)}{d^2}
\end{aligned}$$



**Mathematica** [C] time = 3.12, size = 1487, normalized size = 1.96

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x^2)^2), x]

[Out] 
$$\begin{aligned} &((-8*a*\sqrt{d})/x - (4*a*\sqrt{d}*e*x)/(d + e*x^2) - 12*a*\sqrt{e}*\text{ArcTan}[(\sqrt{e}*x)/\sqrt{d}] + b*(8*c*\sqrt{d}*\sqrt{1 + 1/(c^2*x^2)} - (8*\sqrt{d}*\text{ArcCsch}[c*x])/x - (2*\sqrt{d}*e*\text{ArcCsch}[c*x])/((-I)*\sqrt{d}*\sqrt{e} + e*x) - (2*\sqrt{d}*e*\text{ArcCsch}[c*x])/(I*\sqrt{d}*\sqrt{e} + e*x) - (24*I)*\sqrt{e}*\text{ArcSin}[\sqrt{1 + \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2}]*\text{ArcTan}[(c*\sqrt{d} - \sqrt{e})*\text{Cot}[(\text{Pi} + (2*I)*\text{ArcCsch}[c*x])/4]]/\sqrt{-(c^2*d) + e}] - (24*I)*\sqrt{e}*\text{ArcSin}[\sqrt{1 - \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2}]*\text{ArcTan}[(c*\sqrt{d} + \sqrt{e})*\text{Cot}[(\text{Pi} + (2*I)*\text{ArcCsch}[c*x])/4]]/\sqrt{-(c^2*d) + e}] + 3*\sqrt{e}*Pi*\text{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] - (6*I)*\sqrt{e}*\text{ArcCsch}[c*x]*\text{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] + 12*\sqrt{e}*\text{ArcSin}[\sqrt{1 + \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 - (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] - 3*\sqrt{e}*Pi*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] + (6*I)*\sqrt{e}*\text{ArcCsch}[c*x]*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] - 12*\sqrt{e}*\text{ArcSin}[\sqrt{1 - \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] - 3*\sqrt{e}*Pi*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] + (6*I)*\sqrt{e}*\text{ArcCsch}[c*x]*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] + 12*\sqrt{e}*\text{ArcSin}[\sqrt{1 - \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] + 3*\sqrt{e}*Pi*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] - (6*I)*\sqrt{e}*\text{ArcCsch}[c*x]*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] - 12*\sqrt{e}*\text{ArcSin}[\sqrt{1 + \sqrt{e}/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] + 3*\sqrt{e}*Pi*\text{Log}[\sqrt{e} - (I*\sqrt{d})/x] - 3*\sqrt{e}*Pi*\text{Log}[\sqrt{e} + (I*\sqrt{d})/x] + ((2*I)*e*\text{Log}[(2*\sqrt{d})*\sqrt{e}*(I*\sqrt{e} + c*(c*\sqrt{d} + I*\sqrt{-(c^2*d) + e})*\sqrt{1 + 1/(c^2*x^2)})*x]/(\sqrt{-(c^2*d) + e}*(I*\sqrt{d} + \sqrt{e}*x)))/\sqrt{-(c^2*d) + e} - ((2*I)*e*\text{Log}[(-2*\sqrt{d})*\sqrt{e}*(\sqrt{e} + c*(I*c*\sqrt{d} + \sqrt{-(c^2*d) + e})*\sqrt{1 + 1/(c^2*x^2)})*x]/(\sqrt{-(c^2*d) + e}*(\sqrt{d} + I*\sqrt{e}*x)))/\sqrt{-(c^2*d) + e} + (6*I)*\sqrt{e}*PolyLog[2, ((-I)*(-\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] - (6*I)*\sqrt{e}*PolyLog[2, (I*(-\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] - (6*I)*\sqrt{e}*PolyLog[2, ((-I)*(\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})] + (6*I)*\sqrt{e}*PolyLog[2, (I*(\sqrt{e} + \sqrt{-(c^2*d) + e}))*E^{\text{ArcCsch}[c*x]}/(c*\sqrt{d})])]/(8*d^{(5/2)}) \end{aligned}$$

**fricas** [F] time = 2.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{e^2 x^6 + 2 d e x^4 + d^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e^2\*x^6 + 2\*d\*e\*x^4 + d^2\*x^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)^2\*x^2), x)

**maple** [F] time = 14.66, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 (e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^2,x)

[Out] int((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^2,x)

**maxima** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation \*may\* help (example of legal syntax is 'assume(e-c^2\*d>0)', see `assume?` for more details)Is e-c^2\*d positive or negative?

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2 (e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x^2)^2), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x^2)^2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*2/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.111 \quad \int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^3} dx$$

**Optimal.** Leaf size=694

$$\frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2e^3} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}} + 1\right)}{2e^3} + \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2e^3}$$

```
[Out] 1/4*(-a-b*arccsch(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arccsch(c*x))/e^2/(e+d/x^2)
-(a+b*arccsch(c*x))^2/b/e^3+1/8*b*(c^2*d-2*e)*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))/(c^2*d-e)^(3/2)/e^(5/2)-(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^3+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^3+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d-e)^(1/2)+1/8*b*c*d*(1+1/c^2/x^2)^(1/2)/(c^2*d-e)/e^2/(e+d/x^2)/x
```

**Rubi [A]** time = 1.41, antiderivative size = 676, normalized size of antiderivative = 0.97, number of steps used = 33, number of rules used = 13, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$ , Rules used = {6304, 5791, 5659, 3716, 2190, 2279, 2391, 5787, 382, 377, 205, 5799, 5561}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e} - \sqrt{e - c^2 d}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d} e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2 d} + \sqrt{e}}\right)}{2e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3, x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(8*(c^2*d - e)*e^2*(e + d/x^2)*x) - (a + b*ArcCsch[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcCsch[c*x])/(2*e^2*(e + d/x^2)) + (b*(c^2*d - 2*e)*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)]/(8*(c^2*d - e)^(3/2)*e^(5/2)) + (b*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)]/(2*Sqrt[c^2*d - e]*e^(5/2)) + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3) - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])]/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])]/(2*e^3) + (b*PolyLog[2, ((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])]/(2*e^3)
```

$$\frac{-(c^2d + e)}}{(2e^3) + (b \text{PolyLog}[2, (c \sqrt{-d} E^{\text{ArcCsch}[c x]}) / (\sqrt{e} + \sqrt{-(c^2d + e)})]) / (2e^3) - (b \text{PolyLog}[2, E^{(2 \text{ArcCsch}[c x])})]) / (2e^3)}$$

### Rule 205

$$\text{Int}[(a + (b \cdot x)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$$

### Rule 377

$$\text{Int}[(a + (b \cdot x)^n)^p / ((c + (d \cdot x)^n)), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b \cdot c - a \cdot d) \cdot x^n), x], x, x/(a + b \cdot x^n)^{1/n}] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[n \cdot p + 1, 0] \ \&\& \ \text{IntegerQ}[n]$$

### Rule 382

$$\text{Int}[(a + (b \cdot x)^n)^p \cdot ((c + (d \cdot x)^n)^q), x\_Symbol] \rightarrow -\text{Simp}[(b \cdot x \cdot (a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^{q+1}) / (a \cdot n \cdot (p+1) \cdot (b \cdot c - a \cdot d)), x] + \text{Dist}[(b \cdot c + n \cdot (p+1) \cdot (b \cdot c - a \cdot d)) / (a \cdot n \cdot (p+1) \cdot (b \cdot c - a \cdot d)), \text{Int}[(a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, n, q\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[n \cdot (p+q+2) + 1, 0] \ \&\& \ (\text{LtQ}[p, -1] \ || \ !\text{LtQ}[q, -1]) \ \&\& \ \text{NeQ}[p, -1]$$

### Rule 2190

$$\text{Int}[(F^{(g \cdot (e + f \cdot x))})^{n \cdot ((c + (d \cdot x)^m))} / ((a + (b \cdot x)^n) \cdot (F^{(g \cdot (e + f \cdot x))})^{n \cdot}), x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x))})^n) / a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x))})^n) / a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$$

### Rule 2279

$$\text{Int}[\text{Log}[a + (b \cdot x)^n \cdot (F^{(e \cdot ((c + (d \cdot x)^n))})^n)], x\_Symbol] \rightarrow \text{Dist}[1/(d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$$

### Rule 2391

$$\text{Int}[\text{Log}[(c + (d \cdot x)^n) \cdot (e + f \cdot x)^n] / (x), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)] / n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c \cdot d, 1]$$

### Rule 3716

$$\text{Int}[(c + (d \cdot x)^m) \cdot \tan[(e + \text{Pi} \cdot (k + \text{Complex}[0, fz]) \cdot (f \cdot x))], x\_Symbol] \rightarrow -\text{Simp}[(I \cdot (c + d \cdot x)^{m+1}) / (d \cdot (m+1)), x] + \text{Dist}[2 \cdot I, \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))} / (E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot (1 + E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))} / E^{(2 \cdot I \cdot k \cdot \text{Pi})})], x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \text{IntegerQ}[4 \cdot k] \ \&\& \ \text{IGtQ}[m, 0]$$

### Rule 5561

$$\text{Int}[(\text{Cosh}[(c + (d \cdot x)^n]) \cdot ((e + f \cdot x)^m)) / ((a + (b \cdot x)^n) \cdot \text{Sinh}[(c + (d \cdot x)^n]), x\_Symbol] \rightarrow -\text{Simp}[(e + f \cdot x)^{m+1} / (b \cdot f \cdot (m+1)), x] + (\text{Int}[(e + f \cdot x)^m \cdot E^{(c + d \cdot x)}] / (a - \text{Rt}[a^2 + b^2, 2] + b \cdot E^{(c + d \cdot x)}), x] + \text{Int}[(e + f \cdot x)^m \cdot E^{(c + d \cdot x)}] / (a + \text{Rt}[a^2 + b^2, 2] + b \cdot E^{(c + d \cdot x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$$

### Rule 5659

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[
(a + b*x)^n/Tanh[x], x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
0]
```

#### Rule 5787

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSinh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

#### Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
(f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

#### Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] := Subst[Int[(a + b*x)^n*Cosh[x]/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^3} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x (e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{e^3 x} - \frac{dx (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e (e + dx^2)^3} - \frac{dx (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e^2 (e + dx^2)^2} - \frac{dx (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e^3} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^3} + \frac{d \operatorname{Subst} \left( \int \frac{x (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} + \frac{d \operatorname{Subst} \left( \int \frac{dx (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{e^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left( \int (a + bx) \coth(x) dx, x, \operatorname{csch}^{-1}(cx) \right)}{e^3} \\
&= \frac{bcd \sqrt{1 + \frac{1}{c^2 x^2}}}{8(c^2 d - e) e^2 \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2be^3} \\
&= \frac{bcd \sqrt{1 + \frac{1}{c^2 x^2}}}{8(c^2 d - e) e^2 \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{csch}^{-1}(cx))^2}{2be^3} \\
&= \frac{bcd \sqrt{1 + \frac{1}{c^2 x^2}}}{8(c^2 d - e) e^2 \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b(c^2 d - 2e) \tan^{-1} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{e} \right)}{8(c^2 d - e)} \\
&= \frac{bcd \sqrt{1 + \frac{1}{c^2 x^2}}}{8(c^2 d - e) e^2 \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b(c^2 d - 2e) \tan^{-1} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{e} \right)}{8(c^2 d - e)} \\
&= \frac{bcd \sqrt{1 + \frac{1}{c^2 x^2}}}{8(c^2 d - e) e^2 \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b(c^2 d - 2e) \tan^{-1} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{e} \right)}{8(c^2 d - e)} \\
&= \frac{bcd \sqrt{1 + \frac{1}{c^2 x^2}}}{8(c^2 d - e) e^2 \left( e + \frac{d}{x^2} \right) x} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{csch}^{-1}(cx)}{2e^2 \left( e + \frac{d}{x^2} \right)} + \frac{b(c^2 d - 2e) \tan^{-1} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{e} \right)}{8(c^2 d - e)}
\end{aligned}$$

**Mathematica [C]** time = 7.85, size = 2023, normalized size = 2.91

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^3,x]

[Out] 
$$-1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*\text{Log}[d + e*x^2])/(2*e^3) + b*(-1/16*(d*((I*c*\text{Sqrt}[e]*\text{Sqrt}[1 + 1/(c^2*x^2)])*x)/(\text{Sqrt}[d]*(c^2*d - e)*((-I)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)) - \text{ArcCsch}[c*x]/(\text{Sqrt}[e]*((-I)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)^2) - \text{ArcSinh}[1/(c*x)]/(d*\text{Sqrt}[e]) + (I*(2*c^2*d - e)*\text{Log}[(4*d*\text{Sqrt}[c^2*d - e]*\text{Sqrt}[e]*(\text{Sqrt}[e] + I*c*(c*\text{Sqrt}[d] - \text{Sqrt}[c^2*d - e]*\text{Sqrt}[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(\text{Sqrt}[d] + I*\text{Sqrt}[e]*x)))]/(d*(c^2*d - e)^{(3/2)})))/e^{(5/2)} - (d*((-I)*c*\text{Sqrt}[e]*\text{Sqrt}[1 + 1/(c^2*x^2)])*x)/(\text{Sqrt}[d]*(c^2*d - e)*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)) - \text{ArcCsch}[c*x]/(\text{Sqrt}[e]*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)^2) - \text{ArcSinh}[1/(c*x)]/(d*\text{Sqrt}[e]) + (I*(2*c^2*d - e)*\text{Log}[(4*I*d*\text{Sqrt}[c^2*d - e]*\text{Sqrt}[e]*(I*\text{Sqrt}[e] + c*(c*\text{Sqrt}[d] + \text{Sqrt}[c^2*d - e]*\text{Sqrt}[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(\text{Sqrt}[d] - I*\text{Sqrt}[e]*x)))]/(d*(c^2*d - e)^{(3/2)})))/(16*e^{(5/2)}) - (((7*I)/16)*\text{Sqrt}[d]*(-\text{ArcCsch}[c*x]/(I*\text{Sqrt}[d]*\text{Sqrt}[e] + e*x)) - (I*(\text{ArcSinh}[1/(c*x)]/\text{Sqrt}[e] - \text{Log}[(2*\text{Sqrt}[d]*\text{Sqrt}[e]*(I*\text{Sqrt}[e] + c*(c*\text{Sqrt}[d] + I*\text{Sqrt}[-(c^2*d) + e]*\text{Sqrt}[1 + 1/(c^2*x^2)])*x))/(\text{Sqrt}[-(c^2*d) + e]*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)))/\text{Sqrt}[-(c^2*d) + e]))/\text{Sqrt}[d])/e^{(5/2)} + (((7*I)/16)*\text{Sqrt}[d]*(-\text{ArcCsch}[c*x]/((-I)*\text{Sqrt}[d]*\text{Sqrt}[e] + e*x)) + (I*(\text{ArcSinh}[1/(c*x)]/\text{Sqrt}[e] - \text{Log}[-2*\text{Sqrt}[d]*\text{Sqrt}[e]*(\text{Sqrt}[e] + c*(I*c*\text{Sqrt}[d] + \text{Sqrt}[-(c^2*d) + e]*\text{Sqrt}[1 + 1/(c^2*x^2)])*x))/(\text{Sqrt}[-(c^2*d) + e]*(\text{Sqrt}[d] + I*\text{Sqrt}[e]*x)))/\text{Sqrt}[-(c^2*d) + e]))/\text{Sqrt}[d])/e^{(5/2)} + (\text{Pi}^2 - (4*I)*\text{Pi}*\text{ArcCsch}[c*x] - 8*\text{ArcCsch}[c*x]^2 + 32*\text{ArcSin}[\text{Sqrt}[1 + \text{Sqrt}[e]/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2])* \text{ArcTan}[(c*\text{Sqrt}[d] - \text{Sqrt}[e])* \text{Cot}[(\text{Pi} + (2*I)*\text{ArcCsch}[c*x])/4]]/\text{Sqrt}[-(c^2*d) + e]) - 8*\text{ArcCsch}[c*x]*\text{Log}[1 - E^{-2*\text{ArcCsch}[c*x]}] + (4*I)*\text{Pi}*\text{Log}[1 - (I*(-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + 8*\text{ArcCsch}[c*x]*\text{Log}[1 - (I*(-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + (16*I)*\text{ArcSin}[\text{Sqrt}[1 + \text{Sqrt}[e]/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 - (I*(-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + (4*I)*\text{Pi}*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + 8*\text{ArcCsch}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] - (16*I)*\text{ArcSin}[\text{Sqrt}[1 + \text{Sqrt}[e]/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] - (4*I)*\text{Pi}*\text{Log}[\text{Sqrt}[e] + (I*\text{Sqrt}[d])/x] + 4*\text{PolyLog}[2, E^{-2*\text{ArcCsch}[c*x]}] + 8*\text{PolyLog}[2, (I*(-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + 8*\text{PolyLog}[2, ((-I)*( \text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])])]/(16*e^3) + (\text{Pi}^2 - (4*I)*\text{Pi}*\text{ArcCsch}[c*x] - 8*\text{ArcCsch}[c*x]^2 - 32*\text{ArcSin}[\text{Sqrt}[1 - \text{Sqrt}[e]/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2])* \text{ArcTan}[(c*\text{Sqrt}[d] + \text{Sqrt}[e])* \text{Cot}[(\text{Pi} + (2*I)*\text{ArcCsch}[c*x])/4]]/\text{Sqrt}[-(c^2*d) + e]) - 8*\text{ArcCsch}[c*x]*\text{Log}[1 - E^{-2*\text{ArcCsch}[c*x]}] + (4*I)*\text{Pi}*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + 8*\text{ArcCsch}[c*x]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + (16*I)*\text{ArcSin}[\text{Sqrt}[1 - \text{Sqrt}[e]/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + (4*I)*\text{Pi}*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + 8*\text{ArcCsch}[c*x]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] - (16*I)*\text{ArcSin}[\text{Sqrt}[1 - \text{Sqrt}[e]/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] - (4*I)*\text{Pi}*\text{Log}[\text{Sqrt}[e] - (I*\text{Sqrt}[d])/x] + 4*\text{PolyLog}[2, E^{-2*\text{ArcCsch}[c*x]}] + 8*\text{PolyLog}[2, ((-I)*(-\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])] + 8*\text{PolyLog}[2, (I*(\text{Sqrt}[e] + \text{Sqrt}[-(c^2*d) + e])*E^{\text{ArcCsch}[c*x]})/(c*\text{Sqrt}[d])])]/(16*e^3))$$

**fricas** [F] time = 2.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^5 \operatorname{arcsch}(cx) + ax^5}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*x^5\*arccsch(c\*x) + a\*x^5)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^5/(e\*x^2 + d)^3, x)

**maple** [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

[Out] int(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} a \left( \frac{4dex^2 + 3d^2}{e^5x^4 + 2de^4x^2 + d^2e^3} + \frac{2 \log(ex^2 + d)}{e^3} \right) + b \int \frac{x^5 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/4\*a\*((4\*d\*e\*x^2 + 3\*d^2)/(e^5\*x^4 + 2\*d\*e^4\*x^2 + d^2\*e^3) + 2\*log(e\*x^2 + d)/e^3) + b\*integrate(x^5\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out



$$3.112 \quad \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^3} dx$$

**Optimal.** Leaf size=167

$$\frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{4d (d + ex^2)^2} + \frac{bcx (c^2d - 2e) \tanh^{-1} \left( \frac{\sqrt{e} \sqrt{-c^2x^2 - 1}}{\sqrt{c^2d - e}} \right)}{8de^{3/2} \sqrt{-c^2x^2} (c^2d - e)^{3/2}} - \frac{bcx \sqrt{-c^2x^2 - 1}}{8e \sqrt{-c^2x^2} (c^2d - e) (d + ex^2)}$$

[Out] 1/4\*x^4\*(a+b\*arccsch(c\*x))/d/(e\*x^2+d)^2+1/8\*b\*c\*(c^2\*d-2\*e)\*x\*arctanh(e^(1/2)\*(-c^2\*x^2-1)^(1/2)/(c^2\*d-e)^(1/2))/d/(c^2\*d-e)^(3/2)/e^(3/2)/(-c^2\*x^2)^(1/2)-1/8\*b\*c\*x\*(-c^2\*x^2-1)^(1/2)/(c^2\*d-e)/e/(e\*x^2+d)/(-c^2\*x^2)^(1/2)

**Rubi [A]** time = 0.22, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {264, 6302, 12, 446, 78, 63, 208}

$$\frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{4d (d + ex^2)^2} + \frac{bcx (c^2d - 2e) \tanh^{-1} \left( \frac{\sqrt{e} \sqrt{-c^2x^2 - 1}}{\sqrt{c^2d - e}} \right)}{8de^{3/2} \sqrt{-c^2x^2} (c^2d - e)^{3/2}} - \frac{bcx \sqrt{-c^2x^2 - 1}}{8e \sqrt{-c^2x^2} (c^2d - e) (d + ex^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^3,x]

[Out] -(b\*c\*x\*Sqrt[-1 - c^2\*x^2])/(8\*(c^2\*d - e)\*e\*Sqrt[-(c^2\*x^2)]\*(d + e\*x^2)) + (x^4\*(a + b\*ArcCsch[c\*x]))/(4\*d\*(d + e\*x^2)^2) + (b\*c\*(c^2\*d - 2\*e)\*x\*ArcTanh[(Sqrt[e]\*Sqrt[-1 - c^2\*x^2])/Sqrt[c^2\*d - e]])/(8\*d\*(c^2\*d - e)^(3/2)\*e^(3/2)\*Sqrt[-(c^2\*x^2)])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || (EqQ[e, 0] || (EqQ[c, 0] || LtQ[p, n]))))

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 264

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

#### Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 6302

```
Int[((a_) + ArcCsch[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_)*((d_) + (e_)*(
x_)^2)^(p_)), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^3} dx &= \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(bcx) \int \frac{x^3}{4d \sqrt{-1 - c^2 x^2} (d + ex^2)^2} dx}{\sqrt{-c^2 x^2}} \\
&= \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(bcx) \int \frac{x^3}{\sqrt{-1 - c^2 x^2} (d + ex^2)^2} dx}{4d \sqrt{-c^2 x^2}} \\
&= \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{x}{\sqrt{-1 - c^2 x} (d + ex)^2} dx, x, x^2\right)}{8d \sqrt{-c^2 x^2}} \\
&= -\frac{bcx \sqrt{-1 - c^2 x^2}}{8 (c^2 d - e) e \sqrt{-c^2 x^2} (d + ex^2)} + \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{4d (d + ex^2)^2} - \frac{(bc (c^2 d - 2e) x) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-1 - c^2 x} (d + ex)^2} dx, x, x^2\right)}{16d (c^2 d - e) \sqrt{-c^2 x^2}} \\
&= -\frac{bcx \sqrt{-1 - c^2 x^2}}{8 (c^2 d - e) e \sqrt{-c^2 x^2} (d + ex^2)} + \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{4d (d + ex^2)^2} + \frac{(b (c^2 d - 2e) x) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-1 - c^2 x} (d + ex)^2} dx, x, x^2\right)}{8cd (c^2 d - e) \sqrt{-c^2 x^2}} \\
&= -\frac{bcx \sqrt{-1 - c^2 x^2}}{8 (c^2 d - e) e \sqrt{-c^2 x^2} (d + ex^2)} + \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{4d (d + ex^2)^2} + \frac{bc (c^2 d - 2e) x \tanh^{-1}\left(\frac{\sqrt{-1 - c^2 x}}{\sqrt{d + ex}}\right)}{8d (c^2 d - e)^{3/2} e^{3/2} \sqrt{-c^2 x^2}}
\end{aligned}$$

**Mathematica** [C] time = 1.48, size = 375, normalized size = 2.25

$$\frac{8a}{d+ex^2} - \frac{4ad}{(d+ex^2)^2} + \frac{b\sqrt{e}(2e-c^2d) \log\left(\frac{16de^{3/2}\sqrt{e-c^2d}\left(\sqrt{e+cx}\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d}-ic\sqrt{d}\right)\right)}{b(2e-c^2d)(\sqrt{e+ix}\sqrt{d})}\right)}{d(e-c^2d)^{3/2}} + \frac{b\sqrt{e}(2e-c^2d) \log\left(\frac{16ide^{3/2}\sqrt{e-c^2d}\left(\sqrt{e+cx}\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d}+ic\sqrt{d}\right)\right)}{b(c^2d-2e)(\sqrt{d+ix}\sqrt{e})}\right)}{d(e-c^2d)^{3/2}}$$

16e<sup>2</sup>

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^3,x]

[Out] 
$$-1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*b*c*e*\text{Sqrt}[1 + 1/(c^2*x^2)])*x)/((-c^2*d) + e)*(d + e*x^2) + (4*b*(d + 2*e*x^2)*\text{ArcCsch}[c*x])/(d + e*x^2)^2 - (4*b*\text{ArcSinh}[1/(c*x)])/d + (b*\text{Sqrt}[e]*(-c^2*d) + 2*e)*\text{Log}[(16*d*e^{3/2}*\text{Sqrt}[-(c^2*d) + e]*(\text{Sqrt}[e] + c*(-I)*c*\text{Sqrt}[d] + \text{Sqrt}[-(c^2*d) + e])*\text{Sqrt}[1 + 1/(c^2*x^2)])*x)/(b*(-c^2*d) + 2*e)*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)]/(d*(-c^2*d) + e)^{3/2} + (b*\text{Sqrt}[e]*(-c^2*d) + 2*e)*\text{Log}[((-16*I)*d*e^{3/2}*\text{Sqrt}[-(c^2*d) + e]*(\text{Sqrt}[e] + c*(I*c*\text{Sqrt}[d] + \text{Sqrt}[-(c^2*d) + e])*\text{Sqrt}[1 + 1/(c^2*x^2)])*x)/(b*(c^2*d - 2*e)*(\text{Sqrt}[d] + I*\text{Sqrt}[e]*x))]/(d*(-c^2*d) + e)^{3/2})/e^2$$

**fricas** [B] time = 3.05, size = 1381, normalized size = 8.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] 
$$[-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(b*c^2*d^2*e - 2*b*d*e^2)*x^2)*\text{sqrt}(-c^2*d*e + e^2)*\text{log}((c^2*e*x^2 - c^2*d - 2*\text{sqrt}(-c^2*d*e + e^2)*c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) + 2*e)/(e*x^2 + d)) - 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\text{log}(c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\text{log}(c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\text{log}((c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^2)*x)*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2))]/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 + 4*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(b*c^2*d^2*e - 2*b*d*e^2)*x^2)*\text{sqrt}(c^2*d*e - e^2)*\text{arctan}(-\text{sqrt}(c^2*d*e - e^2)*c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*d - e)) - 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\text{log}(c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\text{log}(c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*\text{log}((c*x*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + ((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^2)*x)*\text{sqrt}((c^2*x^2 + 1)/(c^2*x^2))]/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2)]$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^3/(e\*x^2 + d)^3, x)



$\log(c) + 4*d*e^3*\log(c) + d*e^3 - (8*d^2*e^2*\log(c) + d^2*e^2)*c^2*x^2 + (c^4*d^4 - 2*c^2*d^3*e + (c^4*d^2*e^2 - 2*c^2*d*e^3)*x^4 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2)*x^2)*\log(c^2*x^2 + 1) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2 + d*e^3)*x^2)*\log(\sqrt{c^2*x^2 + 1} + 1)/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2) + \log(e*x^2 + d)/(c^4*d^3 - 2*c^2*d^2*e + d*e^2) - 8*\integrate(1/4*(2*c^2*e*x^3 + c^2*d*x)/(c^2*e^4*x^6 + (2*c^2*d*e^3 + e^4)*x^4 + d^2*e^2 + (c^2*d^2*e^2 + 2*d*e^3)*x^2 + (c^2*e^4*x^6 + (2*c^2*d*e^3 + e^4)*x^4 + d^2*e^2 + (c^2*d^2*e^2 + 2*d*e^3)*x^2)*\sqrt{c^2*x^2 + 1}), x) - 1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(e x^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

$$3.113 \quad \int \frac{x(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^3} dx$$

**Optimal.** Leaf size=205

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e(d+ex^2)^2} + \frac{bcx \tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{4d^2e\sqrt{-c^2x^2}} + \frac{bcx(3c^2d-2e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8d^2\sqrt{e}\sqrt{-c^2x^2}(c^2d-e)^{3/2}} + \frac{bcx\sqrt{-c^2x^2-1}}{8d\sqrt{-c^2x^2}(c^2d-e)(d+ex^2)}$$

[Out]  $1/4*(-a-b*\operatorname{arccsch}(c*x))/e/(e*x^2+d)^2+1/4*b*c*x*\operatorname{arctan}((-c^2*x^2-1)^{(1/2)})/d^2/e/(-c^2*x^2)^{(1/2)}+1/8*b*c*(3*c^2*d-2*e)*x*\operatorname{arctanh}(e^{(1/2)}*(-c^2*x^2-1)^{(1/2)})/(c^2*d-e)^{(1/2)}/d^2/(c^2*d-e)^{(3/2)}/e^{(1/2)}/(-c^2*x^2)^{(1/2)}+1/8*b*c*x*(-c^2*x^2-1)^{(1/2)}/d/(c^2*d-e)/(e*x^2+d)/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.20, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {6300, 446, 103, 156, 63, 205, 208}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e(d+ex^2)^2} + \frac{bcx \tan^{-1}\left(\sqrt{-c^2x^2-1}\right)}{4d^2e\sqrt{-c^2x^2}} + \frac{bcx(3c^2d-2e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8d^2\sqrt{e}\sqrt{-c^2x^2}(c^2d-e)^{3/2}} + \frac{bcx\sqrt{-c^2x^2-1}}{8d\sqrt{-c^2x^2}(c^2d-e)(d+ex^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x*(a+b*\operatorname{ArcCsch}[c*x]))/(d+e*x^2)^3,x]$

[Out]  $(b*c*x*\operatorname{Sqrt}[-1-c^2*x^2])/(8*d*(c^2*d-e)*\operatorname{Sqrt}[-(c^2*x^2)]*(d+e*x^2)) - (a+b*\operatorname{ArcCsch}[c*x])/(4*e*(d+e*x^2)^2) + (b*c*x*\operatorname{ArcTan}[\operatorname{Sqrt}[-1-c^2*x^2]])/(4*d^2*e*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*c*(3*c^2*d-2*e)*x*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1-c^2*x^2])/\operatorname{Sqrt}[c^2*d-e]])/(8*d^2*(c^2*d-e)^{(3/2)}*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-(c^2*x^2)])$

### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 103

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] := \operatorname{Simp}[(b*(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})/((m+1)*(b*c - a*d)*(b*e - a*f)), x] + \operatorname{Dist}[1/((m+1)*(b*c - a*d)*(b*e - a*f)), \operatorname{Int}[(a + b*x)^{(m+1)}*(c + d*x)^n*(e + f*x)^p \operatorname{Simp}[a*d*f*(m+1) - b*(d*e*(m+n+2) + c*f*(m+p+2)) - b*d*f*(m+n+p+3)*x, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, n, p\}, x] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegerQ}[m] \&\& (\operatorname{IntegerQ}[n] || \operatorname{IntegersQ}[2*n, 2*p])$

### Rule 156

$\operatorname{Int}[(e_. + (f_.)*(x_.))^{(p_.)}*((g_.) + (h_.)*(x_.))]/((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.)), x\_Symbol] := \operatorname{Dist}[(b*g - a*h)/(b*c - a*d), \operatorname{Int}[(e + f*x)^p/(a + b*x), x], x] - \operatorname{Dist}[(d*g - c*h)/(b*c - a*d), \operatorname{Int}[(e + f*x)^p/(c + d*x), x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, g, h\}, x]$

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 446

Int[(x\_)^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 6300

Int[((a\_) + ArcCsch[(c\_)\*(x\_)])\*(b\_)\*(x\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsch[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[-(c^2\*x^2)]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[-1 - c^2\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{x(a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^3} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bcx) \int \frac{1}{x\sqrt{-1-c^2x^2}(d+ex^2)^2} dx}{4e\sqrt{-c^2x^2}} \\
 &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}(d+ex)^2} dx, x, x^2\right)}{8e\sqrt{-c^2x^2}} \\
 &= \frac{bcx\sqrt{-1-c^2x^2}}{8d(c^2d - e)\sqrt{-c^2x^2}(d + ex^2)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{c^2d - e - \frac{1}{2}c^2ex}{x\sqrt{-1-c^2x}(d+ex)} dx, x, x^2\right)}{8d(c^2d - e)e\sqrt{-c^2x^2}} \\
 &= \frac{bcx\sqrt{-1-c^2x^2}}{8d(c^2d - e)\sqrt{-c^2x^2}(d + ex^2)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e(d + ex^2)^2} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}} dx, x, x^2\right)}{8d^2e\sqrt{-c^2x^2}} \\
 &= \frac{bcx\sqrt{-1-c^2x^2}}{8d(c^2d - e)\sqrt{-c^2x^2}(d + ex^2)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e(d + ex^2)^2} - \frac{(bx) \operatorname{Subst}\left(\int \frac{1}{-\frac{1}{c^2} - x^2} dx, x, \sqrt{-1-c^2x^2}\right)}{4cd^2e\sqrt{-c^2x^2}} \\
 &= \frac{bcx\sqrt{-1-c^2x^2}}{8d(c^2d - e)\sqrt{-c^2x^2}(d + ex^2)} - \frac{a + b \operatorname{csch}^{-1}(cx)}{4e(d + ex^2)^2} + \frac{bcx \tan^{-1}\left(\sqrt{-1-c^2x^2}\right)}{4d^2e\sqrt{-c^2x^2}} + \frac{b}{4cd^2e}
 \end{aligned}$$

**Mathematica [C]** time = 0.96, size = 368, normalized size = 1.80

$$\frac{1}{16} \left[ -\frac{4a}{e(d+ex^2)^2} + \frac{b(3c^2d-2e) \log\left(\frac{16d^2\sqrt{e}\sqrt{e-c^2d}\left(\sqrt{e+cx}\left(\sqrt{\frac{1}{c^2x^2}+1}\sqrt{e-c^2d}-ic\sqrt{d}\right)\right)}{b(2e-3c^2d)(\sqrt{e}x+i\sqrt{d})}\right)}{d^2\sqrt{e}(e-c^2d)^{3/2}} + \frac{b(3c^2d-2e) \log\left(\frac{16id^2\sqrt{e}\sqrt{e-c^2d}}{d^2\sqrt{e}}\right)}{d^2\sqrt{e}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcSch[c\*x]))/(d + e\*x^2)^3,x]

[Out] ((-4\*a)/(e\*(d + e\*x^2)^2) + (2\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x)/(d\*(c^2\*d - e)\*(d + e\*x^2)) - (4\*b\*ArcSch[c\*x])/(e\*(d + e\*x^2)^2) + (4\*b\*ArcSinh[1/(c\*x)])/(d^2\*e) + (b\*(3\*c^2\*d - 2\*e)\*Log[(16\*d^2\*Sqrt[e]\*Sqrt[-(c^2\*d) + e]\*(Sqrt[e] + c\*((-I)\*c\*Sqrt[d] + Sqrt[-(c^2\*d) + e])\*Sqrt[1 + 1/(c^2\*x^2)])\*x])/(b\*(-3\*c^2\*d + 2\*e)\*(I\*Sqrt[d] + Sqrt[e]\*x)))/(d^2\*Sqrt[e]\*(-(c^2\*d) + e)^(3/2)) + (b\*(3\*c^2\*d - 2\*e)\*Log[(-16\*I)\*d^2\*Sqrt[e]\*Sqrt[-(c^2\*d) + e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) + e])\*Sqrt[1 + 1/(c^2\*x^2)])\*x])/(b\*(3\*c^2\*d - 2\*e)\*(Sqrt[d] + I\*Sqrt[e]\*x)))/(d^2\*Sqrt[e]\*(-(c^2\*d) + e)^(3/2)))/16

**fricas [B]** time = 1.43, size = 1256, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/16\*(4\*a\*c^4\*d^4 - 8\*a\*c^2\*d^3\*e + 4\*a\*d^2\*e^2 + (3\*b\*c^2\*d^3 + (3\*b\*c^2\*d\*e^2 - 2\*b\*e^3)\*x^4 - 2\*b\*d^2\*e + 2\*(3\*b\*c^2\*d^2\*e - 2\*b\*d\*e^2)\*x^2)\*sqrt(-c^2\*d\*e + e^2)\*log((c^2\*e\*x^2 - c^2\*d - 2\*sqrt(-c^2\*d\*e + e^2)\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 2\*e)/(e\*x^2 + d)) - 4\*(b\*c^4\*d^4 - 2\*b\*c^2\*d^3\*e + b\*d^2\*e^2 + (b\*c^4\*d^2\*e^2 - 2\*b\*c^2\*d\*e^3 + b\*e^4)\*x^4 + 2\*(b\*c^4\*d^3\*e - 2\*b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) + 4\*(b\*c^4\*d^4 - 2\*b\*c^2\*d^3\*e + b\*d^2\*e^2 + (b\*c^4\*d^2\*e^2 - 2\*b\*c^2\*d\*e^3 + b\*e^4)\*x^4 + 2\*(b\*c^4\*d^3\*e - 2\*b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x) - 2\*((b\*c^3\*d^2\*e^2 - b\*c\*d\*e^3)\*x^3 + (b\*c^3\*d^3\*e - b\*c\*d^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^4\*d^6\*e - 2\*c^2\*d^5\*e^2 + d^4\*e^3 + (c^4\*d^4\*e^3 - 2\*c^2\*d^3\*e^4 + d^2\*e^5)\*x^4 + 2\*(c^4\*d^5\*e^2 - 2\*c^2\*d^4\*e^3 + d^3\*e^4)\*x^2), -1/8\*(2\*a\*c^4\*d^4 - 4\*a\*c^2\*d^3\*e + 2\*a\*d^2\*e^2 + (3\*b\*c^2\*d^3 + (3\*b\*c^2\*d\*e^2 - 2\*b\*e^3)\*x^4 - 2\*b\*d^2\*e + 2\*(3\*b\*c^2\*d^2\*e - 2\*b\*d\*e^2)\*x^2)\*sqrt(c^2\*d\*e - e^2)\*arctan(-sqrt(c^2\*d\*e - e^2)\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d - e) - 2\*(b\*c^4\*d^4 - 2\*b\*c^2\*d^3\*e + b\*d^2\*e^2 + (b\*c^4\*d^2\*e^2 - 2\*b\*c^2\*d\*e^3 + b\*e^4)\*x^4 + 2\*(b\*c^4\*d^3\*e - 2\*b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - c\*x + 1) + 2\*(b\*c^4\*d^4 - 2\*b\*c^2\*d^3\*e + b\*d^2\*e^2 + (b\*c^4\*d^2\*e^2 - 2\*b\*c^2\*d\*e^3 + b\*e^4)\*x^4 + 2\*(b\*c^4\*d^3\*e - 2\*b\*c^2\*d^2\*e^2 + b\*d\*e^3)\*x^2)\*log(c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x) - ((b\*c^3\*d^2\*e^2 - b\*c\*d\*e^3)\*x^3 + (b\*c^3\*d^3\*e - b\*c\*d^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^4\*d^6\*e - 2\*c^2\*d^5\*e^2 + d^4\*e^3 + (c^4\*d^4\*e^3 - 2\*c^2\*d^3\*e^4 + d^2\*e^5)\*x^4 + 2\*(c^4\*d^5\*e^2 - 2\*c^2\*d^4\*e^3 + d^3\*e^4)\*x^2)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex^2 + d)^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x/(e\*x^2 + d)^3, x)

**maple [B]** time = 0.08, size = 1892, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

[Out] 
$$\begin{aligned} & -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*arccsch(c*x) \\ & -1/4*c^3*b*(c^2*x^2+1)^{(1/2)}*e/((c^2*x^2+1)/c^2/x^2)^{(1/2)}*x/d/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})*arctanh(1/(c^2*x^2+1)^{(1/2)}) \\ & -1/4*c^3*b*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})*arctanh(1/(c^2*x^2+1)^{(1/2)}) \\ & +3/16*c^3*b*(c^2*x^2+1)^{(1/2)}*e/((c^2*x^2+1)/c^2/x^2)^{(1/2)}*x/d/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c^2*d-e)/e)^{(1/2)}/(-c*x*e+(-c^2*d*e)^{(1/2)})*ln(-2*(-(c^2*x^2+1)^{(1/2)}*(-(c^2*d-e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x-e)/(c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +3/16*c^3*b*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c^2*d-e)/e)^{(1/2)}/(-c*x*e+(-c^2*d*e)^{(1/2)})*ln(-2*(-(c^2*x^2+1)^{(1/2)}*(-(c^2*d-e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x-e)/(c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +3/16*c^3*b*(c^2*x^2+1)^{(1/2)}*e/((c^2*x^2+1)/c^2/x^2)^{(1/2)}*x/d/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c^2*d-e)/e)^{(1/2)}/(-c*x*e+(-c^2*d*e)^{(1/2)})*ln(-2*((c^2*x^2+1)^{(1/2)}*(-(c^2*d-e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e)/(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +3/16*c^3*b*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c^2*d-e)/e)^{(1/2)}/(-c*x*e+(-c^2*d*e)^{(1/2)})*ln(-2*((c^2*x^2+1)^{(1/2)}*(-(c^2*d-e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e)/(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & -1/8*c^3*b/((c^2*x^2+1)/c^2/x^2)^{(1/2)}*x/d/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})*e-1/8*c*b/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})*e+1/4*c*b*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}*x/d^2/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})*arctanh(1/(c^2*x^2+1)^{(1/2)})*e^2+1/4*c*b*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})*arctanh(1/(c^2*x^2+1)^{(1/2)})*e-1/8*c*b*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}*x/d^2/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c^2*d-e)/e)^{(1/2)}/(-c*x*e+(-c^2*d*e)^{(1/2)})*ln(-2*(-(c^2*x^2+1)^{(1/2)}*(-(c^2*d-e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x-e)/(c*x*e+(-c^2*d*e)^{(1/2)})) \\ & *e^2-1/8*c*b*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c^2*d-e)/e)^{(1/2)}/(-c*x*e+(-c^2*d*e)^{(1/2)})*ln(-2*(-(c^2*x^2+1)^{(1/2)}*(-(c^2*d-e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e)/(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & *e^2-1/8*c*b*(c^2*x^2+1)^{(1/2)}/((c^2*x^2+1)/c^2/x^2)^{(1/2)}/x/d/(c^2*d-e)/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /(-c^2*d-e)/e)^{(1/2)}/(-c*x*e+(-c^2*d*e)^{(1/2)})*ln(-2*((c^2*x^2+1)^{(1/2)}*(-(c^2*d-e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e)/(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & *e \end{aligned}$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8} \left( 8c^2 \int \frac{x}{4(c^2e^3x^6 + (2c^2de^2 + e^3)x^4 + d^2e + (c^2d^2e + 2de^2)x^2 + (c^2e^3x^6 + (2c^2de^2 + e^3)x^4 + d^2e + (c^2d^2e} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 
$$-1/8*(8*c^2*integrate(1/4*x/(c^2*e^3*x^6 + (2*c^2*d*e^2 + e^3)*x^4 + d^2*e + (c^2*d^2*e + 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 + e^3)*x^4 + d^2*e + (c^2*d^2*e + 2*d*e^2)*x^2)*sqrt(c^2*x^2 + 1)), x) + (2*c^2*d - e)*log(e*x^2 + d)/(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2) - (2*c^4*d^4*log(c) + 2*d^2*e^2*log(c) - d^2*e^2 - (4*d^3*e*log(c) - d^3*e)*c^2 + (c^2*d^2*e^2 - d*e^3)*x^2 + (c^4*d^2*e^2*x^4 + 2*c^4*d^3*e*x^2 + c^4*d^4)*log(c^2*x^2 + 1) - 2*((c^4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*x^4 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2 + d*e^3)*x^2)*log(x) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2)*log(sqrt(c^2*x^2 + 1) + 1))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2))*b - 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

$$3.114 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^3} dx$$

**Optimal.** Leaf size=657

$$\frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}}\right)}{2d^3} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}} + 1\right)}{2d^3} - \frac{(a + b \operatorname{csch}^{-1}(cx)) \log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}}\right)}{2d^3}$$

[Out]  $\frac{1}{4}e^{2(a+b\operatorname{arccsch}(c*x))/d^3}/(e+d/x^2)^2 - e^{(a+b\operatorname{arccsch}(c*x))/d^3}/(e+d/x^2) + \frac{1}{2}(a+b\operatorname{arccsch}(c*x))^2/b/d^3 - \frac{1}{2}(a+b\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/d^3 - \frac{1}{2}(a+b\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/d^3 - \frac{1}{2}(a+b\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/d^3 - \frac{1}{2}(a+b\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/d^3 - \frac{1}{2}b*\operatorname{polylog}(2, -c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/d^3 - \frac{1}{2}b*\operatorname{polylog}(2, c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/d^3 - \frac{1}{2}b*\operatorname{polylog}(2, -c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/d^3 - \frac{1}{2}b*\operatorname{polylog}(2, c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/d^3 - \frac{1}{8}b*(c^2*d-2*e)*\arctan((c^2*d-e)^{1/2}/c/x/e^{1/2}/(1+1/c^2/x^2)^{1/2})*e^{1/2}/d^3/(c^2*d-e)^{3/2} + b*\arctan((c^2*d-e)^{1/2}/c/x/e^{1/2}/(1+1/c^2/x^2)^{1/2})*e^{1/2}/d^3/(c^2*d-e)^{1/2} - \frac{1}{8}b*c*e*(1+1/c^2/x^2)^{1/2}/d^2/(c^2*d-e)/(e+d/x^2)/x$

**Rubi [A]** time = 1.32, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$ , Rules used = {6304, 5791, 5787, 382, 377, 205, 5799, 5561, 2190, 2279, 2391}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}}\right)}{2d^3} - \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - \sqrt{e - c^2d}}}\right)}{2d^3} - \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2d} + \sqrt{e}}\right)}{2d^3} - \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e - c^2d} + \sqrt{e}}\right)}{2d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^3), x]

[Out]  $-(b*c*e*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(8*d^2*(c^2*d - e)*(e + d/x^2)*x) + (e^{2(a + b*\operatorname{ArcCsch}[c*x])}/(4*d^3*(e + d/x^2)^2) - (e^{(a + b*\operatorname{ArcCsch}[c*x])}/(d^3*(e + d/x^2)) + (a + b*\operatorname{ArcCsch}[c*x])^2/(2*b*d^3) - (b*(c^2*d - 2*e)*\operatorname{Sqrt}[e]*\operatorname{ArcTan}[\operatorname{Sqrt}[c^2*d - e]/(c*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d - e)^{3/2}) + (b*\operatorname{Sqrt}[e]*\operatorname{ArcTan}[\operatorname{Sqrt}[c^2*d - e]/(c*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x)])/(d^3*\operatorname{Sqrt}[c^2*d - e]) - ((a + b*\operatorname{ArcCsch}[c*x])*Log[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(2*d^3) - ((a + b*\operatorname{ArcCsch}[c*x])*Log[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])])/(2*d^3) - ((a + b*\operatorname{ArcCsch}[c*x])*Log[1 - (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(2*d^3) - ((a + b*\operatorname{ArcCsch}[c*x])*Log[1 + (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])])/(2*d^3) - (b*\operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])]))/(2*d^3) - (b*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[-(c^2*d) + e])]))/(2*d^3) - (b*\operatorname{PolyLog}[2, -((c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])]))/(2*d^3) - (b*\operatorname{PolyLog}[2, (c*\operatorname{Sqrt}[-d]*E^{\operatorname{ArcCsch}[c*x]})/(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[-(c^2*d) + e])]))/(2*d^3)$

Rule 205

$\text{Int}[(a_ + (b_ \cdot)(x_ )^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

Rule 377

$\text{Int}[(a_ + (b_ \cdot)(x_ )^{n_})^{p_}/((c_ + (d_ \cdot)(x_ )^{n_}), x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b \cdot c - a \cdot d) \cdot x^n), x], x, x/(a + b \cdot x^n)^{1/n}] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[n \cdot p + 1, 0] \ \&\& \ \text{IntegerQ}[n]$

Rule 382

$\text{Int}[(a_ + (b_ \cdot)(x_ )^{n_})^{p_} \cdot ((c_ + (d_ \cdot)(x_ )^{n_})^{q_}), x\_Symbol] \rightarrow -\text{Simp}[(b \cdot x \cdot (a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^{q+1})/(a \cdot n \cdot (p+1) \cdot (b \cdot c - a \cdot d)), x] + \text{Dist}[(b \cdot c + n \cdot (p+1) \cdot (b \cdot c - a \cdot d))/(a \cdot n \cdot (p+1) \cdot (b \cdot c - a \cdot d)), \text{Int}[(a + b \cdot x^n)^{p+1} \cdot (c + d \cdot x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, n, q\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[n \cdot (p+q+2) + 1, 0] \ \&\& \ (\text{LtQ}[p, -1] \ || \ !\text{LtQ}[q, -1]) \ \&\& \ \text{NeQ}[p, -1]$

Rule 2190

$\text{Int}[(F_ )^{((g_ \cdot)((e_ \cdot) + (f_ \cdot)(x_ )))^{n_} \cdot ((c_ \cdot) + (d_ \cdot)(x_ ))^{m_})}/((a_ + (b_ \cdot)((F_ )^{((g_ \cdot)((e_ \cdot) + (f_ \cdot)(x_ )))^{n_})}), x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F^{g \cdot (e + f \cdot x)})^n)/a]/(b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m)/(b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + (b \cdot (F^{g \cdot (e + f \cdot x)})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2279

$\text{Int}[\text{Log}[(a_ + (b_ \cdot)((F_ )^{((e_ \cdot)((c_ \cdot) + (d_ \cdot)(x_ )))^{n_})}], x\_Symbol] \rightarrow \text{Dist}[1/(d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x]/x, x], x, (F^{e \cdot (c + d \cdot x)})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$

Rule 2391

$\text{Int}[\text{Log}[(c_ \cdot)((d_ + (e_ \cdot)(x_ )^{n_})]/(x_ ), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c \cdot d, 1]$

Rule 5561

$\text{Int}[(\text{Cosh}[(c_ \cdot) + (d_ \cdot)(x_ )] \cdot ((e_ \cdot) + (f_ \cdot)(x_ ))^{m_})/((a_ + (b_ \cdot)\text{Sinh}[(c_ \cdot) + (d_ \cdot)(x_ )]), x\_Symbol] \rightarrow -\text{Simp}[(e + f \cdot x)^{m+1}/(b \cdot f \cdot (m+1)), x] + (\text{Int}[(e + f \cdot x)^m \cdot E^{c + d \cdot x})/(a - \text{Rt}[a^2 + b^2, 2] + b \cdot E^{c + d \cdot x}), x] + \text{Int}[(e + f \cdot x)^m \cdot E^{c + d \cdot x})/(a + \text{Rt}[a^2 + b^2, 2] + b \cdot E^{c + d \cdot x}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

Rule 5787

$\text{Int}[(a_ \cdot) + \text{ArcSinh}[(c_ \cdot)(x_ )] \cdot (b_ \cdot)] \cdot (x_ ) \cdot ((d_ + (e_ \cdot)(x_ )^2)^{p_}), x\_Symbol] \rightarrow \text{Simp}[(d + e \cdot x^2)^{p+1} \cdot (a + b \cdot \text{ArcSinh}[c \cdot x])]/(2 \cdot e \cdot (p+1)), x] - \text{Dist}[(b \cdot c)/(2 \cdot e \cdot (p+1)), \text{Int}[(d + e \cdot x^2)^{p+1}/\text{Sqrt}[1 + c^2 \cdot x^2], x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[e, c^2 \cdot d] \ \&\& \ \text{NeQ}[p, -1]$

Rule 5791

$\text{Int}[(a_ \cdot) + \text{ArcSinh}[(c_ \cdot)(x_ )] \cdot (b_ \cdot)]^{n_} \cdot ((f_ \cdot)(x_ ))^{m_} \cdot ((d_ + (e_ \cdot)(x_ )^2)^{p_}), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcSinh}[c \cdot x])^n, (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[e, c^2 \cdot d]$

$2*d]$  && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

Rule 5799

Int[((a\_.) + ArcSinh[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Subst[Int[((a + b\*x)^n\*Cosh[x])/(c\*d + e\*Sinh[x]), x], x, ArcSinh[c\*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 6304

Int[((a\_.) + ArcSch[(c\_.)\*(x\_)]\*(b\_.))^(n\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Subst[Int[((e + d\*x^2)^p\*(a + b\*ArcSinh[x/c])^n)/x^(m + 2\*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^3} dx &= -\operatorname{Subst} \left( \int \frac{x^5 \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{e^2 x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^3} - \frac{2ex \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)^2} + \frac{x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{d^2 (e + dx^2)} \right) dx \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \operatorname{Subst} \left( \int \frac{x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \operatorname{Subst} \left( \int \frac{x \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{e^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{d^3 \left( e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left( \int \left( -\frac{\sqrt{-d} \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} - \sqrt{-d}x)} + \frac{\sqrt{-d} \left( a + b \sinh^{-1} \left( \frac{x}{c} \right) \right)}{2d(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{8d^2 (c^2 d - e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{d^3 \left( e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left( \int \frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{e}} dx, x, \frac{1}{x} \right)}{2d^2} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{8d^2 (c^2 d - e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{b\sqrt{e} \tan^{-1} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{e}} \right)}{d^3 \sqrt{c^2 d - e}} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{8d^2 (c^2 d - e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)}{2bd^3} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{8d^2 (c^2 d - e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)}{2bd^3} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{8d^2 (c^2 d - e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)}{2bd^3} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{8d^2 (c^2 d - e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)}{2bd^3} \\
&= -\frac{bce \sqrt{1 + \frac{1}{c^2 x^2}}}{8d^2 (c^2 d - e) \left( e + \frac{d}{x^2} \right) x} + \frac{e^2 \left( a + b \operatorname{csch}^{-1}(cx) \right)}{4d^3 \left( e + \frac{d}{x^2} \right)^2} - \frac{e \left( a + b \operatorname{csch}^{-1}(cx) \right)}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{\left( a + b \operatorname{csch}^{-1}(cx) \right)}{2bd^3}
\end{aligned}$$

**Mathematica** [F] time = 70.35, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^3), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^3), x]

**fricas** [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{e^3 x^7 + 3 d e^2 x^5 + 3 d^2 e x^3 + d^3 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e^3\*x^7 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^3 + d^3\*x), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)^3\*x), x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x (ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^3,x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4} a \left( \frac{2ex^2 + 3d}{d^2e^2x^4 + 2d^3ex^2 + d^4} - \frac{2 \log(ex^2 + d)}{d^3} + \frac{4 \log(x)}{d^3} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^3x^7 + 3de^2x^5 + 3d^2ex^3 + d^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/4\*a\*((2\*e\*x^2 + 3\*d)/(d^2\*e^2\*x^4 + 2\*d^3\*e\*x^2 + d^4) - 2\*log(e\*x^2 + d)/d^3 + 4\*log(x)/d^3) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e^3\*x^7 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^3 + d^3\*x), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x (ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```



$$3.115 \quad \int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^3} dx$$

Optimal. Leaf size=1106

$$\frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16(c^2d-e)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16(c^2d-e)e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{3(a+b\operatorname{csch}^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{3(a+b\operatorname{csch}^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} +$$

[Out]  $3/16*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/e^{5/2}/(-d)^{1/2}-3/16*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/e^{5/2}/(-d)^{1/2}+3/16*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/e^{5/2}/(-d)^{1/2}-3/16*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/e^{5/2}/(-d)^{1/2}-3/16*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/e^{5/2}/(-d)^{1/2}+3/16*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2}))/e^{5/2}/(-d)^{1/2}-3/16*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/e^{5/2}/(-d)^{1/2}+3/16*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{1/2}))*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2}))/e^{5/2}/(-d)^{1/2}+1/16*b*\operatorname{arctanh}((c^2*d-(-d)^{1/2}*e^{1/2}/x)/c/d^{1/2}/(c^2*d-e)^{1/2}/(1+1/c^2/x^2)^{1/2}))/e^{3/2}/(-d/x+(-d)^{1/2}*e^{1/2})^2+3/16*(a+b*\operatorname{arccsch}(c*x))/e^2/(-d/x+(-d)^{1/2}*e^{1/2})-1/16*(a+b*\operatorname{arccsch}(c*x))*(-d)^{1/2}/e^{3/2}/(d/x+(-d)^{1/2}*e^{1/2})^2-3/16*(a+b*\operatorname{arccsch}(c*x))/e^2/(d/x+(-d)^{1/2}*e^{1/2})-1/16*b*c*(-d)^{1/2}*(1+1/c^2/x^2)^{1/2}/(c^2*d-e)/e^{3/2}/(-d/x+(-d)^{1/2}*e^{1/2})-1/16*b*c*(-d)^{1/2}*(1+1/c^2/x^2)^{1/2}/(c^2*d-e)/e^{3/2}/(d/x+(-d)^{1/2}*e^{1/2})$

**Rubi [A]** time = 1.74, antiderivative size = 1106, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$ , Rules used = {6304, 5706, 5801, 731, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$\frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16(c^2d-e)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16(c^2d-e)e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{3(a+b\operatorname{csch}^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{3(a+b\operatorname{csch}^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} +$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^4*(a + b*\operatorname{ArcCsch}[c*x]))/(d + e*x^2)^3, x]$

[Out]  $-(b*c*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^{3/2}*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)) - (b*c*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^{3/2}*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)) + (\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x]))/(16*e^{3/2}*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)^2) + (3*(a + b*\operatorname{ArcCsch}[c*x]))/(16*e^2*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)) - (\operatorname{Sqrt}[-d]*(a + b*\operatorname{ArcCsch}[c*x]))/(16*e^{3/2}*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)^2) - (3*(a + b*\operatorname{ArcCsch}[c*x]))/(16*e^2*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)) - (3*b*\operatorname{ArcTanh}[(c^2*d - (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x)/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e]*\operatorname{Sqrt}[$

$$\frac{1 + 1/(c^2 x^2)}{(16 \sqrt{d} \sqrt{c^2 d - e} e^2) + (b \operatorname{ArcTanh}[(c^2 d - \sqrt{-d} \sqrt{e})/x] / (c \sqrt{d} \sqrt{c^2 d - e} \sqrt{1 + 1/(c^2 x^2)}))} / (16 \sqrt{d} (c^2 d - e)^{3/2} e - (3 b \operatorname{ArcTanh}[(c^2 d + \sqrt{-d} \sqrt{e})/x] / (c \sqrt{d} \sqrt{c^2 d - e} \sqrt{1 + 1/(c^2 x^2)}))} / (16 \sqrt{d} \sqrt{c^2 d - e} e^2) + (b \operatorname{ArcTanh}[(c^2 d + \sqrt{-d} \sqrt{e})/x] / (c \sqrt{d} \sqrt{c^2 d - e} \sqrt{1 + 1/(c^2 x^2)}))} / (16 \sqrt{d} (c^2 d - e)^{3/2} e) + (3(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 - (c \sqrt{-d} E^{\operatorname{ArcCsch}[c x]}) / (\sqrt{e} - \sqrt{-(c^2 d + e)})]) / (16 \sqrt{-d} e^{5/2}) - (3(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 + (c \sqrt{-d} E^{\operatorname{ArcCsch}[c x]}) / (\sqrt{e} - \sqrt{-(c^2 d + e)})]) / (16 \sqrt{-d} e^{5/2}) + (3(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 - (c \sqrt{-d} E^{\operatorname{ArcCsch}[c x]}) / (\sqrt{e} + \sqrt{-(c^2 d + e)})]) / (16 \sqrt{-d} e^{5/2}) - (3(a + b \operatorname{ArcCsch}[c x]) \operatorname{Log}[1 + (c \sqrt{-d} E^{\operatorname{ArcCsch}[c x]}) / (\sqrt{e} + \sqrt{-(c^2 d + e)})]) / (16 \sqrt{-d} e^{5/2}) - (3 b \operatorname{PolyLog}[2, -(c \sqrt{-d} E^{\operatorname{ArcCsch}[c x]}) / (\sqrt{e} - \sqrt{-(c^2 d + e)})]) / (16 \sqrt{-d} e^{5/2}) + (3 b \operatorname{PolyLog}[2, (c \sqrt{-d} E^{\operatorname{ArcCsch}[c x]}) / (\sqrt{e} - \sqrt{-(c^2 d + e)})]) / (16 \sqrt{-d} e^{5/2}) - (3 b \operatorname{PolyLog}[2, -(c \sqrt{-d} E^{\operatorname{ArcCsch}[c x]}) / (\sqrt{e} + \sqrt{-(c^2 d + e)})]) / (16 \sqrt{-d} e^{5/2}) + (3 b \operatorname{PolyLog}[2, (c \sqrt{-d} E^{\operatorname{ArcCsch}[c x]}) / (\sqrt{e} + \sqrt{-(c^2 d + e)})]) / (16 \sqrt{-d} e^{5/2})$$
Rule 206

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1 \operatorname{ArcTanh}[(Rt[-b, 2] x) / Rt[a, 2]]) / (Rt[a, 2] Rt[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$$
Rule 725

$$\operatorname{Int}[1/((d + (e \cdot x)) \sqrt{(a + (c \cdot x)^2})], x_{\text{Symbol}}] \rightarrow -\operatorname{Subst}[\operatorname{Int}[1/(c d^2 + a e^2 - x^2), x], x, (a e - c d x) / \sqrt{a + c x^2}] /; \operatorname{FreeQ}\{a, c, d, e, x\}$$
Rule 731

$$\operatorname{Int}[(d + (e \cdot x))^m (a + (c \cdot x)^2)^p, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(e(d + e x)^{m+1} (a + c x^2)^{p+1}) / ((m+1)(c d^2 + a e^2)), x] + \operatorname{Dist}[(c d) / (c d^2 + a e^2), \operatorname{Int}[(d + e x)^{m+1} (a + c x^2)^p, x], x] /; \operatorname{FreeQ}\{a, c, d, e, m, p, x\} \ \&\& \operatorname{NeQ}[c d^2 + a e^2, 0] \ \&\& \operatorname{EqQ}[m + 2 p + 3, 0]$$
Rule 2190

$$\operatorname{Int}[(F)^{((g \cdot (e \cdot x) + (f \cdot x)))^{(n \cdot (c \cdot x) + (d \cdot x)^m))} / ((a + (b \cdot x)) \cdot (F)^{((g \cdot (e \cdot x) + (f \cdot x)))^{(n \cdot (c \cdot x) + (d \cdot x)^m))}], x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(c + d x)^m \operatorname{Log}[1 + (b(F^{(g(e + f x)))^n}) / a] / (b f g^n \operatorname{Log}[F]), x] - \operatorname{Dist}[(d m) / (b f g^n \operatorname{Log}[F]), \operatorname{Int}[(c + d x)^{m-1} \operatorname{Log}[1 + (b(F^{(g(e + f x)))^n}) / a], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \ \&\& \operatorname{IGtQ}[m, 0]$$
Rule 2279

$$\operatorname{Int}[\operatorname{Log}[(a + (b \cdot x)) \cdot (F)^{((e \cdot (c \cdot x) + (d \cdot x)))^{(n \cdot x)}}], x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/(d e^n \operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b x] / x, x], x, (F^{(e(c + d x))})^n], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \operatorname{GtQ}[a, 0]$$
Rule 2391

$$\operatorname{Int}[\operatorname{Log}[(c \cdot x) \cdot (d + (e \cdot x)^n)] / (x), x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c e x^n)] / n, x] /; \operatorname{FreeQ}\{c, d, e, n, x\} \ \&\& \operatorname{EqQ}[c d, 1]$$
Rule 5561

$$\operatorname{Int}[(\operatorname{Cosh}[(c \cdot x) + (d \cdot x)] \cdot ((e \cdot x) + (f \cdot x))^m) / ((a + (b \cdot x)) \operatorname{Sin}$$

```
h[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

#### Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
0 || IGtQ[n, 0])
```

#### Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x] - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n - 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

#### Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[m, p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^3} dx &= -\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( -\frac{d^3 (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{8(-d)^{3/2} e^{3/2} (\sqrt{-d} \sqrt{e} - dx)^3} - \frac{3d (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{16e^2 (\sqrt{-d} \sqrt{e} - dx)^2} - \frac{d^3 (a + b \sinh^{-1} \left( \frac{x}{c} \right))}{8(-d)^{3/2} e^{3/2}} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{(3d) \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d} \sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{16e^2} + \frac{(3d) \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1} \left( \frac{x}{c} \right)}{(\sqrt{-d} \sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{16e^2} + \frac{(3d) \operatorname{Subst} \left( \int \frac{1}{\sqrt{-d} \sqrt{e} - dx} dx, x, \frac{1}{x} \right)}{16e^2} \\
&= \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)^2} + \frac{3(a + b \operatorname{csch}^{-1}(cx))}{16e^2 \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)^2} - \frac{3(a + b \operatorname{csch}^{-1}(cx))}{16e^2 \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{-d} \sqrt{1 + \frac{1}{c^2 x^2}}}{16(c^2 d - e) e^{3/2} \left( \sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{-d} (a + b \operatorname{csch}^{-1}(cx))}{16e^{3/2} \left( \sqrt{-d} \sqrt{e} - \frac{d}{x} \right)}
\end{aligned}$$

**Mathematica [C]** time = 6.21, size = 2045, normalized size = 1.85

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^3,x]

[Out] (a\*d\*x)/(4\*e^2\*(d + e\*x^2)^2) - (5\*a\*x)/(8\*e^2\*(d + e\*x^2)) + (3\*a\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*Sqrt[d]\*e^(5/2)) + b\*(((I/16)\*Sqrt[d]\*((I\*c\*Sqrt[e]\*Sqrt[1 + 1/(c^2\*x^2)]\*x)/(Sqrt[d]\*(c^2\*d - e)\*((-I)\*Sqrt[d] + Sqrt[e]\*x)) - ArcCsch[c\*x]/(Sqrt[e]\*((-I)\*Sqrt[d] + Sqrt[e]\*x)^2) - ArcSinh[1/(c\*x)]/(d\*Sqrt[e]) + (I\*(2\*c^2\*d - e)\*Log[(4\*d\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(Sqrt[e] + I\*c\*(c\*Sqrt[d] - Sqrt[c^2\*d - e]\*Sqrt[1 + 1/(c^2\*x^2)]\*x))/((2\*c^2\*d - e)\*(Sqrt[d] + I\*Sqrt[e]\*x)))]/(d\*(c^2\*d - e)^(3/2))))/e^2 - (((I/16)\*Sqrt[d]\*((-I)\*c\*Sqrt[e]\*Sqrt[1 + 1/(c^2\*x^2)]\*x)/(Sqrt[d]\*(c^2\*d - e)\*(I\*Sqrt[d] + Sqrt[e]\*x)) - ArcCsch[c\*x]/(Sqrt[e]\*(I\*Sqrt[d] + Sqrt[e]\*x)^2) - ArcSinh[1/(c\*x)]/(d\*Sqrt[e]) + (I\*(2\*c^2\*d - e)\*Log[(4\*I)\*d\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] + Sqrt[c^2\*d - e]\*Sqrt[1 + 1/(c^2\*x^2)]\*x))/((2\*c^2\*d - e)\*(Sqrt[d] - I\*Sqrt[e]\*x)))]/(d\*(c^2\*d - e)^(3/2))))/e^2 + (5\*(-(ArcCsch[c\*x]/(I\*Sqrt[d]\*Sqrt[e] + e\*x)) - (I\*(ArcSinh[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] + I\*Sqrt[-(c^2\*d) + e]\*Sqrt[1 + 1/(c^2\*x^2)]\*x)))/(Sqrt[-(c^2\*d) + e]\*(I\*Sqrt[d] + Sqrt[e]\*x)))]/Sqrt[-(c^2\*d) + e]))/Sqrt[d]))/(16\*e^2) + (5\*(-(ArcCsch[c\*x]/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x)) + (I\*(ArcSinh[1/(c\*x)]/Sqrt[e] - Log[(-2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) + e]\*Sqrt[1 + 1/(c^2\*x^2)]\*x)))/(Sqrt[-(c^2\*d) + e]\*(Sqrt[d] + I\*Sqrt[e]\*x)))]/Sqrt[-(c^2\*d) + e]))/Sqrt[d]))/(16\*e^2) + (((3\*I)/128)\*(Pi^2 - (4\*I)\*Pi\*ArcCsch[c\*x] - 8\*ArcCsch[c\*x]^2 + 32\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] - Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])] + (4\*I)\*Pi\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 8\*ArcCsch[c\*x]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (16\*I)\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (4\*I)\*Pi\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 8\*ArcCsch[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] - (16\*I)\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] - (4\*I)\*Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 8\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]))/(Sqrt[d]\*e^(5/2)) - (((3\*I)/128)\*(Pi^2 - (4\*I)\*Pi\*ArcCsch[c\*x] - 8\*ArcCsch[c\*x]^2 - 32\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])] + (4\*I)\*Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 8\*ArcCsch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (16\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + (4\*I)\*Pi\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 8\*ArcCsch[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] - (16\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]/Sqrt[2]]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] - (4\*I)\*Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])] + 8\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])/(c\*Sqrt[d])]))/(Sqrt[d]\*e^(5/2)))

**fricas** [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4 \operatorname{arcsch}(cx) + ax^4}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*x^4\*arccsch(c\*x) + a\*x^4)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^4}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^4/(e\*x^2 + d)^3, x)

**maple** [F] time = 3.80, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{arcsch}(cx))}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

[Out] int(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8} a \left( \frac{5ex^3 + 3dx}{e^4x^4 + 2de^3x^2 + d^2e^2} - \frac{3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}e^2} \right) + b \int \frac{x^4 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] -1/8\*a\*((5\*e\*x^3 + 3\*d\*x)/(e^4\*x^4 + 2\*d\*e^3\*x^2 + d^2\*e^2) - 3\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*e^2)) + b\*integrate(x^4\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x^4\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

$$3.116 \quad \int \frac{x^2(a+bcsch^{-1}(cx))}{(d+ex^2)^3} dx$$

Optimal. Leaf size=1106

$$\frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}(c^2d-e)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}(c^2d-e)\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{a+bcsch^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+bcsch^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

[Out]  $-1/16*b*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d-e)^{(1/2)}/(1+1/c^2/x^2)^{(1/2)})/d^{(3/2)}/(c^2*d-e)^{(3/2)}-1/16*b*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d-e)^{(1/2)}/(1+1/c^2/x^2)^{(1/2)})/d^{(3/2)}/(c^2*d-e)^{(3/2)}-1/16*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}-1/16*(a+b*\operatorname{arccsch}(c*x))*\ln(1-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*(a+b*\operatorname{arccsch}(c*x))*\ln(1+c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}-1/16*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}-(-c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}+1/16*b*\operatorname{polylog}(2,-c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}-1/16*b*\operatorname{polylog}(2,c*(1/c/x+(1+1/c^2/x^2)^{(1/2)})*(-d)^{(1/2)}/(e^{(1/2)}+(-c^2*d+e)^{(1/2)}))/(-d)^{(3/2)}/e^{(3/2)}-1/16*b*\operatorname{arctanh}((c^2*d-(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d-e)^{(1/2)}/(1+1/c^2/x^2)^{(1/2)})/d^{(3/2)}/e/(c^2*d-e)^{(1/2)}-1/16*b*\operatorname{arctanh}((c^2*d+(-d)^{(1/2)}*e^{(1/2)}/x)/c/d^{(1/2)}/(c^2*d-e)^{(1/2)}/(1+1/c^2/x^2)^{(1/2)})/d^{(3/2)}/e/(c^2*d-e)^{(1/2)}+1/16*(a+b*\operatorname{arccsch}(c*x))/(-d)^{(1/2)}/e^{(1/2)}/(-d/x+(-d)^{(1/2)}*e^{(1/2)})^2+1/16*(a+b*\operatorname{arccsch}(c*x))/d/e/(-d/x+(-d)^{(1/2)}*e^{(1/2)})+1/16*(-a-b*\operatorname{arccsch}(c*x))/(-d)^{(1/2)}/e^{(1/2)}/(d/x+(-d)^{(1/2)}*e^{(1/2)})^2+1/16*(-a-b*\operatorname{arccsch}(c*x))/d/e/(d/x+(-d)^{(1/2)}*e^{(1/2)})-1/16*b*c*(1+1/c^2/x^2)^{(1/2)}/(c^2*d-e)/(-d)^{(1/2)}/e^{(1/2)}/(-d/x+(-d)^{(1/2)}*e^{(1/2)})-1/16*b*c*(1+1/c^2/x^2)^{(1/2)}/(c^2*d-e)/(-d)^{(1/2)}/e^{(1/2)}/(d/x+(-d)^{(1/2)}*e^{(1/2)})$

**Rubi [A]** time = 3.05, antiderivative size = 1106, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {6304, 5791, 5706, 5801, 731, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$\frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}(c^2d-e)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}(c^2d-e)\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{a+bcsch^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+bcsch^{-1}(cx)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^2*(a + b*\operatorname{ArcCsch}[c*x]))/(d + e*x^2)^3, x]$

[Out]  $-(b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(16*\operatorname{Sqrt}[-d]*(c^2*d - e)*\operatorname{Sqrt}[e]*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)) - (b*c*\operatorname{Sqrt}[1 + 1/(c^2*x^2)])/(16*\operatorname{Sqrt}[-d]*(c^2*d - e)*\operatorname{Sqrt}[e]*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)) + (a + b*\operatorname{ArcCsch}[c*x])/(16*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)^2) + (a + b*\operatorname{ArcCsch}[c*x])/(16*d*e*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)) - (a + b*\operatorname{ArcCsch}[c*x])/(16*\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e]*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)^2) - (a + b*\operatorname{ArcCsch}[c*x])/(16*d*e*(\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)) - (b*\operatorname{ArcTanh}[(c^2*d - (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x)/(c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e]*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]))$

$$\frac{\int \frac{1}{(16d^{3/2}(c^2d - e)^{3/2}) - (b \operatorname{ArcTanh}[\frac{c^2d - (\sqrt{-d}\sqrt{e})}{x}]/(c\sqrt{d}\sqrt{c^2d - e}\sqrt{1 + 1/(c^2x^2)})] \frac{1}{(16d^{3/2}\sqrt{c^2d - e})e} - (b \operatorname{ArcTanh}[\frac{c^2d + (\sqrt{-d}\sqrt{e})}{x}]/(c\sqrt{d}\sqrt{c^2d - e}\sqrt{1 + 1/(c^2x^2)})] \frac{1}{(16d^{3/2}(c^2d - e)^{3/2}) - (b \operatorname{ArcTanh}[\frac{c^2d + (\sqrt{-d}\sqrt{e})}{x}]/(c\sqrt{d}\sqrt{c^2d - e}\sqrt{1 + 1/(c^2x^2)})] \frac{1}{(16d^{3/2}\sqrt{c^2d - e})e} - ((a + b \operatorname{ArcCsch}[cx]) \operatorname{Log}[1 - (c\sqrt{-d}E^{\operatorname{ArcCsch}[cx]})/(\sqrt{e} - \sqrt{-(c^2d + e)})]) \frac{1}{(16(-d)^{3/2}e^{3/2})} + ((a + b \operatorname{ArcCsch}[cx]) \operatorname{Log}[1 + (c\sqrt{-d}E^{\operatorname{ArcCsch}[cx]})/(\sqrt{e} - \sqrt{-(c^2d + e)})]) \frac{1}{(16(-d)^{3/2}e^{3/2})} - ((a + b \operatorname{ArcCsch}[cx]) \operatorname{Log}[1 - (c\sqrt{-d}E^{\operatorname{ArcCsch}[cx]})/(\sqrt{e} + \sqrt{-(c^2d + e)})]) \frac{1}{(16(-d)^{3/2}e^{3/2})} + ((a + b \operatorname{ArcCsch}[cx]) \operatorname{Log}[1 + (c\sqrt{-d}E^{\operatorname{ArcCsch}[cx]})/(\sqrt{e} + \sqrt{-(c^2d + e)})]) \frac{1}{(16(-d)^{3/2}e^{3/2})} + (b \operatorname{PolyLog}[2, -(c\sqrt{-d}E^{\operatorname{ArcCsch}[cx]})/(\sqrt{e} - \sqrt{-(c^2d + e)})]) \frac{1}{(16(-d)^{3/2}e^{3/2})} - (b \operatorname{PolyLog}[2, (c\sqrt{-d}E^{\operatorname{ArcCsch}[cx]})/(\sqrt{e} - \sqrt{-(c^2d + e)})]) \frac{1}{(16(-d)^{3/2}e^{3/2})} + (b \operatorname{PolyLog}[2, -(c\sqrt{-d}E^{\operatorname{ArcCsch}[cx]})/(\sqrt{e} + \sqrt{-(c^2d + e)})]) \frac{1}{(16(-d)^{3/2}e^{3/2})} - (b \operatorname{PolyLog}[2, (c\sqrt{-d}E^{\operatorname{ArcCsch}[cx]})/(\sqrt{e} + \sqrt{-(c^2d + e)})]) \frac{1}{(16(-d)^{3/2}e^{3/2})}}$$

### Rule 206

$$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(1 \cdot \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$$

### Rule 725

$$\operatorname{Int}[1/((d + (e \cdot x)) \sqrt{a + (c \cdot x)^2}), x_{\text{Symbol}}] \rightarrow -\operatorname{Subst}[\operatorname{Int}[1/(c \cdot d^2 + a \cdot e^2 - x^2), x], x, (a \cdot e - c \cdot d \cdot x)/\sqrt{a + c \cdot x^2}] /; \operatorname{FreeQ}\{a, c, d, e, x\}$$

### Rule 731

$$\operatorname{Int}[(d + (e \cdot x))^m \cdot (a + (c \cdot x)^2)^p, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(e \cdot (d + e \cdot x)^{m+1} \cdot (a + c \cdot x^2)^{p+1})]/((m+1) \cdot (c \cdot d^2 + a \cdot e^2)), x] + \operatorname{Dist}[(c \cdot d)/(c \cdot d^2 + a \cdot e^2), \operatorname{Int}[(d + e \cdot x)^{m+1} \cdot (a + c \cdot x^2)^p, x], x] /; \operatorname{FreeQ}\{a, c, d, e, m, p, x\} \ \&\& \operatorname{NeQ}[c \cdot d^2 + a \cdot e^2, 0] \ \&\& \operatorname{EqQ}[m + 2 \cdot p + 3, 0]$$

### Rule 2190

$$\operatorname{Int}[(F)^{((g \cdot (e + (f \cdot x)))^n) \cdot ((c + (d \cdot x))^m) / ((a + (b \cdot x)) \cdot (F)^{((g \cdot (e + (f \cdot x)))^n))}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(c + d \cdot x)^m \cdot \operatorname{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x)))^n})/a]] / (b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]), x] - \operatorname{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \operatorname{Log}[F]), \operatorname{Int}[(c + d \cdot x)^{m-1} \cdot \operatorname{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x)))^n})/a]], x], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \ \&\& \operatorname{IGtQ}[m, 0]$$

### Rule 2279

$$\operatorname{Int}[\operatorname{Log}[(a + (b \cdot x)) \cdot (F)^{((e + (c \cdot x))^n)}], x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/(d \cdot e \cdot n \cdot \operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b \cdot x]/x, x], x, (F^{(e + (c + d \cdot x))^n})], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n, x\} \ \&\& \operatorname{GtQ}[a, 0]$$

### Rule 2391

$$\operatorname{Int}[\operatorname{Log}[(c + (d + (e \cdot x)^n))]/(x), x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c \cdot e \cdot x^n)]/n, x] /; \operatorname{FreeQ}\{c, d, e, n, x\} \ \&\& \operatorname{EqQ}[c \cdot d, 1]$$

### Rule 5561

$$\operatorname{Int}[(\operatorname{Cosh}[(c + (d \cdot x))] \cdot ((e + (f \cdot x))^m) / ((a + (b \cdot x)) \cdot \operatorname{Sin}[\dots])$$



```
h[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

#### Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_.)]*(b_.))^n_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
0 || IGtQ[n, 0])
```

#### Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_.)]*(b_.))^n_)*((f_.)*(x_)^(m_.))*((d_.) + (e
_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
(f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

#### Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_.)]*(b_.))^n_/((d_.) + (e_.)*(x_)), x_Symbo
l] := Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x]
] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_.)]*(b_.))^n_)*((d_.) + (e_.)*(x_)^(m_.), x
_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
- Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
- 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
0] && NeQ[m, -1]
```

#### Rule 6304

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))^n_)*(x_)^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegerQ[m, p]
```

#### Rubi steps



**Mathematica [C]** time = 6.15, size = 2053, normalized size = 1.86

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^3,x]

[Out] 
$$\begin{aligned} & -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*((( -1/16*I)*((I*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)]*x)))/((2*c^2*d - e)*(Sqrt[d] + I*Sqrt[e]*x)))]/(d*(c^2*d - e)^(3/2)))/(Sqrt[d]*e) + ((I/16)*((( -I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)]*x)))/((2*c^2*d - e)*(Sqrt[d] - I*Sqrt[e]*x)))]/(d*(c^2*d - e)^(3/2)))/(Sqrt[d]*e) - ((ArcCsch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)]*x)))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x)))/Sqrt[-(c^2*d) + e]))/Sqrt[d])/((16*d*e) - ((ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)]*x)))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x)))/Sqrt[-(c^2*d) + e]))/Sqrt[d])/((16*d*e) + ((I/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])])/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])])/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])])/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d])] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d])]))/(d^(3/2)*e^(3/2)) - ((I/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])])/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])])/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])])/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d]) + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])]/(c*Sqrt[d])]))/(d^(3/2)*e^(3/2))) \end{aligned}$$

**fricas [F]** time = 0.66, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{bx^2 \operatorname{arcsch}(cx) + ax^2}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsch(c\*x) + a\*x^2)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2/(e\*x^2 + d)^3, x)

**maple** [F] time = 5.01, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arcsch}(cx))}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

[Out] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} a \left( \frac{ex^3 - dx}{de^3x^4 + 2d^2e^2x^2 + d^3e} + \frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}de} \right) + b \int \frac{x^2 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/8\*a\*((e\*x^3 - d\*x)/(d\*e^3\*x^4 + 2\*d^2\*e^2\*x^2 + d^3\*e) + arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d\*e) + b\*integrate(x^2\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3,x)

[Out] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

$$3.117 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex^2)^3} dx$$

Optimal. Leaf size=1096

$$\frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(c^2d-e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{5(a+b\operatorname{csch}^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5(a+b\operatorname{csch}^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

[Out]  $\frac{1}{16}b*e*\operatorname{arctanh}\left(\frac{c^2*d-(-d)^{1/2}*e^{1/2}/x}{c/d^{1/2}/(c^2*d-e)^{1/2}/(1+1/c^2/x^2)^{1/2}}\right)/d^{5/2}/(c^2*d-e)^{3/2} + \frac{1}{16}b*e*\operatorname{arctanh}\left(\frac{c^2*d+(-d)^{1/2}*e^{1/2}/x}{c/d^{1/2}/(c^2*d-e)^{1/2}/(1+1/c^2/x^2)^{1/2}}\right)/d^{5/2}/(c^2*d-e)^{3/2} + \frac{5}{16}b*\operatorname{arctanh}\left(\frac{c^2*d-(-d)^{1/2}*e^{1/2}/x}{c/d^{1/2}/(c^2*d-e)^{1/2}/(1+1/c^2/x^2)^{1/2}}\right)/d^{5/2}/(c^2*d-e)^{1/2} + \frac{5}{16}b*\operatorname{arctanh}\left(\frac{c^2*d+(-d)^{1/2}*e^{1/2}/x}{c/d^{1/2}/(c^2*d-e)^{1/2}/(1+1/c^2/x^2)^{1/2}}\right)/d^{5/2}/(c^2*d-e)^{1/2} + \frac{3}{16}*(a+b*\operatorname{arccsch}(c*x))*\ln\left(\frac{1-c*(1/c/x+(1+1/c^2/x^2)^{1/2})}{(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2})}\right)/(-d)^{5/2}/e^{1/2} - \frac{3}{16}*(a+b*\operatorname{arccsch}(c*x))*\ln\left(\frac{1+c*(1/c/x+(1+1/c^2/x^2)^{1/2})}{(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2})}\right)/(-d)^{5/2}/e^{1/2} + \frac{3}{16}*(a+b*\operatorname{arccsch}(c*x))*\ln\left(\frac{1-c*(1/c/x+(1+1/c^2/x^2)^{1/2})}{(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2})}\right)/(-d)^{5/2}/e^{1/2} - \frac{3}{16}*(a+b*\operatorname{arccsch}(c*x))*\ln\left(\frac{1+c*(1/c/x+(1+1/c^2/x^2)^{1/2})}{(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2})}\right)/(-d)^{5/2}/e^{1/2} - \frac{3}{16}b*\operatorname{polylog}\left(2, -c*(1/c/x+(1+1/c^2/x^2)^{1/2})\right)*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2})/(-d)^{5/2}/e^{1/2} + \frac{3}{16}b*\operatorname{polylog}\left(2, c*(1/c/x+(1+1/c^2/x^2)^{1/2})\right)*(-d)^{1/2}/(e^{1/2}-(-c^2*d+e)^{1/2})/(-d)^{5/2}/e^{1/2} - \frac{3}{16}b*\operatorname{polylog}\left(2, -c*(1/c/x+(1+1/c^2/x^2)^{1/2})\right)*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2})/(-d)^{5/2}/e^{1/2} + \frac{3}{16}b*\operatorname{polylog}\left(2, c*(1/c/x+(1+1/c^2/x^2)^{1/2})\right)*(-d)^{1/2}/(e^{1/2}+(-c^2*d+e)^{1/2})/(-d)^{5/2}/e^{1/2} + \frac{1}{16}*(a+b*\operatorname{arccsch}(c*x))*e^{1/2}/(-d)^{3/2}/(d/x+(-d)^{1/2}*e^{1/2})^2 - \frac{5}{16}*(a+b*\operatorname{arccsch}(c*x))/d^2/(-d/x+(-d)^{1/2}*e^{1/2}) - \frac{1}{16}*(a+b*\operatorname{arccsch}(c*x))*e^{1/2}/(-d)^{3/2}/(d/x+(-d)^{1/2}*e^{1/2})^2 + \frac{5}{16}*(a+b*\operatorname{arccsch}(c*x))/d^2/(d/x+(-d)^{1/2}*e^{1/2}) - \frac{1}{16}b*c*e^{1/2}*(1+1/c^2/x^2)^{1/2}/(-d)^{3/2}/(c^2*d-e)/(-d/x+(-d)^{1/2}*e^{1/2}) - \frac{1}{16}b*c*e^{1/2}*(1+1/c^2/x^2)^{1/2}/(-d)^{3/2}/(c^2*d-e)/(d/x+(-d)^{1/2}*e^{1/2})$

**Rubi [A]** time = 3.74, antiderivative size = 1096, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$ , Rules used = {6294, 5791, 5706, 5801, 731, 725, 206, 5799, 5561, 2190, 2279, 2391}

$$\frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}(c^2d-e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{5(a+b\operatorname{csch}^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5(a+b\operatorname{csch}^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{a + b*\operatorname{ArcCsch}[c*x]}{(d + e*x^2)^3}, x\right]$

[Out]  $-(b*c*\operatorname{Sqrt}[e]*\operatorname{Sqrt}\left[1 + \frac{1}{(c^2*x^2)}\right])/((16*(-d)^{3/2}*(c^2*d - e)*( \operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)) - (b*c*\operatorname{Sqrt}[e]*\operatorname{Sqrt}\left[1 + \frac{1}{(c^2*x^2)}\right])/((16*(-d)^{3/2}*(c^2*d - e)*( \operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)) + (\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcCsch}[c*x]))/(16*(-d)^{3/2}*( \operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)^2) - (5*(a + b*\operatorname{ArcCsch}[c*x]))/(16*d^2*( \operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] - d/x)) - (\operatorname{Sqrt}[e]*(a + b*\operatorname{ArcCsch}[c*x]))/(16*(-d)^{3/2}*( \operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)^2) + (5*(a + b*\operatorname{ArcCsch}[c*x]))/(16*d^2*( \operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e] + d/x)) + (5*b*\operatorname{ArcTanh}\left[\frac{c^2*d - (\operatorname{Sqrt}[-d]*\operatorname{Sqrt}[e])/x}{c*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[c^2*d - e}\right])$

```
e]*Sqrt[1 + 1/(c^2*x^2)]])]/(16*d^(5/2)*Sqrt[c^2*d - e]) + (b*e*ArcTanh[(c^
2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)
]))]/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[
e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)]])]/(16*d^(5/2)*Sqrt
[c^2*d - e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)]])]/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (3*(a
+ b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2
*d) + e])])]/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])]/(16*(-d)^(5/2)*Sqrt
[e]) + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
+ Sqrt[-(c^2*d) + e])])]/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x])*
Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])]/(16*(-
d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
- Sqrt[-(c^2*d) + e])])]/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[
-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])]/(16*(-d)^(5/2)*Sqrt[e]
) - (3*b*PolyLog[2, -(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
+ e])])]/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*
x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])]/(16*(-d)^(5/2)*Sqrt[e])
```

### Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rule 725

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

### Rule 731

```
Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[
(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + D
ist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 5561

```
Int[(Cosh[(c_) + (d_)*(x_)]*((e_) + (f_)*(x_))^(m_))/((a_) + (b_)*Sin
```

```
h[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

#### Rule 5706

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
0 || IGtQ[n, 0])
```

#### Rule 5791

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
(f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

#### Rule 5799

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] := Subst[Int[((a + b*x)^n*Cosh[x])/(c*d + e*Sinh[x]), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

#### Rule 5801

```
Int[((a_.) + ArcSinh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^n)/(e*(m + 1)), x]
- Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSinh[c*x])^(n
- 1))/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
0] && NeQ[m, -1]
```

#### Rule 6294

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcSinh[x/c])^n)/x^(2*(p + 1
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex^2)^3} dx &= -\operatorname{Subst} \left( \int \frac{x^4 (a + b \sinh^{-1}(\frac{x}{c}))}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left( \int \left( \frac{e^2 (a + b \sinh^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)^3} - \frac{2e (a + b \sinh^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)^2} + \frac{a + b \sinh^{-1}(\frac{x}{c})}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}(\frac{x}{c})}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}(\frac{x}{c})}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}(\frac{x}{c})}{(e + dx^2)^3} dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{\operatorname{Subst} \left( \int \left( \frac{a + b \sinh^{-1}(\frac{x}{c})}{2\sqrt{e}(\sqrt{e} - \sqrt{-d}x)} + \frac{a + b \sinh^{-1}(\frac{x}{c})}{2\sqrt{e}(\sqrt{e} + \sqrt{-d}x)} \right) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \operatorname{Subst} \left( \int \left( -\frac{d(a + b \sinh^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e} - dx)} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{3 \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{16d} + \frac{3 \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{16d} + \frac{3 \operatorname{Subst} \left( \int \frac{a + b \sinh^{-1}(\frac{x}{c})}{-de} dx, x, \frac{1}{x} \right)}{8d} \\
&= \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)^2} - \frac{5(a + b \operatorname{csch}^{-1}(cx))}{16d^2 \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)^2} + \frac{5(a + b \operatorname{csch}^{-1}(cx))}{16d^2 \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= -\frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{bc\sqrt{e} \sqrt{1 + \frac{1}{c^2x^2}}}{16(-d)^{3/2} (c^2d - e) \left( \sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{\sqrt{e} (a + b \operatorname{csch}^{-1}(cx))}{16(-d)^{3/2} \left( \sqrt{-d}\sqrt{e} - \frac{d}{x} \right)}
\end{aligned}$$



**Mathematica [C]** time = 6.07, size = 2038, normalized size = 1.86

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x^2)^3,x]

[Out] (a\*x)/(4\*d\*(d + e\*x^2)^2) + (3\*a\*x)/(8\*d^2\*(d + e\*x^2)) + (3\*a\*ArcTan[(Sqrt[e]\*x)/Sqrt[d]])/(8\*d^(5/2)\*Sqrt[e]) + b\*(((I/16)\*((I\*c\*Sqrt[e]\*Sqrt[1 + 1/(c^2\*x^2)])\*x)/(Sqrt[d]\*(c^2\*d - e)\*((-I)\*Sqrt[d] + Sqrt[e]\*x)) - ArcCsch[c\*x]/(Sqrt[e]\*((-I)\*Sqrt[d] + Sqrt[e]\*x)^2) - ArcSinh[1/(c\*x)]/(d\*Sqrt[e]) + (I\*(2\*c^2\*d - e)\*Log[(4\*d\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(Sqrt[e] + I\*c\*(c\*Sqrt[d] - Sqrt[c^2\*d - e]\*Sqrt[1 + 1/(c^2\*x^2)])\*x))/((2\*c^2\*d - e)\*(Sqrt[d] + I\*Sqrt[e]\*x)))]/(d\*(c^2\*d - e)^(3/2)))/d^(3/2) - ((I/16)\*(((I)\*c\*Sqrt[e]\*Sqrt[1 + 1/(c^2\*x^2)]\*x)/(Sqrt[d]\*(c^2\*d - e)\*(I\*Sqrt[d] + Sqrt[e]\*x)) - ArcCsch[c\*x]/(Sqrt[e]\*(I\*Sqrt[d] + Sqrt[e]\*x)^2) - ArcSinh[1/(c\*x)]/(d\*Sqrt[e]) + (I\*(2\*c^2\*d - e)\*Log[((4\*I)\*d\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] + Sqrt[c^2\*d - e]\*Sqrt[1 + 1/(c^2\*x^2)])\*x))/((2\*c^2\*d - e)\*(Sqrt[d] - I\*Sqrt[e]\*x)))]/(d\*(c^2\*d - e)^(3/2)))/d^(3/2) - (3\*(-ArcCsch[c\*x]/(I\*Sqrt[d]\*Sqrt[e] + e\*x)) - (I\*(ArcSinh[1/(c\*x)]/Sqrt[e] - Log[(2\*Sqrt[d]\*Sqrt[e]\*(I\*Sqrt[e] + c\*(c\*Sqrt[d] + I\*Sqrt[-(c^2\*d) + e]\*Sqrt[1 + 1/(c^2\*x^2)])\*x))/(Sqrt[-(c^2\*d) + e]\*(I\*Sqrt[d] + Sqrt[e]\*x))]/Sqrt[-(c^2\*d) + e]))/Sqrt[d]))/(16\*d^2) - (3\*(-ArcCsch[c\*x]/((-I)\*Sqrt[d]\*Sqrt[e] + e\*x)) + (I\*(ArcSinh[1/(c\*x)]/Sqrt[e] - Log[(-2\*Sqrt[d]\*Sqrt[e]\*(Sqrt[e] + c\*(I\*c\*Sqrt[d] + Sqrt[-(c^2\*d) + e]\*Sqrt[1 + 1/(c^2\*x^2)])\*x))/(Sqrt[-(c^2\*d) + e]\*(Sqrt[d] + I\*Sqrt[e]\*x))]/Sqrt[-(c^2\*d) + e]))/Sqrt[d]))/(16\*d^2) + (((3\*I)/128)\*(Pi^2 - (4\*I)\*Pi\*ArcCsch[c\*x] - 8\*ArcCsch[c\*x]^2 + 32\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]]/Sqrt[2])\*ArcTan[((c\*Sqrt[d] - Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])] + (4\*I)\*Pi\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + 8\*ArcCsch[c\*x]\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + (16\*I)\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 - (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + (4\*I)\*Pi\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + 8\*ArcCsch[c\*x]\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] - (16\*I)\*ArcSin[Sqrt[1 + Sqrt[e]/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 + (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])) - (4\*I)\*Pi\*Log[Sqrt[e] + (I\*Sqrt[d])/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + 8\*PolyLog[2, ((-I)\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d]))]/(d^(5/2)\*Sqrt[e]) - (((3\*I)/128)\*(Pi^2 - (4\*I)\*Pi\*ArcCsch[c\*x] - 8\*ArcCsch[c\*x]^2 - 32\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]]/Sqrt[2])\*ArcTan[((c\*Sqrt[d] + Sqrt[e])\*Cot[(Pi + (2\*I)\*ArcCsch[c\*x])/4])/Sqrt[-(c^2\*d) + e]] - 8\*ArcCsch[c\*x]\*Log[1 - E^(-2\*ArcCsch[c\*x])] + (4\*I)\*Pi\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + 8\*ArcCsch[c\*x]\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + (16\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 + (I\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + (4\*I)\*Pi\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + 8\*ArcCsch[c\*x]\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] - (16\*I)\*ArcSin[Sqrt[1 - Sqrt[e]/(c\*Sqrt[d])]]/Sqrt[2])\*Log[1 - (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])) - (4\*I)\*Pi\*Log[Sqrt[e] - (I\*Sqrt[d])/x] + 4\*PolyLog[2, E^(-2\*ArcCsch[c\*x])] + 8\*PolyLog[2, ((-I)\*(-Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d])] + 8\*PolyLog[2, (I\*(Sqrt[e] + Sqrt[-(c^2\*d) + e])\*E^ArcCsch[c\*x])]/(c\*Sqrt[d]))]/(d^(5/2)\*Sqrt[e]))

**fricas [F]** time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x^2 + d)^3, x)

**maple** [F] time = 3.77, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsch}(cx)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

[Out] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} a \left( \frac{3ex^3 + 5dx}{d^2e^2x^4 + 2d^3ex^2 + d^4} + \frac{3 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}d^2} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^3,x, algorithm="maxima")

[Out] 1/8\*a\*((3\*e\*x^3 + 5\*d\*x)/(d^2\*e^2\*x^4 + 2\*d^3\*e\*x^2 + d^4) + 3\*arctan(e\*x/sqrt(d\*e))/(sqrt(d\*e)\*d^2)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(d + e\*x^2)^3,x)

[Out] int((a + b\*asinh(1/(c\*x)))/(d + e\*x^2)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*3,x)

[Out] Timed out

### 3.118 $\int x^5 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=413

$$\frac{d^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^3} + \frac{8bcd^{7/2}}{e^3}$$

[Out]  $1/3*d^2*(e*x^2+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^3-2/5*d*(e*x^2+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^3+1/7*(e*x^2+d)^{(7/2)}*(a+b*\operatorname{arccsch}(c*x))/e^3+1/1680*b*(105*c^6*d^3+35*c^4*d^2*e+63*c^2*d*e^2-75*e^3)*x*\operatorname{arctan}(e^{(1/2)}*(-c^2*x^2-1)^{(1/2)})/c/(e*x^2+d)^{(1/2)}/c^6/e^{(5/2)}/(-c^2*x^2)^{(1/2)}+8/105*b*c*d^{(7/2)}*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(-c^2*x^2-1)^{(1/2)})/e^3/(-c^2*x^2)^{(1/2)}-1/840*b*(29*c^2*d+25*e)*x*(e*x^2+d)^{(3/2)}*(-c^2*x^2-1)^{(1/2)}/c^3/e^2/(-c^2*x^2)^{(1/2)}+1/42*b*x*(e*x^2+d)^{(5/2)}*(-c^2*x^2-1)^{(1/2)}/c/e^2/(-c^2*x^2)^{(1/2)}-1/1680*b*(23*c^4*d^2-12*c^2*d*e-75*e^2)*x*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c^5/e^2/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 1.41, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 6302, 12, 1615, 154, 157, 63, 217, 203, 93, 204}

$$\frac{d^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^3} - \frac{bx\sqrt{-c^2}}{e^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^5*\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcCsCh}[c*x]), x]$

[Out]  $-(b*(23*c^4*d^2 - 12*c^2*d*e - 75*e^2)*x*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(1680*c^5*e^2*\operatorname{Sqrt}[-(c^2*x^2)]) - (b*(29*c^2*d + 25*e)*x*\operatorname{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/(840*c^3*e^2*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*x*\operatorname{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(5/2)})/(42*c*e^2*\operatorname{Sqrt}[-(c^2*x^2)]) + (d^2*(d + e*x^2)^{(3/2)}*(a + b*\operatorname{ArcCsCh}[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^{(5/2)}*(a + b*\operatorname{ArcCsCh}[c*x]))/(5*e^3) + ((d + e*x^2)^{(7/2)}*(a + b*\operatorname{ArcCsCh}[c*x]))/(7*e^3) + (b*(105*c^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(1680*c^6*e^{(5/2)}*\operatorname{Sqrt}[-(c^2*x^2)]) + (8*b*c*d^{(7/2)}*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 - c^2*x^2])])/(105*e^3*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 43

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ (!\operatorname{IntegerQ}[n] \ || \ (\operatorname{EqQ}[c, 0] \ \&\& \ \operatorname{LeQ}[7*m + 4*n + 4, 0])) \ || \ \operatorname{LtQ}[9*m + 5*(n + 1), 0] \ || \ \operatorname{GtQ}[m + n + 2, 0]$

#### Rule 63

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] := \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[-1, m, 0] \ \&\& \ \operatorname{LeQ}[-1, n, 0] \ \&\& \ \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]]$

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 154

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(h\*(a + b\*x)^m\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 2)), x] + Dist[1/(d\*f\*(m + n + p + 2)), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*g\*(m + n + p + 2) - h\*(b\*c\*e\*m + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + (b\*d\*f\*g\*(m + n + p + 2) + h\*(a\*d\*f\*m - b\*(d\*e\*(m + n + 1) + c\*f\*(m + p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 266

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 1615

Int[(Px\_)\*((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k\*(a + b\*x)^(m + q - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*b^(q - 1)\*(m + n + p + q + 1)), x] + Dist[1/(d\*f\*b^q\*(m + n + p + q + 1)), Int[(a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^p\*ExpandToSum[d\*f\*b^q\*(m + n + p + q + 1)\*Px - d\*f\*k\*(m + n + p + q + 1)\*(a + b\*x)^q + k\*(a + b\*x)^(q -

2)\*(a^2\*d\*f\*(m + n + p + q + 1) - b\*(b\*c\*e\*(m + q - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*(m + q) + n + p) - b\*(d\*e\*(m + q + n) + c\*f\*(m + q + p)))\*x), x], x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 6302

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
 \int x^5 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx &= \frac{d^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} + \frac{d^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= -\frac{b(29c^2d + 25e)x\sqrt{-1 - c^2x^2} (d + ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} \\
 &= -\frac{b(23c^4d^2 - 12c^2de - 75e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d + 25e)}{840} \\
 &= -\frac{b(23c^4d^2 - 12c^2de - 75e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d + 25e)}{840} \\
 &= -\frac{b(23c^4d^2 - 12c^2de - 75e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d + 25e)}{840} \\
 &= -\frac{b(23c^4d^2 - 12c^2de - 75e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d + 25e)}{840} \\
 &= -\frac{b(23c^4d^2 - 12c^2de - 75e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d + 25e)}{840}
 \end{aligned}$$

**Mathematica** [C] time = 0.70, size = 345, normalized size = 0.84

$$\frac{\sqrt{d+ex^2} \left( 16ac^5 (8d^3 - 4d^2ex^2 + 3de^2x^4 + 15e^3x^6) + 16bc^5 \operatorname{csch}^{-1}(cx) (8d^3 - 4d^2ex^2 + 3de^2x^4 + 15e^3x^6) + bex\sqrt{d+ex^2} \right)}{1680c^5e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] -1/3360*(b*(128*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))] - (e*(105*c^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)]/Sqrt[1 + c^2*x^2]))/(c^5*e^3*x*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(16*a*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6) + b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(75*e^2 - 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)*ArcCsch[c*x]))/(1680*c^5*e^3)
```

**fricas** [A] time = 5.10, size = 1951, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] [1/6720*(128*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 + 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 64*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3), 1/3360*(64*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 + 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e) + 32*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^7*e^3), 1/6720*(256*b*c^7*sqrt(-d)*d^3*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2) - (105*b*c^6*d^3 + 35*b*c^4*d^2*e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 64*(15*b*c^7*e^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(11*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
```

$$\left. \right) \sqrt{e x^2 + d} / (c^7 e^3), 1/3360 * (128 * b * c^7 * \sqrt{-d} * d^3 * \arctan(1/2 * (c^3 * d + c * e) * x^3 + 2 * c * d * x) * \sqrt{e x^2 + d} * \sqrt{-d} * \sqrt{(c^2 * x^2 + 1) / (c^2 * x^2)}) / (c^2 * d * e * x^4 + (c^2 * d^2 + d * e) * x^2 + d^2) - (105 * b * c^6 * d^3 + 35 * b * c^4 * d^2 * e + 63 * b * c^2 * d * e^2 - 75 * b * e^3) * \sqrt{-e} * \arctan(1/2 * (2 * c^2 * e * x^3 + (c^2 * d + e) * x) * \sqrt{e x^2 + d} * \sqrt{-e} * \sqrt{(c^2 * x^2 + 1) / (c^2 * x^2)}) / (c^2 * e^2 * x^4 + (c^2 * d * e + e^2) * x^2 + d * e) + 32 * (15 * b * c^7 * e^3 * x^6 + 3 * b * c^7 * d * e^2 * x^4 - 4 * b * c^7 * d^2 * e * x^2 + 8 * b * c^7 * d^3) * \sqrt{e x^2 + d} * \log((c * x * \sqrt{(c^2 * x^2 + 1) / (c^2 * x^2)}) + 1) / (c * x)) + 2 * (240 * a * c^7 * e^3 * x^6 + 48 * a * c^7 * d * e^2 * x^4 - 64 * a * c^7 * d^2 * e * x^2 + 128 * a * c^7 * d^3 + (40 * b * c^6 * e^3 * x^5 + 2 * (11 * b * c^6 * d * e^2 - 25 * b * c^4 * e^3) * x^3 - (41 * b * c^6 * d^2 * e + 38 * b * c^4 * d * e^2 - 75 * b * c^2 * e^3) * x) * \sqrt{(c^2 * x^2 + 1) / (c^2 * x^2)}) * \sqrt{e x^2 + d} / (c^7 * e^3)]$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e x^2 + d} (b \operatorname{arcsch}(c x) + a) x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*x^5, x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int x^5 (a + b \operatorname{arccsch}(c x)) \sqrt{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x)

[Out] int(x^5\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{105} \left( \frac{15 (e x^2 + d)^{\frac{3}{2}} x^4}{e} - \frac{12 (e x^2 + d)^{\frac{3}{2}} d x^2}{e^2} + \frac{8 (e x^2 + d)^{\frac{3}{2}} d^2}{e^3} \right) a + \frac{1}{105} b \left( \frac{(15 e^3 x^6 + 3 d e^2 x^4 - 4 d^2 e x^2 + 8 d^3) \sqrt{e x^2 + d}}{e^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/105\*(15\*(e\*x^2 + d)^(3/2)\*x^4/e - 12\*(e\*x^2 + d)^(3/2)\*d\*x^2/e^2 + 8\*(e\*x^2 + d)^(3/2)\*d^2/e^3)\*a + 1/105\*b\*((15\*e^3\*x^6 + 3\*d\*e^2\*x^4 - 4\*d^2\*e\*x^2 + 8\*d^3)\*sqrt(e\*x^2 + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/e^3 + 105\*integrate(1/105\*(15\*c^2\*e^3\*x^7 + 3\*c^2\*d\*e^2\*x^5 - 4\*c^2\*d^2\*e\*x^3 + 8\*c^2\*d^3\*x)\*sqrt(e\*x^2 + d)/(c^2\*e^3\*x^2 + e^3 + (c^2\*e^3\*x^2 + e^3)\*sqrt(c^2\*x^2 + 1)), x) - 105\*integrate(1/105\*(15\*(7\*e^3\*log(c) + e^3)\*c^2\*x^7 - 4\*c^2\*d^2\*e\*x^3 + 8\*c^2\*d^3\*x + 3\*(c^2\*d\*e^2 + 35\*e^3\*log(c))\*x^5 + 105\*(c^2\*e^3\*x^7 + e^3\*x^5)\*log(x))\*sqrt(e\*x^2 + d)/(c^2\*e^3\*x^2 + e^3), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \sqrt{e x^2 + d} \left( a + b \operatorname{asinh}\left(\frac{1}{c x}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x^5\*(d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsch(c\*x))\*(e\*x\*\*2+d)\*\*(1/2), x)

[Out] Timed out



### 3.119 $\int x^3 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=302

$$\frac{d(d+ex^2)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e^2} - \frac{2bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{15e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}}{20ce}$$

[Out]  $-1/3*d*(e*x^2+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^2+1/5*(e*x^2+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^2-1/120*b*(15*c^4*d^2+10*c^2*d*e-9*e^2)*x*\operatorname{arctan}(e^{(1/2)}*(-c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^4/e^{(3/2)}/(-c^2*x^2)^{(1/2)}-2/15*b*c*d^{(5/2)}*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(-c^2*x^2-1)^{(1/2)})/e^2/(-c^2*x^2)^{(1/2)}+1/20*b*x*(e*x^2+d)^{(3/2)}*(-c^2*x^2-1)^{(1/2)}/c/e/(-c^2*x^2)^{(1/2)}+1/120*b*(c^2*d-9*e)*x*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c^3/e/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.43, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 6302, 12, 573, 154, 157, 63, 217, 203, 93, 204}

$$\frac{d(d+ex^2)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e^2} - \frac{2bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{15e^2\sqrt{-c^2x^2}} - \frac{bx(15c^4d^2 - \dots)}{20ce}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $(b*(c^2*d - 9*e)*x*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(120*c^3*e*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*x*\operatorname{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/(20*c*e*\operatorname{Sqrt}[-(c^2*x^2)]) - (d*(d + e*x^2)^{(3/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(3*e^2) + ((d + e*x^2)^{(5/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(5*e^2) - (b*(15*c^4*d^2 + 10*c^2*d*e - 9*e^2)*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(120*c^4*e^{(3/2)}*\operatorname{Sqrt}[-(c^2*x^2)]) - (2*b*c*d^{(5/2)}*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 - c^2*x^2])])/(15*e^2*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)*(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 43

$\operatorname{Int}[(a_*) + (b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ (\ !\operatorname{IntegerQ}[n] \ || \ (\operatorname{EqQ}[c, 0] \ \&\& \ \operatorname{LeQ}[7*m + 4*n + 4, 0]) \ || \ \operatorname{LtQ}[9*m + 5*(n + 1), 0] \ || \ \operatorname{GtQ}[m + n + 2, 0])$

#### Rule 63

$\operatorname{Int}[(a_*) + (b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[-1, m, 0] \ \&\& \ \operatorname{LeQ}[-1, n, 0] \ \&\& \ \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \ \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 93

$\operatorname{Int}[(a_*) + (b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_)^{(n_*)})/((e_*) + (f_*)*(x_)), x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m + 1) - 1)}]/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}$

], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 154

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(h\*(a + b\*x)^m\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 2)), x] + Dist[1/(d\*f\*(m + n + p + 2)), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*g\*(m + n + p + 2) - h\*(b\*c\*e\*m + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + (b\*d\*f\*g\*(m + n + p + 2) + h\*(a\*d\*f\*m - b\*(d\*e\*(m + n + 1) + c\*f\*(m + p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

#### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_))) / ((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[Rt[-b, 2]\*x]/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 573

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_))^(r\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q\*(e + f\*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]

#### Rule 6302

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[Simplify[Integrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0]))

|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
 \int x^3 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx &= -\frac{d(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} - \dots \\
 &= -\frac{d(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} - \dots \\
 &= -\frac{d(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} - \dots \\
 &= \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} - \dots \\
 &= \frac{b(c^2d - 9e)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{120c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} - \dots \\
 &= \frac{b(c^2d - 9e)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{120c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} - \dots \\
 &= \frac{b(c^2d - 9e)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{120c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} - \dots \\
 &= \frac{b(c^2d - 9e)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{120c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} - \dots
 \end{aligned}$$

**Mathematica [A]** time = 0.75, size = 337, normalized size = 1.12

$$\frac{\sqrt{d + ex^2} \left( 8ac^3 (-2d^2 + dex^2 + 3e^2x^4) + 8bc^3 \operatorname{csch}^{-1}(cx) (-2d^2 + dex^2 + 3e^2x^4) + bex\sqrt{\frac{1}{c^2x^2} + 1} (c^2(7d + 6ex^2)) \right)}{120c^3e^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

[Out] (Sqrt[d + e\*x^2]\*(8\*a\*c^3\*(-2\*d^2 + d\*e\*x^2 + 3\*e^2\*x^4) + b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-9\*e + c^2\*(7\*d + 6\*e\*x^2)) + 8\*b\*c^3\*(-2\*d^2 + d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsch[c\*x]))/(120\*c^3\*e^2) + (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-(Sqrt[c^2]\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(15\*c^4\*d^2 + 10\*c^2\*d\*e - 9\*e^2)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e]]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])) + 16\*c^7\*d^(5/2)\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c

$\sqrt{2x^2})/\sqrt{-d - ex^2}]])/(120c^6e^2\sqrt{1 + c^2x^2})\sqrt{d + ex^2})$

**fricas** [A] time = 1.80, size = 1625, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/480\*(16\*b\*c^5\*d^(5/2)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 + 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) - (15\*b\*c^4\*d^2 + 10\*b\*c^2\*d\*e - 9\*b\*e^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 + 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2) + 32\*(3\*b\*c^5\*e^2\*x^4 + b\*c^5\*d\*e\*x^2 - 2\*b\*c^5\*d^2)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 4\*(24\*a\*c^5\*e^2\*x^4 + 8\*a\*c^5\*d\*e\*x^2 - 16\*a\*c^5\*d^2 + (6\*b\*c^4\*e^2\*x^3 + (7\*b\*c^4\*d\*e - 9\*b\*c^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^5\*e^2), 1/240\*(8\*b\*c^5\*d^(5/2)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 + 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) + (15\*b\*c^4\*d^2 + 10\*b\*c^2\*d\*e - 9\*b\*e^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^3 + (c^2\*d + e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(-e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e) + 16\*(3\*b\*c^5\*e^2\*x^4 + b\*c^5\*d\*e\*x^2 - 2\*b\*c^5\*d^2)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*(24\*a\*c^5\*e^2\*x^4 + 8\*a\*c^5\*d\*e\*x^2 - 16\*a\*c^5\*d^2 + (6\*b\*c^4\*e^2\*x^3 + (7\*b\*c^4\*d\*e - 9\*b\*c^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^5\*e^2), -1/480\*(32\*b\*c^5\*sqrt(-d)\*d^2\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2) + (15\*b\*c^4\*d^2 + 10\*b\*c^2\*d\*e - 9\*b\*e^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 + 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2) - 32\*(3\*b\*c^5\*e^2\*x^4 + b\*c^5\*d\*e\*x^2 - 2\*b\*c^5\*d^2)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - 4\*(24\*a\*c^5\*e^2\*x^4 + 8\*a\*c^5\*d\*e\*x^2 - 16\*a\*c^5\*d^2 + (6\*b\*c^4\*e^2\*x^3 + (7\*b\*c^4\*d\*e - 9\*b\*c^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^5\*e^2), -1/240\*(16\*b\*c^5\*sqrt(-d)\*d^2\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2) - (15\*b\*c^4\*d^2 + 10\*b\*c^2\*d\*e - 9\*b\*e^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^3 + (c^2\*d + e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(-e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e) - 16\*(3\*b\*c^5\*e^2\*x^4 + b\*c^5\*d\*e\*x^2 - 2\*b\*c^5\*d^2)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - 2\*(24\*a\*c^5\*e^2\*x^4 + 8\*a\*c^5\*d\*e\*x^2 - 16\*a\*c^5\*d^2 + (6\*b\*c^4\*e^2\*x^3 + (7\*b\*c^4\*d\*e - 9\*b\*c^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^5\*e^2)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*x^3, x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{arcsch}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)`

[Out] `int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15} \left( \frac{3(ex^2 + d)^{\frac{3}{2}}x^2}{e} - \frac{2(ex^2 + d)^{\frac{3}{2}}d}{e^2} \right) a + \frac{1}{15} b \left( \frac{(3e^2x^4 + dex^2 - 2d^2)\sqrt{ex^2 + d} \log(\sqrt{c^2x^2 + 1} + 1)}{e^2} + 15 \int \frac{1}{15} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `1/15*(3*(e*x^2 + d)^(3/2)*x^2/e - 2*(e*x^2 + d)^(3/2)*d/e^2)*a + 1/15*b*((3*e^2*x^4 + d*e*x^2 - 2*d^2)*sqrt(e*x^2 + d)*log(sqrt(c^2*x^2 + 1) + 1)/e^2 + 15*integrate(1/15*(3*c^2*e^2*x^5 + c^2*d*e*x^3 - 2*c^2*d^2*x)*sqrt(e*x^2 + d)/(c^2*e^2*x^2 + e^2 + (c^2*e^2*x^2 + e^2)*sqrt(c^2*x^2 + 1)), x) - 15*integrate(1/15*(3*(5*e^2*log(c) + e^2)*c^2*x^5 - 2*c^2*d^2*x + (c^2*d*e + 15*e^2*log(c))*x^3 + 15*(c^2*e^2*x^5 + e^2*x^3)*log(x))*sqrt(e*x^2 + d)/(c^2*e^2*x^2 + e^2), x))`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{ex^2 + d} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)`

[Out] `int(x^3*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)`

[Out] `Integral(x**3*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)`

### 3.120 $\int x\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=203

$$\frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{bx(3c^2d-e) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{d+ex^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{-c^2x^2}}$$

[Out]  $1/3*(e*x^2+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e+1/3*b*c*d^{(3/2)}*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(-c^2*x^2-1)^{(1/2)})/e/(-c^2*x^2)^{(1/2)}+1/6*b*(3*c^2*d-e)*x*\operatorname{arctan}(e^{(1/2)}*(-c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^2/e^{(1/2)}/(-c^2*x^2)^{(1/2)}+1/6*b*x*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {6300, 446, 102, 157, 63, 217, 203, 93, 204}

$$\frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{bx(3c^2d-e) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{d+ex^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x*\operatorname{Sqrt}[d+e*x^2]*(a+b*\operatorname{ArcCsch}[c*x]),x]$

[Out]  $(b*x*\operatorname{Sqrt}[-1-c^2*x^2]*\operatorname{Sqrt}[d+e*x^2])/(6*c*\operatorname{Sqrt}[-(c^2*x^2)]) + ((d+e*x^2)^{(3/2)}*(a+b*\operatorname{ArcCsch}[c*x]))/(3*e) + (b*(3*c^2*d-e)*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1-c^2*x^2])/(c*\operatorname{Sqrt}[d+e*x^2])])/(6*c^2*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*c*d^{(3/2)}*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d+e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1-c^2*x^2])])/(3*e*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 63

$\operatorname{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_)^n), x\_Symbol] :> \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 93

$\operatorname{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_)^n)/((e_.) + (f_.)*(x_)^p), x\_Symbol] :> \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)}*(c - (a*d)/b + (d*x^q)/b)^n, x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \operatorname{EqQ}[m + n + 1, 0] \&\& \operatorname{RationalQ}[n] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{SimplerQ}[a + b*x, c + d*x]$

#### Rule 102

$\operatorname{Int}[(a_.) + (b_.)*(x_)^m*((c_.) + (d_.)*(x_)^n)*((e_.) + (f_.)*(x_)^p), x\_Symbol] :> \operatorname{Simp}[(b*(a + b*x)^{(m-1)}*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})/(d*f*(m+n+p+1)), x] + \operatorname{Dist}[1/(d*f*(m+n+p+1)), \operatorname{Int}[(a + b*x)^{(m-2)}*(c + d*x)^n*(e + f*x)^p*\operatorname{Simp}[a^2*d*f*(m+n+p+1) - b*(b*c*e*(m-1) + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(2*m+n+p) - b*(d*e*(m+n) + c*f*(m+p))]*x, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, n, p\}, x] \&\& \operatorname{GtQ}[m, 1] \&\& \operatorname{NeQ}[m+n+p+1, 0] \&\& \operatorname{IntegersQ}[2*m, 2*n, 2*p]$

#### Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

#### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)), x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

#### Rubi steps

$$\begin{aligned}
\int x\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx)) dx &= \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} - \frac{(bcx) \int \frac{(d+ex^2)^{3/2}}{x\sqrt{-1-c^2x^2}} dx}{3e\sqrt{-c^2x^2}} \\
&= \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{(d+ex^2)^{3/2}}{x\sqrt{-1-c^2x}} dx, x, x^2\right)}{6e\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{(bx) \operatorname{Subst}\left(\int \frac{(d+ex^2)^{3/2}}{x\sqrt{-1-c^2x}} dx, x, x^2\right)}{6e\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} - \frac{(b(3c^2d-e)x)}{6c\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{(b(3c^2d-e)x)}{6c\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{3e\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}} + \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{3e} + \frac{b(3c^2d-e)x}{6c^2\sqrt{-c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.51, size = 278, normalized size = 1.37

$$\frac{\sqrt{d+ex^2} \left( 2ac(d+ex^2) + bex\sqrt{\frac{1}{c^2x^2}+1} + 2bccsch^{-1}(cx)(d+ex^2) \right) bx\sqrt{\frac{1}{c^2x^2}+1} \left( 2c^5d^{3/2}\sqrt{-d-ex^2} \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right) \right)}{6ce}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

[Out] (Sqrt[d + e\*x^2]\*(b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x + 2\*a\*c\*(d + e\*x^2) + 2\*b\*c\*(d + e\*x^2)\*ArcCsch[c\*x]))/(6\*c\*e) - (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-(Sqrt[c^2]\*Sqrt[c^2\*d - e]\*(3\*c^2\*d - e)\*Sqrt[e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e]]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])]) + 2\*c^5\*d^(3/2)\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]]))/(6\*c^4\*e\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [A]** time = 1.10, size = 1342, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] [1/24\*(2\*b\*c^3\*d^(3/2)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 - 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) - (3\*b\*c^2\*d - b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 - 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) +



$$e^2) + 8*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d)/(c^3*e), 1/12*(b*c^3*d^(3/2)*log((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d)/(c^3*e), 1/24*(4*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 8*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d)/(c^3*e), 1/12*(2*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d)/(c^3*e]$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*x, x)

**maple** [F] time = 0.48, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{arccsch}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x)

[Out] int(x\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} \left( \frac{(ex^2 + d)^{\frac{3}{2}} \log(\sqrt{c^2x^2 + 1} + 1)}{e} + 3 \int \frac{(c^2ex^3 + c^2dx)\sqrt{ex^2 + d}}{3(c^2ex^2 + (c^2ex^2 + e)\sqrt{c^2x^2 + 1} + e)} dx - 3 \int \frac{((3e \log(c) + e)c^2x^3 + (3e \log(c) + e)c^2d + 3e^2)x^2}{3(c^2ex^2 + (c^2ex^2 + e)\sqrt{c^2x^2 + 1} + e)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/3\*((e\*x^2 + d)^(3/2)\*log(sqrt(c^2\*x^2 + 1) + 1)/e + 3\*integrate(1/3\*(c^2\*e\*x^3 + c^2\*d\*x)\*sqrt(e\*x^2 + d)/(c^2\*e\*x^2 + (c^2\*e\*x^2 + e)\*sqrt(c^2\*x^2 + 1) + e), x) - 3\*integrate(1/3\*((3\*e\*log(c) + e)\*c^2\*x^3 + (c^2\*d + 3\*e\*log(c) + e)\*c^2\*x^2 + (3\*e\*log(c) + e)\*c^2\*d + 3\*e^2), x))

$g(c)) * x + 3 * (c^2 * e * x^3 + e * x) * \log(x) * \sqrt{e * x^2 + d} / (c^2 * e * x^2 + e), x) * b + 1/3 * (e * x^2 + d)^{(3/2)} * a / e$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x \sqrt{e x^2 + d} \left( a + b \operatorname{asinh} \left( \frac{1}{c x} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)`

[Out] `int(x*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x (a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*acsch(c*x))*(e*x**2+d)**(1/2), x)`

[Out] `Integral(x*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)`

$$3.121 \quad \int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x}, x \right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x,x)

**Rubi** [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x,x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x} dx = \int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x} dx$$

**Mathematica** [A] time = 5.62, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x,x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x, x]

**fricas** [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2+d} (b \operatorname{arcsch}(cx) + a)}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d} (b \operatorname{arcsch}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x, x)

**maple** [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx)) \sqrt{ex^2 + d}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x,x)

[Out] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{d} \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right) - \sqrt{ex^2 + d}\right)a + b \int \frac{\sqrt{ex^2 + d} \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x,x, algorithm="maxima")

[Out] -(sqrt(d)\*arcsinh(d/(sqrt(d\*e)\*abs(x)))) - sqrt(e\*x^2 + d))\*a + b\*integrate(sqrt(e\*x^2 + d)\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{ex^2 + d} \left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x,x)

[Out] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x,x)

[Out] Integral((a + b\*acsch(c\*x))\*sqrt(d + e\*x\*\*2)/x, x)

$$3.122 \quad \int \frac{\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx))}{x^3} dx$$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx))}{x^3}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^3, x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^3, x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^3, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx))}{x^3} dx = \int \frac{\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx))}{x^3} dx$$

Mathematica [A] time = 5.40, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^3, x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^3, x]

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex^2+d} (b \operatorname{arcsch}(cx) + a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^3, x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^3, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d} (b \operatorname{arcsch}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^3, x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^3, x)

**maple** [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx)) \sqrt{ex^2 + d}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x)

[Out] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( \frac{e \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{\sqrt{d}} - \frac{\sqrt{ex^2 + d} e}{d} + \frac{(ex^2 + d)^{\frac{3}{2}}}{dx^2} \right) a + b \int \frac{\sqrt{ex^2 + d} \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^3,x, algorithm="maxima")

[Out] -1/2\*(e\*arcsinh(d/(sqrt(d\*e)\*abs(x)))/sqrt(d) - sqrt(e\*x^2 + d)\*e/d + (e\*x^2 + d)^(3/2)/(d\*x^2))\*a + b\*integrate(sqrt(e\*x^2 + d)\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/x^3, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{ex^2 + d} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x^3,x)

[Out] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x^3, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*3,x)

[Out] Integral((a + b\*acsch(c\*x))\*sqrt(d + e\*x\*\*2)/x\*\*3, x)

### 3.123 $\int x^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(x^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)), x\right)$$

[Out] Unintegrable(x^2\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x)

**Rubi** [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

[Out] Defer[Int][x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

Rubi steps

$$\int x^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx = \int x^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

**Mathematica** [A] time = 9.31, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[x^2\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

**fricas** [A] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(bx^2 \operatorname{arcsch}(cx) + ax^2\right) \sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsch(c\*x) + a\*x^2)\*sqrt(e\*x^2 + d), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a) x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*x^2, x)

**maple** [A] time = 0.43, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{arccsch}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)`

[Out] `int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} \left( \frac{2(ex^2 + d)^{\frac{3}{2}}x}{e} - \frac{\sqrt{ex^2 + d} dx}{e} - \frac{d^2 \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}} \right) a + b \int \sqrt{ex^2 + d} x^2 \log \left( \sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `1/8*(2*(e*x^2 + d)^(3/2)*x/e - sqrt(e*x^2 + d)*d*x/e - d^2*arcsinh(e*x/sqrt(d*e))/e^(3/2))*a + b*integrate(sqrt(e*x^2 + d)*x^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x)), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \sqrt{ex^2 + d} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)`

[Out] `int(x^2*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)`

[Out] `Integral(x**2*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)`



### 3.124 $\int \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)), x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x)

**Rubi** [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

[Out] Defer[Int][Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

Rubi steps

$$\int \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx = \int \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

**Mathematica** [A] time = 2.66, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

**fricas** [A] time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a), x)

**maple** [A] time = 0.42, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccsch}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)`

[Out] `int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( \sqrt{ex^2 + d} x + \frac{d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}} \right) a + b \int \sqrt{ex^2 + d} \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `1/2*(sqrt(e*x^2 + d)*x + d*arcsinh(e*x/sqrt(d*e))/sqrt(e))*a + b*integrate(sqrt(e*x^2 + d)*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x)), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{ex^2 + d} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)`

[Out] `int((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)`

[Out] `Integral((a + b*acsch(c*x))*sqrt(d + e*x**2), x)`

$$3.125 \quad \int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^2}, x \right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x)

**Rubi** [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^2,x]

[Out] Defer[Int] [(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^2, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^2} dx = \int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^2} dx$$

**Mathematica** [A] time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^2,x]

[Out] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^2, x]

**fricas** [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2+d} (b \operatorname{arcsch}(cx) + a)}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^2, x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d} (b \operatorname{arcsch}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^2, x)

**maple** [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx)) \sqrt{ex^2 + d}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x)

[Out] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\left( \sqrt{e} \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right) - \frac{\sqrt{ex^2 + d}}{x} \right) a + b \int \frac{\sqrt{ex^2 + d} \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^2,x, algorithm="maxima")

[Out] (sqrt(e)\*arcsinh(e\*x/sqrt(d\*e)) - sqrt(e\*x^2 + d)/x)\*a + b\*integrate(sqrt(e\*x^2 + d)\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/x^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{ex^2 + d} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x^2,x)

[Out] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*2,x)

[Out] Integral((a + b\*acsch(c\*x))\*sqrt(d + e\*x\*\*2)/x\*\*2, x)

$$3.126 \quad \int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^4} dx$$

**Optimal.** Leaf size=389

$$\frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3} - \frac{bex(c^2d-3e)\sqrt{d+ex^2} F(\tan^{-1}(cx)|1-\frac{e}{c^2d})}{9d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} - \frac{2bc\sqrt{-c^2x^2-1}(c^2d-2e)\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}}$$

[Out]  $-1/3*(e*x^2+d)^{(3/2)}*(a+b*arccsch(c*x))/d/x^3-2/9*b*c^3*(c^2*d-2*e)*x^2*(e*x^2+d)^{(1/2)}/d/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}-2/9*b*c*(c^2*d-2*e)*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/(-c^2*x^2)^{(1/2)}+1/9*b*c*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/x^2/(-c^2*x^2)^{(1/2)}+2/9*b*c^2*(c^2*d-2*e)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticE(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}-1/9*b*(c^2*d-3*e)*e*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticF(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}$

**Rubi [A]** time = 0.43, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {264, 6302, 12, 474, 583, 531, 418, 492, 411}

$$\frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3} - \frac{bex(c^2d-3e)\sqrt{d+ex^2} F(\tan^{-1}(cx)|1-\frac{e}{c^2d})}{9d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} - \frac{2bc^3x^2(c^2d-2e)\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^4, x]

[Out]  $(-2*b*c^3*(c^2*d-2*e)*x^2*\text{Sqrt}[d+e*x^2])/(9*d*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1-c^2*x^2]) - (2*b*c*(c^2*d-2*e)*\text{Sqrt}[-1-c^2*x^2]*\text{Sqrt}[d+e*x^2])/(9*d*\text{Sqrt}[-(c^2*x^2)]) + (b*c*\text{Sqrt}[-1-c^2*x^2]*\text{Sqrt}[d+e*x^2])/(9*x^2*\text{Sqrt}[-(c^2*x^2)]) - ((d+e*x^2)^{(3/2)}*(a+b*ArcCsch[c*x]))/(3*d*x^3) + (2*b*c^2*(c^2*d-2*e)*x*\text{Sqrt}[d+e*x^2]*\text{EllipticE}[ArcTan[c*x], 1-e/(c^2*d)])/(9*d*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1-c^2*x^2]*\text{Sqrt}[(d+e*x^2)/(d*(1+c^2*x^2))]) - (b*(c^2*d-3*e)*e*x*\text{Sqrt}[d+e*x^2]*\text{EllipticF}[ArcTan[c*x], 1-e/(c^2*d)])/(9*d^2*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1-c^2*x^2]*\text{Sqrt}[(d+e*x^2)/(d*(1+c^2*x^2))])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 264

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

#### Rule 411

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/((c\_) + (d\_)\*(x\_)^2)^(3/2), x\_Symbol] := Simp[(Sqrt[a+b\*x^2]\*EllipticE[ArcTan[Rt[d/c, 2]\*x], 1-(b\*c)/(a\*d)])/(c\*Rt[d/c, 2]\*Sqrt[c+d\*x^2]\*Sqrt[(c\*(a+b\*x^2))/(a\*(c+d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rule 418

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 474

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^ (q_), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(
q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
+ a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (
f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 583

```
Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^ (q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
] && LtQ[m, -1]
```

Rule 6302

```
Int[((a_) + ArcCsch[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(
x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^4} dx &= -\frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3} - \frac{(bcx) \int -\frac{(d+ex^2)^{3/2}}{3dx^4\sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\
&= -\frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3} + \frac{(bcx) \int \frac{(d+ex^2)^{3/2}}{x^4\sqrt{-1-c^2x^2}} dx}{3d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2} \sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3} - \frac{(bcx) \int \frac{2d(c^2x^2)}{x^2\sqrt{-1-c^2x^2}} dx}{9d} \\
&= -\frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3} \\
&= -\frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3} \\
&= -\frac{2bc^3(c^2d-2e)x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3} \\
&= -\frac{2bc^3(c^2d-2e)x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{3dx^3}
\end{aligned}$$

**Mathematica [C]** time = 0.62, size = 237, normalized size = 0.61

$$\frac{\sqrt{d+ex^2} \left( 3a(d+ex^2) + bcx\sqrt{\frac{1}{c^2x^2}+1} (2c^2dx^2-d-4ex^2) + 3bcsch^{-1}(cx)(d+ex^2) \right)}{9dx^3} - \frac{ibcx\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{ex^2}{d}}}{9d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^4, x]

[Out] -1/9\*(Sqrt[d + e\*x^2]\*(b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-d + 2\*c^2\*d\*x^2 - 4\*e\*x^2) + 3\*a\*(d + e\*x^2) + 3\*b\*(d + e\*x^2)\*ArcCsch[c\*x]))/(d\*x^3) - ((I/9)\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(2\*c^2\*d\*(c^2\*d - 2\*e)\*EllipticE[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)] + (-2\*c^4\*d^2 + 5\*c^2\*d\*e - 3\*e^2)\*EllipticF[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)))/(Sqrt[c^2]\*d\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [F]** time = 0.66, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^4, x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^4, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^4, x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx)) \sqrt{e x^2 + d}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x)

[Out] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} b \left( \frac{(e x^2 + d)^{\frac{3}{2}} \log(\sqrt{c^2 x^2 + 1} + 1)}{d x^3} + 3 \int -\frac{(c^2 e x^4 - (3 d \log(c) - d) c^2 x^2 - 3 d \log(c) - 3 (c^2 d x^2 + d) \log(x)) \sqrt{e x^2 + d}}{3 (c^2 d x^6 + d x^4)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^4,x, algorithm="maxima")

[Out] -1/3\*b\*((e\*x^2 + d)^(3/2)\*log(sqrt(c^2\*x^2 + 1) + 1)/(d\*x^3) + 3\*integrate(-1/3\*(c^2\*e\*x^4 - (3\*d\*log(c) - d)\*c^2\*x^2 - 3\*d\*log(c) - 3\*(c^2\*d\*x^2 + d)\*log(x))\*sqrt(e\*x^2 + d)/(c^2\*d\*x^6 + d\*x^4), x) + 3\*integrate(1/3\*(c^2\*e\*x^2 + c^2\*d)\*sqrt(e\*x^2 + d)/(c^2\*d\*x^4 + d\*x^2 + (c^2\*d\*x^4 + d\*x^2)\*sqrt(c^2\*x^2 + 1)), x) - 1/3\*(e\*x^2 + d)^(3/2)\*a/(d\*x^3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e x^2 + d} \left( a + b \operatorname{asinh}\left(\frac{1}{c x}\right) \right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x^4,x)

[Out] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x^4, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) \sqrt{d + e x^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*4,x)

[Out] Integral((a + b\*acsch(c\*x))\*sqrt(d + e\*x\*\*2)/x\*\*4, x)



$$3.127 \quad \int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=527

$$\frac{2e(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{15d^2x^3} - \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{5dx^5} - \frac{bc\sqrt{-c^2x^2-1} (12c^2d+e)\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}}$$

[Out]  $-1/5*(e*x^2+d)^{(3/2)}*(a+b*arccsch(c*x))/d/x^5+2/15*e*(e*x^2+d)^{(3/2)}*(a+b*arccsch(c*x))/d^2/x^3-1/45*b*c^3*(2*c^2*d-e)*e*x^2*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}-2/15*b*c^3*e^2*x^2*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}+1/75*b*c^3*(8*c^4*d^2-3*c^2*d*e-2*e^2)*x^2*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}-1/45*b*c*(2*c^2*d-e)*e*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}-2/15*b*c*e^2*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}+1/75*b*c*(8*c^4*d^2-3*c^2*d*e-2*e^2)*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}+1/25*b*c*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/x^4/(-c^2*x^2)^{(1/2)}-1/75*b*c*(4*c^2*d-e)*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^2/(-c^2*x^2)^{(1/2)}+1/45*b*c*e*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^2/(-c^2*x^2)^{(1/2)}+1/45*b*c^2*(2*c^2*d-e)*e*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticE(c*x/(c^2*x^2+1))^{(1/2)}, (1-e/c^2/d)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}+2/15*b*c^2*e^2*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticE(c*x/(c^2*x^2+1))^{(1/2)}, (1-e/c^2/d)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}-1/75*b*c^2*(8*c^4*d^2-3*c^2*d*e-2*e^2)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticE(c*x/(c^2*x^2+1))^{(1/2)}, (1-e/c^2/d)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}+1/75*b*c^2*(4*c^2*d-e)*e*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticF(c*x/(c^2*x^2+1))^{(1/2)}, (1-e/c^2/d)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}-1/45*b*c^2*e^2*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticF(c*x/(c^2*x^2+1))^{(1/2)}, (1-e/c^2/d)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}-2/15*b*e^3*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticF(c*x/(c^2*x^2+1))^{(1/2)}, (1-e/c^2/d)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^3/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}$

**Rubi [A]** time = 0.64, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {271, 264, 6302, 12, 580, 583, 531, 418, 492, 411}

$$\frac{2e(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{15d^2x^3} - \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{5dx^5} + \frac{bc^3x^2(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^6, x]

[Out]  $(b*c^3*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*x^2*\text{Sqrt}[d + e*x^2])/(225*d^2*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1 - c^2*x^2]) + (b*c*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])/(225*d^2*\text{Sqrt}[-(c^2*x^2)]) - (b*c*(12*c^2*d + e)*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])/(225*d*x^2*\text{Sqrt}[-(c^2*x^2)]) + (b*c*\text{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/(25*d*x^4*\text{Sqrt}[-(c^2*x^2)]) - ((d + e*x^2)^{(3/2)}*(a + b*ArcCsch[c*x]))/(5*d*x^5) + (2*e*(d + e*x^2)^{(3/2)}*(a + b*ArcCsch[c*x]))/(15*d^2*x^3) - (b*c^2*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*x*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[\text{ArcTan}[c*x], 1 - e/(c^2*d)])/(225*d^2*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (2*$

$b * e * (6 * c^4 * d^2 - 4 * c^2 * d * e - 15 * e^2) * x * \text{Sqrt}[d + e * x^2] * \text{EllipticF}[\text{ArcTan}[c * x], 1 - e / (c^2 * d)] / (225 * d^3 * \text{Sqrt}[-(c^2 * x^2)] * \text{Sqrt}[-1 - c^2 * x^2] * \text{Sqrt}[(d + e * x^2) / (d * (1 + c^2 * x^2))])$

### Rule 12

$\text{Int}[(a_*) * (u_*), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*) * (v_*) /; \text{FreeQ}[b, x]]$

### Rule 264

$\text{Int}[((c_*) * (x_*))^{(m_*)} * ((a_*) + (b_*) * (x_*)^{(n_*)})^{(p_*)}, x\_Symbol] := \text{Simp}[(c * x)^{(m + 1)} * (a + b * x^n)^{(p + 1)} / (a * c * (m + 1)), x] /; \text{FreeQ}[\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[(m + 1) / n + p + 1, 0] \&\& \text{NeQ}[m, -1]$

### Rule 271

$\text{Int}[(x_*)^{(m_*)} * ((a_*) + (b_*) * (x_*)^{(n_*)})^{(p_*)}, x\_Symbol] := \text{Simp}[x^{(m + 1)} * (a + b * x^n)^{(p + 1)} / (a * (m + 1)), x] - \text{Dist}[(b * (m + n * (p + 1) + 1)) / (a * (m + 1)), \text{Int}[x^{(m + n)} * (a + b * x^n)^p, x], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \&\& \text{IntQ}[\text{Simplify}[(m + 1) / n + p + 1], 0] \&\& \text{NeQ}[m, -1]$

### Rule 411

$\text{Int}[\text{Sqrt}[(a_*) + (b_*) * (x_*)^2] / ((c_*) + (d_*) * (x_*)^2)^{(3/2)}, x\_Symbol] := \text{Simp}[(\text{Sqrt}[a + b * x^2] * \text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2] * x], 1 - (b * c) / (a * d)]) / (c * \text{Rt}[d/c, 2] * \text{Sqrt}[c + d * x^2] * \text{Sqrt}[(c * (a + b * x^2)) / (a * (c + d * x^2))]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

### Rule 418

$\text{Int}[1 / (\text{Sqrt}[(a_*) + (b_*) * (x_*)^2] * \text{Sqrt}[(c_*) + (d_*) * (x_*)^2]), x\_Symbol] := \text{Simp}[(\text{Sqrt}[a + b * x^2] * \text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2] * x], 1 - (b * c) / (a * d)]) / (a * \text{Rt}[d/c, 2] * \text{Sqrt}[c + d * x^2] * \text{Sqrt}[(c * (a + b * x^2)) / (a * (c + d * x^2))]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

### Rule 492

$\text{Int}[(x_*)^2 / (\text{Sqrt}[(a_*) + (b_*) * (x_*)^2] * \text{Sqrt}[(c_*) + (d_*) * (x_*)^2]), x\_Symbol] := \text{Simp}[(x * \text{Sqrt}[a + b * x^2]) / (b * \text{Sqrt}[c + d * x^2]), x] - \text{Dist}[c/b, \text{Int}[\text{Sqrt}[a + b * x^2] / (c + d * x^2)^{(3/2)}, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

### Rule 531

$\text{Int}[((a_*) + (b_*) * (x_*)^{(n_*)})^{(p_*)} * ((c_*) + (d_*) * (x_*)^{(n_*)})^{(q_*)} * ((e_*) + (f_*) * (x_*)^{(n_*)}), x\_Symbol] := \text{Dist}[e, \text{Int}[(a + b * x^n)^p * (c + d * x^n)^q, x], x] + \text{Dist}[f, \text{Int}[x^n * (a + b * x^n)^p * (c + d * x^n)^q, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, p, q\}, x]$

### Rule 580

$\text{Int}[((g_*) * (x_*)^{(m_*)} * ((a_*) + (b_*) * (x_*)^{(n_*)})^{(p_*)} * ((c_*) + (d_*) * (x_*)^{(n_*)})^{(q_*)} * ((e_*) + (f_*) * (x_*)^{(n_*)}), x\_Symbol] := \text{Simp}[(e * (g * x)^{(m + 1)} * (a + b * x^n)^{(p + 1)} * (c + d * x^n)^q / (a * g * (m + 1)), x] - \text{Dist}[1 / (a * g^n * (m + 1)), \text{Int}[(g * x)^{(m + n)} * (a + b * x^n)^p * (c + d * x^n)^{(q - 1)} * \text{Simp}[c * (b * e - a * f) * (m + 1) + e * n * (b * c * (p + 1) + a * d * q) + d * ((b * e - a * f) * (m + 1) + b * e * n * (p + q + 1)) * x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[q, 0] \&\& \text{LtQ}[m, -1] \&\& \text{!(EqQ}[q, 1] \&\& \text{SimplerQ}[e + f * x^n, c + d * x^n])$

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcSch[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2} (a+bcsch^{-1}(cx))}{x^6} dx &= -\frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{15d^2x^3} - \frac{bc\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&= -\frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{15d^2x^3} - \frac{bc\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{15d^2x^3} \\
&= \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} - \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}}{225d^2\sqrt{-c^2x^2}} \\
&= \frac{bc(24c^4d^2-19c^2de-31e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} - \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}}{225d^2\sqrt{-c^2x^2}} \\
&= \frac{bc(24c^4d^2-19c^2de-31e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} - \frac{bc(12c^2d+e)\sqrt{-1-c^2x^2}}{225d^2\sqrt{-c^2x^2}} \\
&= \frac{bc^3(24c^4d^2-19c^2de-31e^2)x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc(24c^4d^2-19c^2de-31e^2)}{225d^2\sqrt{-c^2x^2}} \\
&= \frac{bc^3(24c^4d^2-19c^2de-31e^2)x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc(24c^4d^2-19c^2de-31e^2)}{225d^2\sqrt{-c^2x^2}}
\end{aligned}$$

**Mathematica [C]** time = 0.70, size = 314, normalized size = 0.60

$$\frac{\sqrt{d+ex^2} \left( -15a(3d^2+dex^2-2e^2x^4) + bcx\sqrt{\frac{1}{c^2x^2}} + 1 \right) (dex^2(8-19c^2x^2) + 3d^2(8c^4x^4-4c^2x^2+3) - 31e^2x^4)}{225d^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/x^6,x]

[Out] (Sqrt[d + e\*x^2]\*(-15\*a\*(3\*d^2 + d\*e\*x^2 - 2\*e^2\*x^4) + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-31\*e^2\*x^4 + d\*e\*x^2\*(8 - 19\*c^2\*x^2) + 3\*d^2\*(3 - 4\*c^2\*x^2 + 8\*c^4\*x^4)) - 15\*b\*(3\*d^2 + d\*e\*x^2 - 2\*e^2\*x^4)\*ArcCsch[c\*x]))/(225\*d^2\*x^5) + ((I/225)\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(24\*c^4\*d^2 - 19\*c^2\*d\*e - 31\*e^2)\*EllipticE[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)] + (-24\*c^6\*d^3 + 31\*c^4\*d^2\*e + 23\*c^2\*d\*e^2 - 30\*e^3)\*EllipticF[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)))/(Sqrt[c^2]\*d^2\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^6, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/x^6, x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccsch}(cx)) \sqrt{ex^2 + d}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x)

[Out] int((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15} a \left( \frac{2 (ex^2 + d)^{\frac{3}{2}} e}{d^2 x^3} - \frac{3 (ex^2 + d)^{\frac{3}{2}}}{dx^5} \right) + \frac{1}{15} b \left( \frac{(2e^2x^4 - dex^2 - 3d^2)\sqrt{ex^2 + d} \log(\sqrt{c^2x^2 + 1} + 1)}{d^2x^5} - 15 \int \frac{(2c^2e^2x^6}{d^2x^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/x^6,x, algorithm="maxima")

[Out] 1/15\*a\*(2\*(e\*x^2 + d)^(3/2)\*e/(d^2\*x^3) - 3\*(e\*x^2 + d)^(3/2)/(d\*x^5)) + 1/15\*b\*((2\*e^2\*x^4 - d\*e\*x^2 - 3\*d^2)\*sqrt(e\*x^2 + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/(d^2\*x^5) - 15\*integrate(1/15\*(2\*c^2\*e^2\*x^6 - c^2\*d\*e\*x^4 + 3\*(5\*d^2\*log(c) - d^2)\*c^2\*x^2 + 15\*d^2\*log(c) + 15\*(c^2\*d^2\*x^2 + d^2)\*log(x))\*sqrt(e\*x^2 + d)/(c^2\*d^2\*x^8 + d^2\*x^6), x) + 15\*integrate(1/15\*(2\*c^2\*e^2\*x^4 - c^2\*d\*e\*x^2 - 3\*c^2\*d^2)\*sqrt(e\*x^2 + d)/(c^2\*d^2\*x^6 + d^2\*x^4 + (c^2\*d^2\*x^6 + d^2\*x^4)\*sqrt(c^2\*x^2 + 1)), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{ex^2 + d} \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x^6, x)

[Out] int(((d + e\*x^2)^(1/2)\*(a + b\*asinh(1/(c\*x))))/x^6, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))\*(e\*x\*\*2+d)\*\*(1/2)/x\*\*6, x)

[Out] Integral((a + b\*acsch(c\*x))\*sqrt(d + e\*x\*\*2)/x\*\*6, x)

### 3.128 $\int x^3 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=384

$$\frac{d(d+ex^2)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\operatorname{csch}^{-1}(cx))}{7e^2} - \frac{2bcd^{7/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{35e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1}}{42ce\sqrt{-c^2x^2}}$$

[Out]  $-1/5*d*(e*x^2+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^2+1/7*(e*x^2+d)^{(7/2)}*(a+b*\operatorname{arccsch}(c*x))/e^2-1/560*b*(35*c^6*d^3+35*c^4*d^2*e-63*c^2*d*e^2+25*e^3)*x*\operatorname{arctan}(e^{(1/2)}*(-c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^6/e^{(3/2)}/(-c^2*x^2)^{(1/2)}-2/35*b*c*d^{(7/2)}*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(-c^2*x^2-1)^{(1/2)})/e^2/(-c^2*x^2)^{(1/2)}+1/840*b*(13*c^2*d-25*e)*x*(e*x^2+d)^{(3/2)}*(-c^2*x^2-1)^{(1/2)}/c^3/e/(-c^2*x^2)^{(1/2)}+1/42*b*x*(e*x^2+d)^{(5/2)}*(-c^2*x^2-1)^{(1/2)}/c/e/(-c^2*x^2)^{(1/2)}-1/560*b*(3*c^4*d^2+38*c^2*d*e-25*e^2)*x*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c^5/e/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.51, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 6302, 12, 573, 154, 157, 63, 217, 203, 93, 204}

$$\frac{d(d+ex^2)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\operatorname{csch}^{-1}(cx))}{7e^2} - \frac{bx\sqrt{-c^2x^2-1}(3c^4d^2+38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3*(d + e*x^2)^{(3/2)}*(a + b*\operatorname{ArcCsch}[c*x]), x]$

[Out]  $-(b*(3*c^4*d^2 + 38*c^2*d*e - 25*e^2)*x*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(560*c^5*e*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*(13*c^2*d - 25*e)*x*\operatorname{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/(840*c^3*e*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*x*\operatorname{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(5/2)})/(42*c*e*\operatorname{Sqrt}[-(c^2*x^2)]) - (d*(d + e*x^2)^{(5/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(5*e^2) + ((d + e*x^2)^{(7/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(7*e^2) - (b*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^2*d*e^2 + 25*e^3)*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(560*c^6*e^{(3/2)}*\operatorname{Sqrt}[-(c^2*x^2)]) - (2*b*c*d^{(7/2)}*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 - c^2*x^2])])/(35*e^2*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 43

$\operatorname{Int}[(a_*)(x_*)^{(m_*)}*((c_*) + (d_*)(x_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ (\ !\operatorname{IntegerQ}[n] \ || \ (\operatorname{EqQ}[c, 0] \ \&\& \ \operatorname{LeQ}[7*m + 4*n + 4, 0]) \ || \ \operatorname{LtQ}[9*m + 5*(n + 1), 0] \ || \ \operatorname{GtQ}[m + n + 2, 0])$

#### Rule 63

$\operatorname{Int}[(a_*)(x_*)^{(m_*)}*((c_*) + (d_*)(x_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[-1, m, 0] \ \&\& \ \operatorname{LeQ}[-1, n, 0] \ \&\& \ \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \ \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_))^(r_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6302

```

Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int x^3 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx &= -\frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2} \\
&= -\frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2} \\
&= -\frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2} \\
&= \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2} \\
&= \frac{b(13c^2d - 25e)x\sqrt{-1 - c^2x^2} (d + ex^2)^{3/2}}{840c^3e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1 - c^2x^2} (d + ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2} \\
&= -\frac{b(3c^4d^2 + 38c^2de - 25e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b(13c^2d - 25e)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{840c^3e\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2} \\
&= -\frac{b(3c^4d^2 + 38c^2de - 25e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b(13c^2d - 25e)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{840c^3e\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2} \\
&= -\frac{b(3c^4d^2 + 38c^2de - 25e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b(13c^2d - 25e)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{840c^3e\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2} \\
&= -\frac{b(3c^4d^2 + 38c^2de - 25e^2)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{560c^5e\sqrt{-c^2x^2}} + \frac{b(13c^2d - 25e)x\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{840c^3e\sqrt{-c^2x^2}} - \frac{d(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{csch}^{-1}(cx))}{7e^2} - \frac{(d + ex^2)^{9/2} (a + b \operatorname{csch}^{-1}(cx))}{9e^2}
\end{aligned}$$

**Mathematica [C]** time = 0.77, size = 318, normalized size = 0.83

$$\frac{\sqrt{d + ex^2} \left( -48ac^5 (2d - 5ex^2) (d + ex^2)^2 - 48bc^5 \operatorname{csch}^{-1}(cx) (2d - 5ex^2) (d + ex^2)^2 + bex\sqrt{\frac{1}{c^2x^2} + 1} (c^4 (57d^2 + 1) \sqrt{-1 - c^2x^2}) \right)}{1680c^5e^2}$$

Warning: Unable to verify antiderivative.



[In] Integrate[x^3\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]),x]

[Out] 
$$-1/1120*(b*(-32*c^4*d^4*\sqrt{1 + d/(e*x^2)})*\text{AppellF1}[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))]) + (e*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^2*d*e^2 + 25*e^3)*\sqrt{1 + 1/(c^2*x^2)}*x^4*\sqrt{1 + (e*x^2)/d})*\text{AppellF1}[1, 1/2, 1/2, 2, -(c^2*x^2), -(e*x^2)/d])/\sqrt{1 + c^2*x^2})/(c^5*e^2*x*\sqrt{d + e*x^2}) + (\sqrt{d + e*x^2}*(-48*a*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*\sqrt{1 + 1/(c^2*x^2)})*x*(75*e^2 - 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 + 40*e^2*x^4)) - 48*b*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2*\text{ArcCsch}[c*x])/(1680*c^5*e^2)$$

**fricas** [A] time = 3.41, size = 1943, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/6720*(96*b*c^7*d^{(7/2)}*\log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*\sqrt{e*x^2 + d})*\sqrt{d})*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*\sqrt{e}*\log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*\sqrt{e*x^2 + d})*\sqrt{e})*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + e^2) + 192*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*\sqrt{e*x^2 + d})*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}))*\sqrt{e*x^2 + d}]/(c^7*e^2), \\ & 1/3360*(48*b*c^7*d^{(7/2)}*\log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*\sqrt{e*x^2 + d})*\sqrt{d})*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*\sqrt{-e}*\arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*\sqrt{e*x^2 + d})*\sqrt{-e})*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}]/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e) + 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*\sqrt{e*x^2 + d})*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}))*\sqrt{e*x^2 + d}]/(c^7*e^2), \\ & -1/6720*(192*b*c^7*\sqrt{-d})*d^3*\arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*\sqrt{e*x^2 + d})*\sqrt{-d})*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}]/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2) - 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*\sqrt{e}*\log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*\sqrt{e*x^2 + d})*\sqrt{e})*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + e^2) - 192*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*\sqrt{e*x^2 + d})*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)) - 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}))*\sqrt{e*x^2 + d}]/(c^7*e^2), \\ & -1/3360*(96*b*c^7*\sqrt{-d})*d^3*\arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*\sqrt{e*x^2 + d})*\sqrt{-d})*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}]/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2) - 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*\sqrt{-e}*\arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*\sqrt{e*x^2 + d})*\sqrt{-e})*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}]/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e) - 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*\sqrt{e*x^2 + d})*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)) - 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x)*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)}))*\sqrt{e*x^2 + d}]/(c^7*e^2)] \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a) x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)\*x^3, x)

**maple** [F] time = 0.48, size = 0, normalized size = 0.00

$$\int x^3 (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x)

[Out] int(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{35} \left( \frac{5(ex^2 + d)^{\frac{5}{2}} x^2}{e} - \frac{2(ex^2 + d)^{\frac{5}{2}} d}{e^2} \right) a + \frac{1}{35} b \left( \frac{(5e^3 x^6 + 8de^2 x^4 + d^2 ex^2 - 2d^3) \sqrt{ex^2 + d} \log(\sqrt{c^2 x^2 + 1} + 1)}{e^2} + 3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/35\*(5\*(e\*x^2 + d)^(5/2)\*x^2/e - 2\*(e\*x^2 + d)^(5/2)\*d/e^2)\*a + 1/35\*b\*((5\*e^3\*x^6 + 8\*d\*e^2\*x^4 + d^2\*e\*x^2 - 2\*d^3)\*sqrt(e\*x^2 + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/e^2 + 35\*integrate(1/35\*(5\*c^2\*e^3\*x^7 + 8\*c^2\*d\*e^2\*x^5 + c^2\*d^2\*e\*x^3 - 2\*c^2\*d^3\*x)\*sqrt(e\*x^2 + d)/(c^2\*e^2\*x^2 + e^2 + (c^2\*e^2\*x^2 + e^2)\*sqrt(c^2\*x^2 + 1)), x) - 35\*integrate(1/35\*(5\*(7\*e^3\*log(c) + e^3)\*c^2\*x^7 - 2\*c^2\*d^3\*x + (35\*e^3\*log(c) + (35\*d\*e^2\*log(c) + 8\*d\*e^2)\*c^2)\*x^5 + (c^2\*d^2\*e + 35\*d\*e^2\*log(c))\*x^3 + 35\*(c^2\*e^3\*x^7 + d\*e^2\*x^3 + (c^2\*d\*e^2 + e^3)\*x^5)\*log(x))\*sqrt(e\*x^2 + d)/(c^2\*e^2\*x^2 + e^2), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (ex^2 + d)^{3/2} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x^3\*(d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsch(c\*x)),x)

[Out] Timed out

### 3.129 $\int x (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$

**Optimal.** Leaf size=270

$$\frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e} + \frac{bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{5e\sqrt{-c^2x^2}} + \frac{bx\sqrt{-c^2x^2-1} (d + ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{bx(15c^4d^2 - 10c^2de)}{40c^4}$$

[Out] 1/5\*(e\*x^2+d)^(5/2)\*(a+b\*arccsch(c\*x))/e+1/5\*b\*c\*d^(5/2)\*x\*arctan((e\*x^2+d)^(1/2)/d^(1/2)/(-c^2\*x^2-1)^(1/2))/e/(-c^2\*x^2)^(1/2)+1/40\*b\*(15\*c^4\*d^2-10\*c^2\*d\*e+3\*e^2)\*x\*arctan(e^(1/2)\*(-c^2\*x^2-1)^(1/2)/c/(e\*x^2+d)^(1/2))/c^4/e^(1/2)/(-c^2\*x^2)^(1/2)+1/20\*b\*x\*(e\*x^2+d)^(3/2)\*(-c^2\*x^2-1)^(1/2)/c/(-c^2\*x^2)^(1/2)+1/40\*b\*(7\*c^2\*d-3\*e)\*x\*(-c^2\*x^2-1)^(1/2)\*(e\*x^2+d)^(1/2)/c^3/(-c^2\*x^2)^(1/2)

**Rubi [A]** time = 0.28, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {6300, 446, 102, 154, 157, 63, 217, 203, 93, 204}

$$\frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e} + \frac{bx(15c^4d^2 - 10c^2de + 3e^2) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{-c^2x^2}} + \frac{bcd^{5/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{5e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] (b\*(7\*c^2\*d - 3\*e)\*x\*sqrt[-1 - c^2\*x^2]\*sqrt[d + e\*x^2])/(40\*c^3\*sqrt[-(c^2\*x^2)]) + (b\*x\*sqrt[-1 - c^2\*x^2]\*(d + e\*x^2)^(3/2))/(20\*c\*sqrt[-(c^2\*x^2)]) + ((d + e\*x^2)^(5/2)\*(a + b\*ArcCsch[c\*x]))/(5\*e) + (b\*(15\*c^4\*d^2 - 10\*c^2\*d\*e + 3\*e^2)\*x\*ArcTan[(sqrt[e]\*sqrt[-1 - c^2\*x^2])/(c\*sqrt[d + e\*x^2])])/(40\*c^4\*sqrt[e]\*sqrt[-(c^2\*x^2)]) + (b\*c\*d^(5/2)\*x\*ArcTan[sqrt[d + e\*x^2]/(sqrt[d]\*sqrt[-1 - c^2\*x^2])])/(5\*e\*sqrt[-(c^2\*x^2)])

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 93

Int((((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 102

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6300

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)), x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x(d+ex^2)^{3/2}(a+b\operatorname{csch}^{-1}(cx))dx &= \frac{(d+ex^2)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e} - \frac{(bcx)\int\frac{(d+ex^2)^{5/2}}{x\sqrt{-1-c^2x^2}}dx}{5e\sqrt{-c^2x^2}} \\
&= \frac{(d+ex^2)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e} - \frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{10e\sqrt{-c^2x^2}} \\
&= \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{(d+ex^2)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{5e} + \frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{10e\sqrt{-c^2x^2}} \\
&= \frac{b(7c^2d-3e)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{10e\sqrt{-c^2x^2}} \\
&= \frac{b(7c^2d-3e)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{10e\sqrt{-c^2x^2}} \\
&= \frac{b(7c^2d-3e)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{10e\sqrt{-c^2x^2}} \\
&= \frac{b(7c^2d-3e)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{10e\sqrt{-c^2x^2}} \\
&= \frac{b(7c^2d-3e)x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}} + \frac{(bcx)\operatorname{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}dx,x,x^2\right)}{10e\sqrt{-c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.80, size = 314, normalized size = 1.16

$$\frac{\sqrt{d+ex^2}\left(8ac^3(d+ex^2)^2+8bc^3\operatorname{csch}^{-1}(cx)(d+ex^2)^2+bex\sqrt{\frac{1}{c^2x^2}+1}\left(c^2(9d+2ex^2)-3e\right)\right)+bx\sqrt{\frac{1}{c^2x^2}+1}}{40c^3e}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]),x]

[Out] (Sqrt[d + e\*x^2]\*(8\*a\*c^3\*(d + e\*x^2)^2 + b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-3\*e + c^2\*(9\*d + 2\*e\*x^2)) + 8\*b\*c^3\*(d + e\*x^2)^2\*ArcCsch[c\*x]))/(40\*c^3\*e) - (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(Sqrt[c^2]\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(-15\*c^4\*d^2 + 10\*c^2\*d\*e - 3\*e^2)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e])\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])] + 8\*c^7\*d^(5/2)\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]]))/(40\*c^6\*e\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [A]** time = 2.01, size = 1625, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

```
[Out] [1/160*(8*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x))*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e
^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)
*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x))*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*
d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4
*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(
3*b*c^4*d*e - b*c^2*e^2)*x))*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(e*x^2 + d)
/(c^5*e), 1/80*(4*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c
^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x))*sqrt(e*x^2 + d)*sqrt(d)
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 - 10*b*c^2*d*e
+ 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 +
d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^
2 + d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d)
*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8*a*c^5*e^2*x^4 +
16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e - b*c^2
*e^2)*x))*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(e*x^2 + d)/(c^5*e), 1/160*(16
*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x))*sqrt(e*x^2 + d)
)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2
+ d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^
4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4
*d + c^2*e)*x))*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2)
+ 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5
*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e - b*c^2*e^2)*x)*
sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(e*x^2 + d))/(c^5*e), 1/80*(8*b*c^5*sqrt
(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x))*sqrt(e*x^2 + d)*sqrt(-d)*
sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) -
(15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 +
(c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*
e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2
+ b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*
x^3 + 3*(3*b*c^4*d*e - b*c^2*e^2)*x))*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(e*
x^2 + d))/(c^5*e)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x, x)
```

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int x (e x^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ex^2 + d)^{\frac{5}{2}}a}{5e} + \frac{1}{5} \left( \frac{(e^2x^4 + 2dex^2 + d^2)\sqrt{ex^2 + d} \log(\sqrt{c^2x^2 + 1} + 1)}{e} + 5 \int \frac{(c^2e^2x^5 + 2c^2dex^3 + c^2d^2x)\sqrt{ex^2 + d}}{5(c^2ex^2 + (c^2ex^2 + e)\sqrt{c^2x^2 + 1} + e)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] 1/5\*(e\*x^2 + d)^(5/2)\*a/e + 1/5\*((e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(e\*x^2 + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/e + 5\*integrate(1/5\*(c^2\*e^2\*x^5 + 2\*c^2\*d\*e\*x^3 + c^2\*d^2\*x)\*sqrt(e\*x^2 + d)/(c^2\*e\*x^2 + (c^2\*e\*x^2 + e)\*sqrt(c^2\*x^2 + 1) + e), x) - 5\*integrate(1/5\*((5\*e^2\*log(c) + e^2)\*c^2\*x^5 + ((5\*d\*e\*log(c) + 2\*d\*e)\*c^2 + 5\*e^2\*log(c))\*x^3 + (c^2\*d^2 + 5\*d\*e\*log(c))\*x + 5\*(c^2\*e^2\*x^5 + (c^2\*d\*e + e^2)\*x^3 + d\*e\*x)\*log(x))\*sqrt(e\*x^2 + d)/(c^2\*e\*x^2 + e), x))\*b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x (ex^2 + d)^{3/2} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int(x\*(d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsch(c\*x)),x)

[Out] Timed out

$$3.130 \quad \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{x} dx$$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{x}, x\right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x, x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x, x]

[Out] Defer[Int][((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{x} dx = \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{x} dx$$

Mathematica [A] time = 6.56, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{csch}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x, x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x, x]

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arcsch}(cx))\sqrt{ex^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x, x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*sqrt(e\*x^2 + d)/x, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{3/2} (b \operatorname{arcsch}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)/x, x)

**maple** [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsch}(cx))}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} \left( 3d^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right) - (ex^2 + d)^{\frac{3}{2}} - 3\sqrt{ex^2 + d}d \right) a + b \int \frac{(ex^2 + d)^{\frac{3}{2}} \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x,x, algorithm="maxima")

[Out] -1/3\*(3\*d^(3/2)\*arcsinh(d/(sqrt(d\*e)\*abs(x))) - (e\*x^2 + d)^(3/2) - 3\*sqrt(e\*x^2 + d)\*d)\*a + b\*integrate((e\*x^2 + d)^(3/2)\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/x, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x,x)

[Out] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsch(c\*x))/x,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)\*\*(3/2)/x, x)

$$3.131 \quad \int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^3} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^3}, x \right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^3,x)

Rubi [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^3,x]

[Out] Defer[Int] [((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^3, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^3} dx = \int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^3} dx$$

Mathematica [A] time = 5.86, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^3,x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^3, x]

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \text{arcsch}(cx)) \sqrt{ex^2 + d}}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*sqrt(e\*x^2 + d)/x^3, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{3/2} (b \text{arcsch}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)/x^3, x)

**maple** [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsch}(cx))}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^3,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^3,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2} \left( 3 \sqrt{d} e \operatorname{arsinh} \left( \frac{d}{\sqrt{de} |x|} \right) - 3 \sqrt{ex^2 + d} e - \frac{(ex^2 + d)^{\frac{3}{2}} e}{d} + \frac{(ex^2 + d)^{\frac{5}{2}}}{dx^2} \right) a + b \int \frac{(ex^2 + d)^{\frac{3}{2}} \log \left( \sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx} \right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^3,x, algorithm="maxima")

[Out] -1/2\*(3\*sqrt(d)\*e\*arcsinh(d/(sqrt(d\*e)\*abs(x))) - 3\*sqrt(e\*x^2 + d)\*e - (e\*x^2 + d)^(3/2)\*e/d + (e\*x^2 + d)^(5/2)/(d\*x^2))\*a + b\*integrate((e\*x^2 + d)^(3/2)\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/x^3, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^3,x)

[Out] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^3, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsch(c\*x))/x\*\*3,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)\*\*(3/2)/x\*\*3, x)

$$3.132 \quad \int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

**Optimal.** Leaf size=26

$$\operatorname{Int}\left(x^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)), x\right)$$

[Out] Unintegrable(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x)

**Rubi [A]** time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] Defer[Int][x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

Rubi steps

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx = \int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

**Mathematica [A]** time = 9.67, size = 0, normalized size = 0.00

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[x^2\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

**fricas [A]** time = 0.70, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(aex^4 + adx^2 + (bex^4 + bdx^2) \operatorname{arcsch}(cx)\right) \sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral((a\*e\*x^4 + a\*d\*x^2 + (b\*e\*x^4 + b\*d\*x^2)\*arccsch(c\*x))\*sqrt(e\*x^2 + d), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a) x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)\*x^2, x)

**maple** [A] time = 0.61, size = 0, normalized size = 0.00

$$\int x^2 (e x^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)`

[Out] `int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48} \left( \frac{8 (e x^2 + d)^{\frac{5}{2}} x}{e} - \frac{2 (e x^2 + d)^{\frac{3}{2}} dx}{e} - \frac{3 \sqrt{e x^2 + d} d^2 x}{e} - \frac{3 d^3 \operatorname{arsinh}\left(\frac{e x}{\sqrt{d e}}\right)}{e^{\frac{3}{2}}} \right) a + b \int (e x^2 + d)^{\frac{3}{2}} x^2 \log\left(\sqrt{\frac{1}{c^2 x^2}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")`

[Out] `1/48*(8*(e*x^2 + d)^(5/2)*x/e - 2*(e*x^2 + d)^(3/2)*d*x/e - 3*sqrt(e*x^2 + d)*d^2*x/e - 3*d^3*arcsinh(e*x/sqrt(d*e))/e^(3/2))*a + b*integrate((e*x^2 + d)^(3/2)*x^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x)), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 (e x^2 + d)^{3/2} \left( a + b \operatorname{asinh}\left(\frac{1}{c x}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)`

[Out] `int(x^2*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)`

[Out] Timed out

$$3.133 \quad \int (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Optimal. Leaf size=23

$$\operatorname{Int}\left((d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)), x\right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x)

**Rubi [A]** time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] Defer[Int] [(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

Rubi steps

$$\int (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx = \int (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

**Mathematica [A]** time = 3.52, size = 0, normalized size = 0.00

$$\int (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

**fricas [A]** time = 0.43, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \operatorname{arcsch}(cx)\right) \sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*sqrt(e\*x^2 + d), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a), x)

**maple [A]** time = 0.44, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)`

[Out] `int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8} \left( 2 (ex^2 + d)^{\frac{3}{2}} x + 3 \sqrt{ex^2 + d} dx + \frac{3d^2 \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}} \right) a + b \int (ex^2 + d)^{\frac{3}{2}} \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")`

[Out] `1/8*(2*(e*x^2 + d)^(3/2)*x + 3*sqrt(e*x^2 + d)*d*x + 3*d^2*arcsinh(e*x/sqrt(d*e))/sqrt(e))*a + b*integrate((e*x^2 + d)^(3/2)*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x)), x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (ex^2 + d)^{3/2} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)`

[Out] `int((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) (d + ex^2)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)`

[Out] `Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2), x)`

$$3.134 \quad \int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^2}, x \right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^2,x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^2,x]

[Out] Defer[Int][((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^2, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^2} dx = \int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^2} dx$$

Mathematica [A] time = 5.79, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^2,x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^2, x]

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \text{arcsch}(cx)) \sqrt{ex^2 + d}}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*sqrt(e\*x^2 + d)/x^2, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{3/2} (b \text{arcsch}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)/x^2, x)

**maple** [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsch}(cx))}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^2,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( 3 \sqrt{ex^2 + d} ex + 3 d \sqrt{e} \operatorname{arsinh} \left( \frac{ex}{\sqrt{de}} \right) - \frac{2 (ex^2 + d)^{\frac{3}{2}}}{x} \right) a + b \int \frac{(ex^2 + d)^{\frac{3}{2}} \log \left( \sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx} \right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^2,x, algorithm="maxima")

[Out] 1/2\*(3\*sqrt(e\*x^2 + d)\*e\*x + 3\*d\*sqrt(e)\*arcsinh(e\*x/sqrt(d\*e)) - 2\*(e\*x^2 + d)^(3/2)/x)\*a + b\*integrate((e\*x^2 + d)^(3/2)\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/x^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^2,x)

[Out] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^2, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsch(c\*x))/x\*\*2,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)\*\*(3/2)/x\*\*2, x)

$$3.135 \quad \int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^4} dx$$

Optimal. Leaf size=26

$$\text{Int} \left( \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^4}, x \right)$$

[Out] Unintegrable((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^4,x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^4,x]

[Out] Defer[Int][((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^4, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^4} dx = \int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^4} dx$$

Mathematica [A] time = 15.97, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^4,x]

[Out] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^4, x]

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \text{arcsch}(cx)) \sqrt{ex^2 + d}}{x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^4,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*sqrt(e\*x^2 + d)/x^4, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{3/2} (b \text{arcsch}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^4,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)/x^4, x)

**maple** [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsch}(cx))}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^4,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^4,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} \left( \frac{3 \sqrt{ex^2 + d} e^2 x}{d} + 3 e^{\frac{3}{2}} \operatorname{arsinh} \left( \frac{ex}{\sqrt{de}} \right) - \frac{2 (ex^2 + d)^{\frac{3}{2}} e}{dx} - \frac{(ex^2 + d)^{\frac{5}{2}}}{dx^3} \right) a + b \int \frac{(ex^2 + d)^{\frac{3}{2}} \log \left( \sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx} \right)}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^4,x, algorithm="maxima")

[Out] 1/3\*(3\*sqrt(e\*x^2 + d)\*e^2\*x/d + 3\*e^(3/2)\*arcsinh(e\*x/sqrt(d\*e)) - 2\*(e\*x^2 + d)^(3/2)\*e/(d\*x) - (e\*x^2 + d)^(5/2)/(d\*x^3))\*a + b\*integrate((e\*x^2 + d)^(3/2)\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/x^4, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^4,x)

[Out] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^4, x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^{\frac{3}{2}}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsch(c\*x))/x\*\*4,x)

[Out] Integral((a + b\*acsch(c\*x))\*(d + e\*x\*\*2)\*\*(3/2)/x\*\*4, x)

$$3.136 \quad \int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^6} dx$$

**Optimal.** Leaf size=492

$$\frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} - \frac{4bc\sqrt{-c^2x^2-1} (c^2d-2e) \sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-c^2x^2-1} (d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-c^2x^2-1}}{75d\sqrt{-c^2x^2}}$$

[Out]  $-1/5*(e*x^2+d)^{(5/2)}*(a+b*arccsch(c*x))/d/x^5+1/25*b*c*(e*x^2+d)^{(3/2)}*(-c^2*x^2-1)^{(1/2)}/x^4/(-c^2*x^2)^{(1/2)}+1/75*b*c^3*(8*c^4*d^2-23*c^2*d*e+23*e^2)*x^2*(e*x^2+d)^{(1/2)}/d/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}+1/75*b*c*(8*c^4*d^2-23*c^2*d*e+23*e^2)*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/(-c^2*x^2)^{(1/2)}-4/75*b*c*(c^2*d-2*e)*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/x^2/(-c^2*x^2)^{(1/2)}-1/75*b*c^2*(8*c^4*d^2-23*c^2*d*e+23*e^2)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticE(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}+1/75*b*e*(4*c^4*d^2-11*c^2*d*e+15*e^2)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticF(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}$

**Rubi [A]** time = 0.58, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {264, 6302, 12, 474, 580, 583, 531, 418, 492, 411}

$$\frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} + \frac{bc^3x^2(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}} + \frac{bc\sqrt{-c^2x^2-1}(8c^4d^2-23c^2de+23e^2)}{75d\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^6, x]

[Out]  $(b*c^3*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*x^2*\text{Sqrt}[d + e*x^2])/(75*d*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1 - c^2*x^2]) + (b*c*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])/(75*d*\text{Sqrt}[-(c^2*x^2)]) - (4*b*c*(c^2*d - 2*e)*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])/(75*x^2*\text{Sqrt}[-(c^2*x^2)]) + (b*c*\text{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/(25*x^4*\text{Sqrt}[-(c^2*x^2)]) - ((d + e*x^2)^{(5/2)}*(a + b*\text{ArcCsch}[c*x]))/(5*d*x^5) - (b*c^2*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*x*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[\text{ArcTan}[c*x], 1 - e/(c^2*d)])/(75*d*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (b*e*(4*c^4*d^2 - 11*c^2*d*e + 15*e^2)*x*\text{Sqrt}[d + e*x^2]*\text{EllipticF}[\text{ArcTan}[c*x], 1 - e/(c^2*d)])/(75*d^2*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))])$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 264

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m+1)\*(a+b\*x^n)^(p+1))/(a\*c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

### Rule 411

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/((c\_) + (d\_.)\*(x\_)^2)^(3/2), x\_Symbol] := Simp[(Sqrt[a+b\*x^2]\*EllipticE[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)])/(c\*Rt[

$d/c, 2] * \text{Sqrt}[c + d*x^2] * \text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))], x] /;$  FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

#### Rule 418

$\text{Int}[1/(\text{Sqrt}[a_ + (b_)*(x_)^2] * \text{Sqrt}[c_ + (d_)*(x_)^2]), x\_Symbol] :> \text{Simp}[(\text{Sqrt}[a + b*x^2] * \text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*\text{Rt}[d/c, 2] * \text{Sqrt}[c + d*x^2] * \text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /;$  FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

#### Rule 474

$\text{Int}[(e_)*(x_)^m * ((a_ + (b_)*(x_)^n))^p * ((c_ + (d_)*(x_)^n))^q, x\_Symbol] :> \text{Simp}[(c*(e*x)^{m+1} * (a + b*x^n)^{p+1} * (c + d*x^n)^{q-1}) / (a*e^{m+1}), x] - \text{Dist}[1/(a*e^n * (m+1)), \text{Int}[(e*x)^{m+n} * (a + b*x^n)^p * (c + d*x^n)^{q-2} * \text{Simp}[c*(c*b - a*d)*(m+1) + c*n*(b*c*(p+1) + a*d*(q-1)) + d*((c*b - a*d)*(m+1) + c*b*n*(p+q)) * x^n, x], x] /;$  FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

#### Rule 492

$\text{Int}[(x_)^2 / (\text{Sqrt}[a_ + (b_)*(x_)^2] * \text{Sqrt}[c_ + (d_)*(x_)^2]), x\_Symbol] :> \text{Simp}[(x * \text{Sqrt}[a + b*x^2]) / (b * \text{Sqrt}[c + d*x^2]), x] - \text{Dist}[c/b, \text{Int}[\text{Sqrt}[a + b*x^2] / (c + d*x^2)^{3/2}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

#### Rule 531

$\text{Int}[(a_ + (b_)*(x_)^n)^p * ((c_ + (d_)*(x_)^n))^q * ((e_ + (f_)*(x_)^n)), x\_Symbol] :> \text{Dist}[e, \text{Int}[(a + b*x^n)^p * (c + d*x^n)^q, x], x] + \text{Dist}[f, \text{Int}[x^n * (a + b*x^n)^p * (c + d*x^n)^q, x], x] /;$  FreeQ[{a, b, c, d, e, f, n, p, q}, x]

#### Rule 580

$\text{Int}[(g_)*(x_)^m * ((a_ + (b_)*(x_)^n))^p * ((c_ + (d_)*(x_)^n))^q * ((e_ + (f_)*(x_)^n)), x\_Symbol] :> \text{Simp}[(e*(g*x)^{m+1} * (a + b*x^n)^{p+1} * (c + d*x^n)^q) / (a*g^{m+1}), x] - \text{Dist}[1/(a*g^n * (m+1)), \text{Int}[(g*x)^{m+n} * (a + b*x^n)^p * (c + d*x^n)^{q-1} * \text{Simp}[c*(b*e - a*f)*(m+1) + e*n*(b*c*(p+1) + a*d*q) + d*((b*e - a*f)*(m+1) + b*e*n*(p+q+1)) * x^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f\*x^n, c + d\*x^n])

#### Rule 583

$\text{Int}[(g_)*(x_)^m * ((a_ + (b_)*(x_)^n))^p * ((c_ + (d_)*(x_)^n))^q * ((e_ + (f_)*(x_)^n)), x\_Symbol] :> \text{Simp}[(e*(g*x)^{m+1} * (a + b*x^n)^{p+1} * (c + d*x^n)^{q+1}) / (a*c*g^{m+1}), x] + \text{Dist}[1/(a*c*g^n * (m+1)), \text{Int}[(g*x)^{m+n} * (a + b*x^n)^p * (c + d*x^n)^q * \text{Simp}[a*f*c*(m+1) - e*(b*c + a*d)*(m+n+1) - e*n*(b*c*p + a*d*q) - b*e*d*(m+n*(p+q+2) + 1) * x^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

#### Rule 6302

$\text{Int}[(a_ + \text{ArcSch}[(c_)*(x_)]) * (b_)*((f_)*(x_)^m * ((d_ + (e_)*(x_)^2))^p), x\_Symbol] :> \text{With}[\{u = \text{IntHide}[(f*x)^m * (d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcSch}[c*x], u, x] - \text{Dist}[(b*c*x) / \text{Sqrt}[-(c^2*x^2)], \text{Int}[\text{SimplifyI}$

```

ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

```

### Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^6} dx &= -\frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} - \frac{(bcx) \int -\frac{(d+ex^2)^{5/2}}{5dx^6 \sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\
&= -\frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} + \frac{(bcx) \int \frac{(d+ex^2)^{5/2}}{x^6 \sqrt{-1-c^2x^2}} dx}{5d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{x^6 \sqrt{-1-c^2x^2}} dx}{5d\sqrt{-c^2x^2}} \\
&= -\frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} \\
&= \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} - \frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} \\
&= \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} - \frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} \\
&= \frac{bc^3(8c^4d^2-23c^2de+23e^2)x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5} \\
&= \frac{bc^3(8c^4d^2-23c^2de+23e^2)x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{5dx^5}
\end{aligned}$$

**Mathematica [C]** time = 0.70, size = 291, normalized size = 0.59

$$\frac{\sqrt{d+ex^2} \left( -15a(d+ex^2)^2 + bcx\sqrt{\frac{1}{c^2x^2}+1} (dex^2(11-23c^2x^2) + d^2(8c^4x^4-4c^2x^2+3) + 23e^2x^4) - 15bcsch^{-1}(cx) \right)}{75dx^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^6, x]

[Out] (Sqrt[d + e\*x^2]\*(-15\*a\*(d + e\*x^2)^2 + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(23\*e^2\*x^4 + d\*e\*x^2\*(11 - 23\*c^2\*x^2) + d^2\*(3 - 4\*c^2\*x^2 + 8\*c^4\*x^4)) - 15\*b\*(d + e\*x^2)^2\*ArcCsch[c\*x]))/(75\*d\*x^5) + ((I/75)\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(8\*c^4\*d^2 - 23\*c^2\*d\*e + 23\*e^2)\*EllipticE[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)] + (-8\*c^6\*d^3 + 27\*c^4\*d^2\*e - 34\*c^2\*d\*e^2 + 15\*e^3)\*EllipticF[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)]))/(Sqrt[c^2]\*d\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [F]** time = 0.58, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arcsch}(cx))\sqrt{ex^2 + d}}{x^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^6,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*sqrt(e\*x^2 + d)/x^6, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arcsch}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^6,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)/x^6, x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(a + b \operatorname{arccsch}(cx))}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^6,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^6,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{5}b \left( \frac{(e^2x^4 + 2dex^2 + d^2)\sqrt{ex^2 + d} \log(\sqrt{c^2x^2 + 1} + 1)}{dx^5} + 5 \int -\frac{(c^2e^2x^6 - (5de \log(c) - 2de)c^2x^4 - ((5d^2 \log(c) - d^2)c^2 + 5d^2e \log(c))x^2 - 5d^2 \log(c) - 5(c^2d * e * x^4 + (c^2d^2 + d * e) * x^2 + d^2) * \log(x)) * \sqrt{ex^2 + d}}{(c^2d * x^8 + dx^6)}, x) + 5 * \int \frac{1/5 * (c^2 * e^2 * x^4 + 2 * c^2 * d * e * x^2 + c^2 * d^2) * \sqrt{ex^2 + d}}{(c^2 * d * x^6 + d * x^4 + (c^2 * d * x^6 + d * x^4) * \sqrt{c^2 * x^2 + 1})}, x) - 1/5 * (e * x^2 + d)^{(5/2)} * a / (d * x^5) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^6,x, algorithm="maxima")

[Out] -1/5\*b\*((e^2\*x^4 + 2\*d\*e\*x^2 + d^2)\*sqrt(e\*x^2 + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/(d\*x^5) + 5\*integrate(-1/5\*(c^2\*e^2\*x^6 - (5\*d\*e\*log(c) - 2\*d\*e)\*c^2\*x^4 - ((5\*d^2\*log(c) - d^2)\*c^2 + 5\*d\*e\*log(c))\*x^2 - 5\*d^2\*log(c) - 5\*(c^2\*d \* e \* x^4 + (c^2\*d^2 + d \* e) \* x^2 + d^2) \* log(x)) \* sqrt(e\*x^2 + d)/(c^2\*d\*x^8 + d\*x^6), x) + 5\*integrate(1/5\*(c^2\*e^2\*x^4 + 2\*c^2\*d\*e\*x^2 + c^2\*d^2)\*sqrt(e\*x^2 + d)/(c^2\*d\*x^6 + d\*x^4 + (c^2\*d\*x^6 + d\*x^4)\*sqrt(c^2\*x^2 + 1)), x) - 1/5\*(e\*x^2 + d)^(5/2)\*a/(d\*x^5)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^6,x)

[Out] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^6, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**6,x)
```

```
[Out] Timed out
```



$$3.137 \quad \int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^8} dx$$

**Optimal.** Leaf size=643

$$\frac{2e(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{35d^2x^5} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{7dx^7} + \frac{bc\sqrt{-c^2x^2-1} (d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} - \frac{bc\sqrt{-c^2x^2-1}}{1225}$$

[Out]  $-1/7*(e*x^2+d)^{(5/2)}*(a+b*arccsch(c*x))/d/x^7+2/35*e*(e*x^2+d)^{(5/2)}*(a+b*arccsch(c*x))/d^2/x^5-1/1225*b*c*(30*c^2*d-11*e)*(e*x^2+d)^{(3/2)}*(-c^2*x^2-1)^{(1/2)}/d/x^4/(-c^2*x^2)^{(1/2)}+1/49*b*c*(e*x^2+d)^{(5/2)}*(-c^2*x^2-1)^{(1/2)}/d/x^6/(-c^2*x^2)^{(1/2)}-1/3675*b*c^3*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*x^2*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}-1/3675*b*c*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}+1/3675*b*c*(120*c^4*d^2-159*c^2*d*e-37*e^2)*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^2/(-c^2*x^2)^{(1/2)}+1/3675*b*c^2*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticE(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}-1/3675*b*e*(120*c^6*d^3-249*c^4*d^2*e+71*c^2*d*e^2+210*e^3)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*EllipticF(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d^3/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}$

**Rubi [A]** time = 0.87, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {271, 264, 6302, 12, 580, 583, 531, 418, 492, 411}

$$\frac{2e(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{35d^2x^5} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{7dx^7} - \frac{bc^3x^2(-528c^4d^2e+240c^6d^3+193c^2de^2+247e^3)}{3675d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^8,x]

[Out]  $-(b*c^3*(240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x^2*\text{Sqrt}[d + e*x^2])/((3675*d^2*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1 - c^2*x^2]) - (b*c*(240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])/((3675*d^2*\text{Sqrt}[-(c^2*x^2)])) + (b*c*(120*c^4*d^2 - 159*c^2*d*e - 37*e^2)*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[d + e*x^2])/((3675*d*x^2*\text{Sqrt}[-(c^2*x^2)])) - (b*c*(30*c^2*d - 11*e)*\text{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/((1225*d*x^4*\text{Sqrt}[-(c^2*x^2)])) + (b*c*\text{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(5/2)})/((49*d*x^6*\text{Sqrt}[-(c^2*x^2)])) - ((d + e*x^2)^{(5/2)}*(a + b*ArcCsch[c*x]))/((7*d*x^7) + (2*e*(d + e*x^2)^{(5/2)}*(a + b*ArcCsch[c*x]))/((35*d^2*x^5) + (b*c^2*(240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x*\text{Sqrt}[d + e*x^2]*\text{EllipticE}[ArcTan[c*x], 1 - e/(c^2*d)])/((3675*d^2*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(120*c^6*d^3 - 249*c^4*d^2*e + 71*c^2*d*e^2 + 210*e^3)*x*\text{Sqrt}[d + e*x^2]*\text{EllipticF}[ArcTan[c*x], 1 - e/(c^2*d)])/((3675*d^3*\text{Sqrt}[-(c^2*x^2)]*\text{Sqrt}[-1 - c^2*x^2]*\text{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))]))$

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

**Rule 264**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[((c\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

#### Rule 271

Int[(x\_)^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x^(m + 1)\*(a + b\*x^n)^(p + 1))/(a\*(m + 1)), x] - Dist[(b\*(m + n\*(p + 1) + 1))/(a\*(m + 1)), Int[x^(m + n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]

#### Rule 411

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/((c\_) + (d\_.)\*(x\_)^2)^(3/2), x\_Symbol] := Simp[(Sqrt[a + b\*x^2]\*EllipticE[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)])/(c\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

#### Rule 418

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(Sqrt[a + b\*x^2]\*EllipticF[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)])/(a\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

#### Rule 492

Int[(x\_)^2/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(x\*Sqrt[a + b\*x^2])/(b\*Sqrt[c + d\*x^2]), x] - Dist[c/b, Int[Sqrt[a + b\*x^2]/(c + d\*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

#### Rule 531

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[e, Int[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] + Dist[f, Int[x^n\*(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

#### Rule 580

Int[((g\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(e\*(g\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*g\*(m + 1)), x] - Dist[1/(a\*g^n\*(m + 1)), Int[(g\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^(q - 1)\*Simp[c\*(b\*e - a\*f)\*(m + 1) + e\*n\*(b\*c\*(p + 1) + a\*d\*q) + d\*((b\*e - a\*f)\*(m + 1) + b\*e\*n\*(p + q + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f\*x^n, c + d\*x^n])

#### Rule 583

Int[((g\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(e\*(g\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*c\*g\*(m + 1)), x] + Dist[1/(a\*c\*g^n\*(m + 1)), Int[(g\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^q\*Simp[a\*f\*c\*(m + 1) - e\*(b\*c + a\*d)\*(m + n + 1) - e\*n\*(b\*c\*p + a\*d\*q) - b\*e\*d\*(m + n\*(p + q + 2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

## Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

## Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2} (a+bcsch^{-1}(cx))}{x^8} dx &= -\frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{35d^2x^5} \\
&= -\frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{35d^2x^5} \\
&= \frac{bc\sqrt{-1-c^2x^2} (d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2} (a+bcsch^{-1}(cx))}{35d^2x^5} \\
&= -\frac{bc(30c^2d-11e)\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2} (d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&= \frac{bc(120c^4d^2-159c^2de-37e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} - \frac{bc(30c^2d-11e)\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} \\
&= -\frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} + \frac{bc(30c^2d-11e)\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} \\
&= -\frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} + \frac{bc(30c^2d-11e)\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} \\
&= -\frac{bc^3(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} \\
&= -\frac{bc^3(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}}
\end{aligned}$$

**Mathematica [C]** time = 0.83, size = 372, normalized size = 0.58

$$\frac{\sqrt{d+ex^2} \left( 105a(5d-2ex^2)(d+ex^2)^2 + bcx\sqrt{\frac{1}{c^2x^2}+1} (de^2x^4(193c^2x^2-71) - 3d^2ex^2(176c^4x^4-83c^2x^2+1)) \right)}{3675d^2x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/x^8,x]

[Out] -1/3675\*(Sqrt[d + e\*x^2]\*(105\*a\*(5\*d - 2\*e\*x^2)\*(d + e\*x^2)^2 + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(247\*e^3\*x^6 + d\*e^2\*x^4\*(-71 + 193\*c^2\*x^2) - 3\*d^2\*e\*x^2

$$\begin{aligned} &*(61 - 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16* \\ &c^6*x^6)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsch[c*x]))/(d^2*x^7) - \\ &((I/3675)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*d \\ &^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*EllipticE[I*ArcSinh[Sqrt[c^2] \\ &*x], e/(c^2*d)] - 2*(120*c^8*d^4 - 324*c^6*d^3*e + 221*c^4*d^2*e^2 + 88*c^2 \\ &*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)])))/(Sqrt[c^2] \\ &*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2]) \end{aligned}$$

**fricas** [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arcsch}(cx)) \sqrt{ex^2 + d}}{x^8}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^8,x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*sqrt(e\*x^2 + d)/x^8, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arcsch}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^8,x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)/x^8, x)

**maple** [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(a + b \operatorname{arccsch}(cx))}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^8,x)

[Out] int((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^8,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{35} a \left( \frac{2(ex^2 + d)^{\frac{5}{2}} e}{d^2 x^5} - \frac{5(ex^2 + d)^{\frac{5}{2}}}{dx^7} \right) + \frac{1}{35} b \left( \frac{(2e^3 x^6 - de^2 x^4 - 8d^2 ex^2 - 5d^3) \sqrt{ex^2 + d} \log(\sqrt{c^2 x^2 + 1} + 1)}{d^2 x^7} \right) - 35$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/x^8,x, algorithm="maxima")

[Out] 
$$\begin{aligned} &1/35*a*(2*(e*x^2 + d)^{(5/2)}*e/(d^2*x^5) - 5*(e*x^2 + d)^{(5/2)}/(d*x^7)) + 1/ \\ &35*b*((2*e^3*x^6 - d*e^2*x^4 - 8*d^2*e*x^2 - 5*d^3)*sqrt(e*x^2 + d)*log(sqrt \\ &(c^2*x^2 + 1) + 1)/(d^2*x^7) - 35*integrate(1/35*(2*c^2*e^3*x^8 - c^2*d*e^ \\ &2*x^6 + (35*d^2*e*log(c) - 8*d^2*e)*c^2*x^4 + 35*d^3*log(c) + 5*(7*d^2*e*lo \\ &g(c) + (7*d^3*log(c) - d^3)*c^2)*x^2 + 35*(c^2*d^2*e*x^4 + d^3 + (c^2*d^3 + \\ &d^2*e)*x^2)*log(x))*sqrt(e*x^2 + d)/(c^2*d^2*x^10 + d^2*x^8), x) + 35*inte \\ &grate(1/35*(2*c^2*e^3*x^6 - c^2*d*e^2*x^4 - 8*c^2*d^2*e*x^2 - 5*c^2*d^3)*sq \\ &rt(e*x^2 + d)/(c^2*d^2*x^8 + d^2*x^6 + (c^2*d^2*x^8 + d^2*x^6)*sqrt(c^2*x^2 \\ &+ 1)), x) \end{aligned}$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(ex^2 + d)^{3/2} \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^8, x)

[Out] int(((d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))))/x^8, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsch(c\*x))/x\*\*8, x)

[Out] Timed out

$$3.138 \quad \int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Optimal. Leaf size=329

$$\frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} + \frac{8bcd^{5/2} x \tan^{-1}\left(\frac{d + ex^2}{c}\right)}{15e^3}$$

[Out]  $-2/3*d*(e*x^2+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^3+1/5*(e*x^2+d)^{(5/2)}*(a+b*\operatorname{arccsch}(c*x))/e^3+1/120*b*(45*c^4*d^2+10*c^2*d*e+9*e^2)*x*\operatorname{arctan}(e^{(1/2)}*(-c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^4/e^{(5/2)}/(-c^2*x^2)^{(1/2)}+8/15*b*c*d^{(5/2)}*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(-c^2*x^2-1)^{(1/2)})/e^3/(-c^2*x^2)^{(1/2)}+1/20*b*x*(e*x^2+d)^{(3/2)}*(-c^2*x^2-1)^{(1/2)}/c/e^2/(-c^2*x^2)^{(1/2)}+d^2*(a+b*\operatorname{arccsch}(c*x))*(e*x^2+d)^{(1/2)}/e^3-1/120*b*(19*c^2*d+9*e)*x*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c^3/e^2/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 1.18, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 6302, 12, 1615, 154, 157, 63, 217, 203, 93, 204}

$$\frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} + \frac{8bcd^{5/2} x \tan^{-1}\left(\frac{d + ex^2}{c}\right)}{15e^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^5*(a + b*\operatorname{ArcCsch}[c*x]))/\operatorname{Sqrt}[d + e*x^2], x]$

[Out]  $-(b*(19*c^2*d + 9*e)*x*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(120*c^3*e^2*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*x*\operatorname{Sqrt}[-1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/(20*c*e^2*\operatorname{Sqrt}[-(c^2*x^2)]) + (d^2*\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcCsch}[c*x]))/e^3 - (2*d*(d + e*x^2)^{(3/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(3*e^3) + ((d + e*x^2)^{(5/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(5*e^3) + (b*(45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(120*c^4*e^{(5/2)}*\operatorname{Sqrt}[-(c^2*x^2)]) + (8*b*c*d^{(5/2)}*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 - c^2*x^2])])/(15*e^3*\operatorname{Sqrt}[-(c^2*x^2)])$

### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 43

$\operatorname{Int}[(a_*) + (b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 63

$\operatorname{Int}[(a_*) + (b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

#### Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

#### Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

#### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

#### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

#### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

#### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

#### Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p + q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q - 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m + q + p)))*x, x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
```

d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx &= \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= \frac{bx \sqrt{-1 - c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{-c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} \\
 &= -\frac{b (19c^2 d + 9e) x \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{-c^2 x^2}} + \frac{bx \sqrt{-1 - c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{-c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= -\frac{b (19c^2 d + 9e) x \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{-c^2 x^2}} + \frac{bx \sqrt{-1 - c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{-c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= -\frac{b (19c^2 d + 9e) x \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{-c^2 x^2}} + \frac{bx \sqrt{-1 - c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{-c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3} \\
 &= -\frac{b (19c^2 d + 9e) x \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{120c^3 e^2 \sqrt{-c^2 x^2}} + \frac{bx \sqrt{-1 - c^2 x^2} (d + ex^2)^{3/2}}{20ce^2 \sqrt{-c^2 x^2}} + \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{5e^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.79, size = 339, normalized size = 1.03

$$\frac{\sqrt{d + ex^2} \left( 8ac^3 (8d^2 - 4dex^2 + 3e^2x^4) + 8bc^3 \operatorname{csch}^{-1}(cx) (8d^2 - 4dex^2 + 3e^2x^4) + bex \sqrt{\frac{1}{c^2 x^2} + 1} (c^2 (6ex^2 - 13d) - 2d) \right)}{120c^3 e^3}$$

Antiderivative was successfully verified.



[In] Integrate[(x^5\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2],x]

[Out] (Sqrt[d + e\*x^2]\*(8\*a\*c^3\*(8\*d^2 - 4\*d\*e\*x^2 + 3\*e^2\*x^4) + b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-9\*e + c^2\*(-13\*d + 6\*e\*x^2)) + 8\*b\*c^3\*(8\*d^2 - 4\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsch[c\*x]))/(120\*c^3\*e^3) - (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-(Sqrt[c^2]\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(45\*c^4\*d^2 + 10\*c^2\*d\*e + 9\*e^2)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])]) + 64\*c^7\*d^(5/2)\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]]))/(120\*c^6\*e^3\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [A] time = 1.82, size = 1633, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/480\*(64\*b\*c^5\*d^(5/2)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 - 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) + (45\*b\*c^4\*d^2 + 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 + 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2) + 32\*(3\*b\*c^5\*e^2\*x^4 - 4\*b\*c^5\*d\*e\*x^2 + 8\*b\*c^5\*d^2)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 4\*(24\*a\*c^5\*e^2\*x^4 - 32\*a\*c^5\*d\*e\*x^2 + 64\*a\*c^5\*d^2 + (6\*b\*c^4\*e^2\*x^3 - (13\*b\*c^4\*d\*e + 9\*b\*c^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^5\*e^3), 1/240\*(32\*b\*c^5\*d^(5/2)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 - 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) - (45\*b\*c^4\*d^2 + 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^3 + (c^2\*d + e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(-e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e) + 16\*(3\*b\*c^5\*e^2\*x^4 - 4\*b\*c^5\*d\*e\*x^2 + 8\*b\*c^5\*d^2)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*(24\*a\*c^5\*e^2\*x^4 - 32\*a\*c^5\*d\*e\*x^2 + 64\*a\*c^5\*d^2 + (6\*b\*c^4\*e^2\*x^3 - (13\*b\*c^4\*d\*e + 9\*b\*c^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^5\*e^3), 1/480\*(128\*b\*c^5\*sqrt(-d)\*d^2\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2) + (45\*b\*c^4\*d^2 + 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 + 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2) + 32\*(3\*b\*c^5\*e^2\*x^4 - 4\*b\*c^5\*d\*e\*x^2 + 8\*b\*c^5\*d^2)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 4\*(24\*a\*c^5\*e^2\*x^4 - 32\*a\*c^5\*d\*e\*x^2 + 64\*a\*c^5\*d^2 + (6\*b\*c^4\*e^2\*x^3 - (13\*b\*c^4\*d\*e + 9\*b\*c^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^5\*e^3), 1/240\*(64\*b\*c^5\*sqrt(-d)\*d^2\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2) - (45\*b\*c^4\*d^2 + 10\*b\*c^2\*d\*e + 9\*b\*e^2)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^3 + (c^2\*d + e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(-e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e) + 16\*(3\*b\*c^5\*e^2\*x^4 - 4\*b\*c^5\*d\*e\*x^2 + 8\*b\*c^5\*d^2)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*(24\*a\*c^5\*e^2\*x^4 - 32\*a\*c^5\*d\*e\*x^2 + 64\*a\*c^5\*d^2 + (6\*b\*c^4\*e^2\*x^3 - (13\*b\*c^4\*d\*e + 9\*b\*c^2\*e^2)\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^5\*e^3)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^5/sqrt(e\*x^2 + d), x)

**maple** [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arccsch}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15} \left( \frac{3 \sqrt{ex^2 + d} x^4}{e} - \frac{4 \sqrt{ex^2 + d} dx^2}{e^2} + \frac{8 \sqrt{ex^2 + d} d^2}{e^3} \right) a + \frac{1}{15} b \left( \frac{(3e^3x^6 - de^2x^4 + 4d^2ex^2 + 8d^3) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex^2 + d} e^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/15\*(3\*sqrt(e\*x^2 + d)\*x^4/e - 4\*sqrt(e\*x^2 + d)\*d\*x^2/e^2 + 8\*sqrt(e\*x^2 + d)\*d^2/e^3)\*a + 1/15\*b\*((3\*e^3\*x^6 - d\*e^2\*x^4 + 4\*d^2\*e\*x^2 + 8\*d^3)\*log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x^2 + d)\*e^3) + 15\*integrate(1/15\*(3\*c^2\*e^3\*x^7 - c^2\*d\*e^2\*x^5 + 4\*c^2\*d^2\*e\*x^3 + 8\*c^2\*d^3\*x)/((c^2\*e^3\*x^2 + e^3)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x^2 + d) + (c^2\*e^3\*x^2 + e^3)\*sqrt(e\*x^2 + d)), x) - 15\*integrate(1/15\*(3\*(5\*e^3\*log(c) + e^3)\*c^2\*x^7 + 4\*c^2\*d^2\*e\*x^3 + 8\*c^2\*d^3\*x - (c^2\*d\*e^2 - 15\*e^3\*log(c))\*x^5 + 15\*(c^2\*e^3\*x^7 + e^3\*x^5)\*log(x))/((c^2\*e^3\*x^2 + e^3)\*sqrt(e\*x^2 + d)), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(1/2),x)

[Out] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{acsch}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral(x\*\*5\*(a + b\*acsch(c\*x))/sqrt(d + e\*x\*\*2), x)

$$3.139 \quad \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=229

$$\frac{d\sqrt{d+ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^2\sqrt{-c^2x^2}} - \frac{bx(3c^2d+e)}{6c^2e^3}$$

[Out]  $1/3*(e*x^2+d)^{(3/2)}*(a+b*\operatorname{arccsch}(c*x))/e^2-1/6*b*(3*c^2*d+e)*x*\operatorname{arctan}(e^{(1/2)}*(-c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/c^2/e^{(3/2)}/(-c^2*x^2)^{(1/2)}-2/3*b*c*d^{(3/2)}*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(-c^2*x^2-1)^{(1/2)})/e^2/(-c^2*x^2)^{(1/2)}-d*(a+b*\operatorname{arccsch}(c*x))*(e*x^2+d)^{(1/2)}/e^2+1/6*b*x*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/c/e/(-c^2*x^2)^{(1/2)}$

**Rubi [A]** time = 0.32, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$ , Rules used = {266, 43, 6302, 12, 573, 154, 157, 63, 217, 203, 93, 204}

$$\frac{d\sqrt{d+ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} - \frac{2bcd^{3/2}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3e^2\sqrt{-c^2x^2}} - \frac{bx(3c^2d+e)}{6c^2e^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^3*(a + b*\operatorname{ArcCsch}[c*x]))/\operatorname{Sqrt}[d + e*x^2], x]$

[Out]  $(b*x*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(6*c*e*\operatorname{Sqrt}[-(c^2*x^2)]) - (d*\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcCsch}[c*x]))/e^2 + ((d + e*x^2)^{(3/2)}*(a + b*\operatorname{ArcCsch}[c*x]))/(3*e^2) - (b*(3*c^2*d + e)*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(6*c^2*e^{(3/2)}*\operatorname{Sqrt}[-(c^2*x^2)]) - (2*b*c*d^{(3/2)}*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 - c^2*x^2])])/(3*e^2*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_*)(v_)] /; \operatorname{FreeQ}[b, x]$

#### Rule 43

$\operatorname{Int}[(a_*) + (b_*)(x_*)^{(m_*)}*((c_*) + (d_*)(x_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ (\ !\operatorname{IntegerQ}[n] \ || \ (\operatorname{EqQ}[c, 0] \ \&\& \ \operatorname{LeQ}[7*m + 4*n + 4, 0]) \ || \ \operatorname{LtQ}[9*m + 5*(n + 1), 0] \ || \ \operatorname{GtQ}[m + n + 2, 0])$

#### Rule 63

$\operatorname{Int}[(a_*) + (b_*)(x_*)^{(m_*)}*((c_*) + (d_*)(x_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[-1, m, 0] \ \&\& \ \operatorname{LeQ}[-1, n, 0] \ \&\& \ \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \ \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 93

$\operatorname{Int}[(a_*) + (b_*)(x_*)^{(m_*)}*((c_*) + (d_*)(x_*)^{(n_*)})/((e_*) + (f_*)(x_*)^{(q_*)}), x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m+1)-1)}*(c - (a*d)/b + (d*x^q)/b)^n, x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \operatorname{EqQ}[m + n + 1, 0] \ \&\& \ \operatorname{RationalQ}[n]$

&& LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 154

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(h\*(a + b\*x)^m\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 2)), x] + Dist[1/(d\*f\*(m + n + p + 2)), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*g\*(m + n + p + 2) - h\*(b\*c\*e\*m + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + (b\*d\*f\*g\*(m + n + p + 2) + h\*(a\*d\*f\*m - b\*(d\*e\*(m + n + 1) + c\*f\*(m + p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_)), x\_Symbol] := Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 573

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_))^(r\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q\*(e + f\*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 6302

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_)^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[Simplify[Integrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[

$(m + 2*p + 1)/2, 0]$  && !ILtQ $[(m - 1)/2, 0])$

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} - \frac{(bcx) \int \frac{(-2d + ex^2)}{3e^2 \sqrt{-d - ex^2}} dx}{\sqrt{-d - ex^2}} \\
 &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} - \frac{(bcx) \int \frac{(-2d + ex^2)}{3e^2 \sqrt{-d - ex^2}} dx}{\sqrt{-d - ex^2}} \\
 &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} - \frac{(bcx) \operatorname{Subst}\left(\int \frac{(-2d + ex^2)}{3e^2 \sqrt{-d - ex^2}} dx, x, \sqrt{-d - ex^2}\right)}{\sqrt{-d - ex^2}} \\
 &= \frac{bx\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} \\
 &= \frac{bx\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} \\
 &= \frac{bx\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} \\
 &= \frac{bx\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2} \\
 &= \frac{bx\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.56, size = 280, normalized size = 1.22

$$\frac{\sqrt{d + ex^2} \left( -4acd + 2acex^2 + bex\sqrt{\frac{1}{c^2x^2} + 1} + 2bccsch^{-1}(cx)(ex^2 - 2d) \right)}{6ce^2} + \frac{bx\sqrt{\frac{1}{c^2x^2} + 1} \left( 4c^5d^{3/2}\sqrt{-d - ex^2} \operatorname{atan}\left(\frac{\sqrt{-d - ex^2}}{\sqrt{-d - ex^2}}\right) \right)}{6ce^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[d + e\*x^2]\*(-4\*a\*c\*d + b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x + 2\*a\*c\*e\*x^2 + 2\*b\*c\*(-2\*d + e\*x^2)\*ArcCsch[c\*x]))/(6\*c\*e^2) + (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-(Sqrt[c^2]\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(3\*c^2\*d + e)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])]) + 4\*c^5\*d^(3/2)\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]]))/(6\*c^4\*e^2\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [A]** time = 1.02, size = 1341, normalized size = 5.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/24\*(4\*b\*c^3\*d^(3/2)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 + 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) + (3\*b\*c^2\*d + b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 - 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2) + 8\*(b\*c^3\*e\*x^2 - 2\*b\*c^3\*d)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 4\*(2\*a\*c^3\*e\*x^2 + b\*c^2\*e\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - 4\*a\*c^3\*d)\*sqrt(e\*x^2 + d))/(c^3\*e^2), 1/12\*(2\*b\*c^3\*d^(3/2)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 + 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) + (3\*b\*c^2\*d + b\*e)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^3 + (c^2\*d + e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(-e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e) + 4\*(b\*c^3\*e\*x^2 - 2\*b\*c^3\*d)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*(2\*a\*c^3\*e\*x^2 + b\*c^2\*e\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - 4\*a\*c^3\*d)\*sqrt(e\*x^2 + d))/(c^3\*e^2), -1/24\*(8\*b\*c^3\*sqrt(-d)\*d\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2)) - (3\*b\*c^2\*d + b\*e)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 - 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2) - 8\*(b\*c^3\*e\*x^2 - 2\*b\*c^3\*d)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - 4\*(2\*a\*c^3\*e\*x^2 + b\*c^2\*e\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - 4\*a\*c^3\*d)\*sqrt(e\*x^2 + d))/(c^3\*e^2), -1/12\*(4\*b\*c^3\*sqrt(-d)\*d\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2)) - (3\*b\*c^2\*d + b\*e)\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^3 + (c^2\*d + e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(-e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e) - 4\*(b\*c^3\*e\*x^2 - 2\*b\*c^3\*d)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - 2\*(2\*a\*c^3\*e\*x^2 + b\*c^2\*e\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - 4\*a\*c^3\*d)\*sqrt(e\*x^2 + d))/(c^3\*e^2)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^3/sqrt(e\*x^2 + d), x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arcsch}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} \left( \frac{\sqrt{ex^2 + d} x^2}{e} - \frac{2\sqrt{ex^2 + d} d}{e^2} \right) a + \frac{1}{3} b \left( \frac{(e^2 x^4 - dex^2 - 2d^2) \log(\sqrt{c^2 x^2 + 1} + 1)}{\sqrt{ex^2 + d} e^2} \right) + 3 \int \frac{c^2 e^2 x^5}{3((c^2 e^2 x^2 + e^2)\sqrt{c^2 x^2 + 1})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/3\*(sqrt(e\*x^2 + d)\*x^2/e - 2\*sqrt(e\*x^2 + d)\*d/e^2)\*a + 1/3\*b\*((e^2\*x^4 - d\*e\*x^2 - 2\*d^2)\*log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x^2 + d)\*e^2) + 3\*integrate(1/3\*(c^2\*e^2\*x^5 - c^2\*d\*e\*x^3 - 2\*c^2\*d^2\*x)/((c^2\*e^2\*x^2 + e^2)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x^2 + d) + (c^2\*e^2\*x^2 + e^2)\*sqrt(e\*x^2 + d)), x) - 3\*integrate(1/3\*((3\*e^2\*log(c) + e^2)\*c^2\*x^5 - 2\*c^2\*d^2\*x - (c^2\*d\*e - 3\*e^2\*log(c))\*x^3 + 3\*(c^2\*e^2\*x^5 + e^2\*x^3)\*log(x))/((c^2\*e^2\*x^2 + e^2)\*sqrt(e\*x^2 + d)), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{\sqrt{e x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(1/2),x)

[Out] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acsch}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral(x\*\*3\*(a + b\*acsch(c\*x))/sqrt(d + e\*x\*\*2), x)

$$3.140 \quad \int \frac{x(a+b\operatorname{csch}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

**Optimal.** Leaf size=135

$$\frac{\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx))}{e} + \frac{bx \tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}} + \frac{bc\sqrt{d}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e\sqrt{-c^2x^2}}$$

[Out] b\*c\*x\*arctan((e\*x^2+d)^(1/2)/d^(1/2)/(-c^2\*x^2-1)^(1/2))\*d^(1/2)/e/(-c^2\*x^2)^(1/2)+b\*x\*arctan(e^(1/2)\*(-c^2\*x^2-1)^(1/2)/c/(e\*x^2+d)^(1/2))/e^(1/2)/(-c^2\*x^2)^(1/2)+(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/e

**Rubi [A]** time = 0.15, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {6300, 446, 105, 63, 217, 203, 93, 204}

$$\frac{\sqrt{d+ex^2} (a+b\operatorname{csch}^{-1}(cx))}{e} + \frac{bx \tan^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}} + \frac{bc\sqrt{d}x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/e + (b\*x\*ArcTan[(Sqrt[e]\*Sqrt[-1 - c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(Sqrt[e]\*Sqrt[-(c^2\*x^2)]) + (b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 - c^2\*x^2])])/(e\*Sqrt[-(c^2\*x^2)])

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 105

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> Dist[b/f, Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n, x], x] - Dist[(b\*e - a\*f)/f, Int[((a + b\*x)^(m - 1)\*(c + d\*x)^n)/(e + f\*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))

#### Rule 203

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])



Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6300

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.)\*(x\_)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsch[c\*x]))/(2\*e\*(p + 1)), x] - Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[-(c^2\*x^2)]), Int[(d + e\*x^2)^(p + 1)/(x\*Sqrt[-1 - c^2\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{x(a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx &= \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{x\sqrt{-1-c^2x^2}} dx}{e\sqrt{-c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e} - \frac{(bcx) \operatorname{Subst} \left( \int \frac{\sqrt{d+ex^2}}{x\sqrt{-1-c^2x^2}} dx, x, x^2 \right)}{2e\sqrt{-c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e} - \frac{(bcx) \operatorname{Subst} \left( \int \frac{1}{\sqrt{-1-c^2x^2} \sqrt{d+ex^2}} dx, x, x^2 \right)}{2\sqrt{-c^2x^2}} - \frac{(bcdx) \operatorname{Subst} \left( \int \frac{1}{\sqrt{-1-c^2x^2}} dx, x, x^2 \right)}{2\sqrt{-c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{(bx) \operatorname{Subst} \left( \int \frac{1}{\sqrt{d - \frac{e}{c^2} - \frac{ex^2}{c^2}}} dx, x, \sqrt{-1 - c^2x^2} \right)}{c\sqrt{-c^2x^2}} - \frac{(bcdx) \operatorname{Subst} \left( \int \frac{1}{\sqrt{-1-c^2x^2}} dx, x, x^2 \right)}{2\sqrt{-c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{bc\sqrt{d} x \tan^{-1} \left( \frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-1-c^2x^2}} \right)}{e\sqrt{-c^2x^2}} + \frac{(bx) \operatorname{Subst} \left( \int \frac{1}{1 + \frac{ex^2}{c^2}} dx, x, x^2 \right)}{c\sqrt{-c^2x^2}} \\
 &= \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e} + \frac{bx \tan^{-1} \left( \frac{\sqrt{e} \sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}} \right)}{\sqrt{e} \sqrt{-c^2x^2}} + \frac{bc\sqrt{d} x \tan^{-1} \left( \frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-1-c^2x^2}} \right)}{e\sqrt{-c^2x^2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 223, normalized size = 1.65

$$\frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e} - \frac{bx \sqrt{\frac{1}{c^2x^2} + 1} \left( c^3 \sqrt{d} \sqrt{-d - ex^2} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{c^2x^2 + 1}}{\sqrt{-d - ex^2}} \right) - \sqrt{c^2} \sqrt{e} \sqrt{c^2d - e} \sqrt{\frac{c^2(d+ex^2)}{c^2d - e}} \right)}{c^2e\sqrt{c^2x^2 + 1} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] (Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/e - (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-(Sqrt[c^2]\*Sqrt[c^2\*d - e]\*Sqrt[e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e)]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])]) + c^3\*Sqrt[d]\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]]))/(c^2\*e\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [B] time = 0.69, size = 1064, normalized size = 7.88

$$\frac{4\sqrt{ex^2+d}bc\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}+1}}{cx}\right) + bc\sqrt{d}\log\left(\frac{(c^4d^2+6c^2de+e^2)x^4+8(c^2d^2+de)x^2-4((c^3d+ce)x^3+2cdx)\sqrt{ex^2+d}\sqrt{d}\sqrt{\frac{c^2x^2+1}{c^2x^2}+8d^2}}{x^4}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] [1/4\*(4\*sqrt(e\*x^2 + d)\*b\*c\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + b\*c\*sqrt(d)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 - 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) + 4\*sqrt(e\*x^2 + d)\*a\*c + b\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 + 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2))/(c\*e), 1/4\*(4\*sqrt(e\*x^2 + d)\*b\*c\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + b\*c\*sqrt(d)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 - 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) + 4\*sqrt(e\*x^2 + d)\*a\*c - 2\*b\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^3 + (c^2\*d + e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(-e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e))/(c\*e), 1/4\*(2\*b\*c\*sqrt(-d)\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2)) + 4\*sqrt(e\*x^2 + d)\*b\*c\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 4\*sqrt(e\*x^2 + d)\*a\*c + b\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 + 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2))/(c\*e), 1/2\*(b\*c\*sqrt(-d)\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2)) + 2\*sqrt(e\*x^2 + d)\*b\*c\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*sqrt(e\*x^2 + d)\*a\*c - b\*sqrt(-e)\*arctan(1/2\*(2\*c^2\*e\*x^3 + (c^2\*d + e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(-e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e))/(c\*e)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x/sqrt(e\*x^2 + d), x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{arcsch}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)`

[Out] `int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \left( \frac{\sqrt{ex^2 + d} \log\left(\sqrt{c^2x^2 + 1} + 1\right)}{e} + \int \frac{c^2ex^3 + c^2dx}{(c^2ex^2 + e)\sqrt{c^2x^2 + 1}\sqrt{ex^2 + d} + (c^2ex^2 + e)\sqrt{ex^2 + d}} dx - \int \frac{(e \log(c))}{\dots} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `b*(sqrt(e*x^2 + d)*log(sqrt(c^2*x^2 + 1) + 1)/e + integrate((c^2*e*x^3 + c^2*d*x)/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (c^2*e*x^2 + e)*sqrt(e*x^2 + d)), x) - integrate(((e*log(c) + e)*c^2*x^3 + (c^2*d + e*log(c))*x + (c^2*e*x^3 + e*x)*log(x))/((c^2*e*x^2 + e)*sqrt(e*x^2 + d)), x) + sqrt(e*x^2 + d)*a/e`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)`

[Out] `int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{acsch}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)`

[Out] `Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)`

$$3.141 \quad \int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{d + ex^2}} dx$$

**Optimal.** Leaf size=26

$$\operatorname{Int}\left(\frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{d + ex^2}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(1/2), x)

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

[Out] Defer[Int] [(a + b\*ArcCsch[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

Rubi steps

$$\int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{d + ex^2}} dx = \int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{d + ex^2}} dx$$

**Mathematica [A]** time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[d + e\*x^2]), x]

**fricas [A]** time = 0.67, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{ex^3 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e\*x^3 + d\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(sqrt(e\*x^2 + d)\*x), x)

**maple** [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(1/2), x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(1/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{\sqrt{ex^2 + d}} dx - \frac{a \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(1/2), x, algorithm="maxima")

[Out] b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(sqrt(e\*x^2 + d)\*x), x) - a\*arcsinh(d/(sqrt(d\*e)\*abs(x)))/sqrt(d)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)^(1/2)), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(e\*x\*\*2+d)\*\*(1/2), x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*sqrt(d + e\*x\*\*2)), x)

$$3.142 \quad \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(1/2), x)

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

[Out] Defer[Int] [(a + b\*ArcCsch[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx = \int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx$$

Mathematica [A] time = 23.25, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x^3\*Sqrt[d + e\*x^2]), x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)}{ex^5+dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e\*x^5 + d\*x^3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcsch}(cx)+a}{\sqrt{ex^2+d}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(sqrt(e\*x^2 + d)\*x^3), x)

**maple** [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^3 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(1/2), x)

[Out] int((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(1/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{e \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{3}{2}}} - \frac{\sqrt{ex^2 + d}}{dx^2} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{\sqrt{ex^2 + d} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(1/2), x, algorithm="maxima")

[Out] 1/2\*a\*(e\*arsinh(d/(sqrt(d\*e)\*abs(x)))/d^(3/2) - sqrt(e\*x^2 + d)/(d\*x^2)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(sqrt(e\*x^2 + d)\*x^3), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^3 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^3\*(d + e\*x^2)^(1/2)), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^3\*(d + e\*x^2)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^3 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*3/(e\*x\*\*2+d)\*\*(1/2), x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*\*3\*sqrt(d + e\*x\*\*2)), x)

$$3.143 \quad \int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}}, x\right)$$

[Out] Unintegrable(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^2\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Defer[Int] [(x^2\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

Rubi steps

$$\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx = \int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Mathematica [A] time = 6.33, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{bx^2 \operatorname{arcsch}(cx) + ax^2}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsch(c\*x) + a\*x^2)/sqrt(e\*x^2 + d), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2/sqrt(e\*x^2 + d), x)



**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccsch}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x)

[Out] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{\sqrt{ex^2 + d} x}{e} - \frac{d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}}\right) + b \int \frac{x^2 \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/2\*a\*(sqrt(e\*x^2 + d)\*x/e - d\*arcsinh(e\*x/sqrt(d\*e))/e^(3/2)) + b\*integrate(x^2\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/sqrt(e\*x^2 + d), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(1/2),x)

[Out] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(1/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acsch}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral(x\*\*2\*(a + b\*acsch(c\*x))/sqrt(d + e\*x\*\*2), x)

$$3.144 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}} dx$$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/Sqrt[d + e\*x^2], x]

[Out] Defer[Int] [(a + b\*ArcCsch[c\*x])/Sqrt[d + e\*x^2], x]

Rubi steps

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}} dx = \int \frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}} dx$$

Mathematica [A] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/Sqrt[d + e\*x^2], x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/Sqrt[d + e\*x^2], x]

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)/sqrt(e\*x^2 + d), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/sqrt(e\*x^2 + d), x)

**maple** [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x)

[Out] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{\sqrt{ex^2 + d}} dx + \frac{a \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="maxima")

[Out] b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/sqrt(e\*x^2 + d), x) + a\*arcsinh(e\*x/sqrt(d\*e))/sqrt(e)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(d + e\*x^2)^(1/2), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(d + e\*x^2)^(1/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(1/2), x)

[Out] Integral((a + b\*acsch(c\*x))/sqrt(d + e\*x\*\*2), x)

$$3.145 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 \sqrt{d + ex^2}} dx$$

**Optimal.** Leaf size=294

$$\frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} + \frac{bex\sqrt{d + ex^2} F\left(\tan^{-1}(cx) \left| 1 - \frac{e}{c^2 d} \right. \right)}{d^2 \sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1} \sqrt{\frac{d+ex^2}{d(c^2 x^2 + 1)}}} + \frac{bc\sqrt{-c^2 x^2 - 1} \sqrt{d + ex^2}}{d\sqrt{-c^2 x^2}} - \frac{bc^2 x \sqrt{d + ex^2} E\left(\tan^{-1}(cx) \left| 1 - \frac{e}{c^2 d} \right. \right)}{d\sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1}}$$

[Out]  $-(a + b \operatorname{arccsch}(c x)) (e x^2 + d)^{1/2} / d x + b c^3 x^2 (e x^2 + d)^{1/2} / d (-c^2 x^2)^{1/2} / (-c^2 x^2 - 1)^{1/2} + b c (-c^2 x^2 - 1)^{1/2} (e x^2 + d)^{1/2} / d (-c^2 x^2)^{1/2} - b c^2 x (1 / (c^2 x^2 + 1))^{1/2} (c^2 x^2 + 1)^{1/2} \operatorname{EllipticE}(c x / (c^2 x^2 + 1)^{1/2}, (1 - e / c^2 d)^{1/2}) (e x^2 + d)^{1/2} / d (-c^2 x^2)^{1/2} / (-c^2 x^2 - 1)^{1/2} / ((e x^2 + d) / d / (c^2 x^2 + 1))^{1/2} + b e x x (1 / (c^2 x^2 + 1))^{1/2} (c^2 x^2 + 1)^{1/2} \operatorname{EllipticF}(c x / (c^2 x^2 + 1)^{1/2}, (1 - e / c^2 d)^{1/2}) (e x^2 + d)^{1/2} / d^2 / (-c^2 x^2)^{1/2} / (-c^2 x^2 - 1)^{1/2} / ((e x^2 + d) / d / (c^2 x^2 + 1))^{1/2}$

**Rubi [A]** time = 0.25, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {264, 6302, 12, 475, 21, 422, 418, 492, 411}

$$\frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} + \frac{bex\sqrt{d + ex^2} F\left(\tan^{-1}(cx) \left| 1 - \frac{e}{c^2 d} \right. \right)}{d^2 \sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1} \sqrt{\frac{d+ex^2}{d(c^2 x^2 + 1)}}} + \frac{bc^3 x^2 \sqrt{d + ex^2}}{d\sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1}} + \frac{bc\sqrt{-c^2 x^2 - 1} \sqrt{d + ex^2}}{d\sqrt{-c^2 x^2}}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]`

[Out]  $(b c^3 x^2 \operatorname{Sqrt}[d + e x^2]) / (d \operatorname{Sqrt}[-(c^2 x^2)] \operatorname{Sqrt}[-1 - c^2 x^2]) + (b c \operatorname{Sqrt}[-1 - c^2 x^2] \operatorname{Sqrt}[d + e x^2]) / (d \operatorname{Sqrt}[-(c^2 x^2)]) - (\operatorname{Sqrt}[d + e x^2] (a + b \operatorname{ArcCsch}[c x])) / (d x) - (b c^2 x \operatorname{Sqrt}[d + e x^2] \operatorname{EllipticE}[\operatorname{ArcTan}[c x], 1 - e / (c^2 d)]) / (d \operatorname{Sqrt}[-(c^2 x^2)] \operatorname{Sqrt}[-1 - c^2 x^2] \operatorname{Sqrt}[(d + e x^2) / (d (1 + c^2 x^2))]) + (b e x \operatorname{Sqrt}[d + e x^2] \operatorname{EllipticF}[\operatorname{ArcTan}[c x], 1 - e / (c^2 d)]) / (d^2 \operatorname{Sqrt}[-(c^2 x^2)] \operatorname{Sqrt}[-1 - c^2 x^2] \operatorname{Sqrt}[(d + e x^2) / (d (1 + c^2 x^2))])$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 21

`Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])`

#### Rule 264

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]`

#### Rule 411

`Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[`

$d/c, 2] \cdot \sqrt{c + dx^2} \cdot \sqrt{(c(a + bx^2))/(a(c + dx^2))}, x] /;$  FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

#### Rule 418

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)^2]\*Sqrt[(c\_) + (d\_)\*(x\_)^2]), x\_Symbol] := Simp[1/(Sqrt[a + b\*x^2]\*EllipticF[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)]/(a\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

#### Rule 422

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/Sqrt[(c\_) + (d\_)\*(x\_)^2], x\_Symbol] := Dist[a, Int[1/(Sqrt[a + b\*x^2]\*Sqrt[c + d\*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[a + b\*x^2]\*Sqrt[c + d\*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a]

#### Rule 475

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*e\*(m + 1)), x] - Dist[1/(a\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a + b\*x^n)^p\*(c + d\*x^n)^(q - 1)\*Simp[c\*b\*(m + 1) + n\*(b\*c\*(p + 1) + a\*d\*q) + d\*(b\*(m + 1) + b\*n\*(p + q + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

#### Rule 492

Int[(x\_)^2/(Sqrt[(a\_) + (b\_)\*(x\_)^2]\*Sqrt[(c\_) + (d\_)\*(x\_)^2]), x\_Symbol] := Simp[(x\*Sqrt[a + b\*x^2])/(b\*Sqrt[c + d\*x^2]), x] - Dist[c/b, Int[Sqrt[a + b\*x^2]/(c + d\*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

#### Rule 6302

Int[((a\_) + ArcCsch[(c\_)\*(x\_)])\*(b\_)\*((f\_)\*(x\_))^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(p\_), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

#### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 \sqrt{d + ex^2}} dx &= -\frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} + \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{dx^2 \sqrt{-1-c^2x^2}} dx}{\sqrt{-c^2x^2}} \\
&= -\frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} + \frac{(bcx) \int \frac{\sqrt{d+ex^2}}{x^2 \sqrt{-1-c^2x^2}} dx}{d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2} \sqrt{d + ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} - \frac{(bcx) \int \frac{-e^{-c^2ex^2}}{\sqrt{-1-c^2x^2} \sqrt{d+ex^2}} dx}{d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2} \sqrt{d + ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} - \frac{(bcex) \int \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}} dx}{d\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1-c^2x^2} \sqrt{d + ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} + \frac{(bcex) \int \frac{1}{\sqrt{-1-c^2x^2} \sqrt{d+ex^2}} dx}{d\sqrt{-c^2x^2}} + \\
&= \frac{bc^3x^2\sqrt{d + ex^2}}{d\sqrt{-c^2x^2} \sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2} \sqrt{d + ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} + \frac{bcex}{d^2\sqrt{-}} \\
&= \frac{bc^3x^2\sqrt{d + ex^2}}{d\sqrt{-c^2x^2} \sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2} \sqrt{d + ex^2}}{d\sqrt{-c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{dx} - \frac{bc^2x}{d\sqrt{-}}
\end{aligned}$$

**Mathematica [A]** time = 0.21, size = 139, normalized size = 0.47

$$\frac{\sqrt{d + ex^2} \left( -a + bcx \sqrt{\frac{1}{c^2x^2} + 1} - b \operatorname{csch}^{-1}(cx) \right)}{dx} - \frac{bcex \sqrt{\frac{1}{c^2x^2} + 1} \sqrt{\frac{ex^2}{d} + 1} E \left( \sin^{-1} \left( \sqrt{-\frac{e}{d}} x \right) \middle| \frac{c^2d}{e} \right)}{d\sqrt{c^2x^2 + 1} \sqrt{-\frac{e}{d}} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*Sqrt[d + e\*x^2]), x]

[Out] (Sqrt[d + e\*x^2]\*(-a + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x - b\*ArcCsch[c\*x]))/(d\*x) - (b\*c\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*EllipticE[ArcSin[Sqrt[-(e/d)]\*x], (c^2\*d)/e])/(d\*Sqrt[-(e/d)]\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [F]** time = 0.61, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{ex^4 + dx^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e\*x^4 + d\*x^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(sqrt(e\*x^2 + d)\*x^2), x)

**maple** [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 \sqrt{e x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(1/2),x)

[Out] int((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-b \left( \frac{\sqrt{ex^2 + d} \log(\sqrt{c^2 x^2 + 1} + 1)}{dx} + \int \frac{c^2 ex^2 + c^2 d}{(c^2 dx^2 + d) \sqrt{c^2 x^2 + 1} \sqrt{ex^2 + d} + (c^2 dx^2 + d) \sqrt{ex^2 + d}} dx + \int -\frac{c^2 e}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] -b\*(sqrt(e\*x^2 + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/(d\*x) + integrate((c^2\*e\*x^2 + c^2\*d)/((c^2\*d\*x^2 + d)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x^2 + d) + (c^2\*d\*x^2 + d)\*sqrt(e\*x^2 + d)), x) + integrate(-(c^2\*e\*x^4 - (d\*log(c) - d)\*c^2\*x^2 - d\*log(c) - (c^2\*d\*x^2 + d)\*log(x))/((c^2\*d\*x^4 + d\*x^2)\*sqrt(e\*x^2 + d)), x)) - sqrt(e\*x^2 + d)\*a/(d\*x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2 \sqrt{e x^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x^2)^(1/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^2 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*2/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*\*2\*sqrt(d + e\*x\*\*2)), x)

$$3.146 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^4 \sqrt{d + ex^2}} dx$$

**Optimal.** Leaf size=425

$$\frac{2e\sqrt{d+ex^2} (a + b \operatorname{csch}^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2} (a + b \operatorname{csch}^{-1}(cx))}{3dx^3} - \frac{bex (c^2d + 6e) \sqrt{d+ex^2} F\left(\tan^{-1}(cx) \mid 1 - \frac{e}{c^2d}\right)}{9d^3 \sqrt{-c^2x^2} \sqrt{-c^2x^2 - 1}} - \frac{bc\sqrt{d+ex^2}}{\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$

[Out]  $-1/3*(a+b*\operatorname{arccsch}(c*x))*(e*x^2+d)^{(1/2)}/d/x^3+2/3*e*(a+b*\operatorname{arccsch}(c*x))*(e*x^2+d)^{(1/2)}/d^2/x-1/9*b*c^3*(2*c^2*d+5*e)*x^2*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}-1/9*b*c*(2*c^2*d+5*e)*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}+1/9*b*c*(-c^2*x^2-1)^{(1/2)}*(e*x^2+d)^{(1/2)}/d/x^2/(-c^2*x^2)^{(1/2)}+1/9*b*c^2*(2*c^2*d+5*e)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*\operatorname{EllipticE}(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d^2/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}-1/9*b*e*(c^2*d+6*e)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*\operatorname{EllipticF}(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d^3/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}$

**Rubi [A]** time = 0.49, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {271, 264, 6302, 12, 580, 583, 531, 418, 492, 411}

$$\frac{2e\sqrt{d+ex^2} (a + b \operatorname{csch}^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2} (a + b \operatorname{csch}^{-1}(cx))}{3dx^3} - \frac{bc^3x^2 (2c^2d + 5e) \sqrt{d+ex^2}}{9d^2 \sqrt{-c^2x^2} \sqrt{-c^2x^2 - 1}} - \frac{bc\sqrt{-c^2x^2 - 1} (2c^2d - 5e)}{9d^2 \sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{ArcCsch}[c*x])/(x^4*\operatorname{Sqrt}[d + e*x^2]),x]$

[Out]  $-(b*c^3*(2*c^2*d + 5*e)*x^2*\operatorname{Sqrt}[d + e*x^2])/(9*d^2*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[-1 - c^2*x^2]) - (b*c*(2*c^2*d + 5*e)*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(9*d^2*\operatorname{Sqrt}[-(c^2*x^2)]) + (b*c*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(9*d*x^2*\operatorname{Sqrt}[-(c^2*x^2)]) - (\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcCsch}[c*x]))/(3*d*x^3) + (2*e*\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcCsch}[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d + 5*e)*x*\operatorname{Sqrt}[d + e*x^2]*\operatorname{EllipticE}[\operatorname{ArcTan}[c*x], 1 - e/(c^2*d)])/(9*d^2*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(c^2*d + 6*e)*x*\operatorname{Sqrt}[d + e*x^2]*\operatorname{EllipticF}[\operatorname{ArcTan}[c*x], 1 - e/(c^2*d)])/(9*d^3*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))])$

**Rule 12**

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \&\& !\operatorname{MatchQ}[u, (b_*)(v_)] /;$   $\operatorname{FreeQ}[b, x]$

**Rule 264**

$\operatorname{Int}[((c_*)(x_))^{(m_)}*((a_*) + (b_*)(x_)^{(n_)})^{(p_)}, x\_Symbol] := \operatorname{Simp}[((c*x)^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*c*(m+1)), x] /;$   $\operatorname{FreeQ}\{a, b, c, m, n, p\}, x] \&\& \operatorname{EqQ}[(m+1)/n + p + 1, 0] \&\& \operatorname{NeQ}[m, -1]$

**Rule 271**

$\operatorname{Int}[(x_*)^{(m_)}*((a_*) + (b_*)(x_*)^{(n_)})^{(p_)}, x\_Symbol] := \operatorname{Simp}[x^{(m+1)}*(a + b*x^n)^{(p+1)})/(a*(m+1)), x] - \operatorname{Dist}[(b*(m+n*(p+1)+1))/(a*(m+1)), \operatorname{Int}[x^{(m+n)}*(a + b*x^n)^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, m, n, p\}, x] \&\& \operatorname{ILtQ}[\operatorname{Simplify}[(m+1)/n + p + 1], 0] \&\& \operatorname{NeQ}[m, -1]$



Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
&& LtQ[m, -1]
```

Rule 6302

```
Int[((a_.) + ArcSch[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x^4 \sqrt{d + ex^2}} dx &= -\frac{\sqrt{d + ex^2} (a + b \operatorname{arcsch}^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \operatorname{arcsch}^{-1}(cx))}{3d^2x} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{3d^2x^4\sqrt{-1-c^2x^2}}}{\sqrt{-c^2x^2}} \\
&= -\frac{\sqrt{d + ex^2} (a + b \operatorname{arcsch}^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \operatorname{arcsch}^{-1}(cx))}{3d^2x} - \frac{(bcx) \int \frac{\sqrt{d+ex^2}(-d+2ex^2)}{x^4\sqrt{-1-c^2x^2}}}{3d^2\sqrt{-c^2x^2}} \\
&= \frac{bc\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{-c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \operatorname{arcsch}^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \operatorname{arcsch}^{-1}(cx))}{3d^2x} \\
&= -\frac{bc(2c^2d + 5e) \sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{-c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \operatorname{arcsch}^{-1}(cx))}{3dx^3} \\
&= -\frac{bc(2c^2d + 5e) \sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{-c^2x^2}} - \frac{\sqrt{d + ex^2} (a + b \operatorname{arcsch}^{-1}(cx))}{3dx^3} \\
&= -\frac{bc^3(2c^2d + 5e) x^2 \sqrt{d + ex^2}}{9d^2\sqrt{-c^2x^2} \sqrt{-1 - c^2x^2}} - \frac{bc(2c^2d + 5e) \sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{-c^2x^2}} \\
&= -\frac{bc^3(2c^2d + 5e) x^2 \sqrt{d + ex^2}}{9d^2\sqrt{-c^2x^2} \sqrt{-1 - c^2x^2}} - \frac{bc(2c^2d + 5e) \sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9d^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1 - c^2x^2} \sqrt{d + ex^2}}{9dx^2\sqrt{-c^2x^2}}
\end{aligned}$$

**Mathematica** [C] time = 0.64, size = 239, normalized size = 0.56

$$\frac{\sqrt{d + ex^2} \left( 3a(d - 2ex^2) + bcx \sqrt{\frac{1}{c^2x^2} + 1} (2c^2dx^2 - d + 5ex^2) + 3b \operatorname{arcsch}^{-1}(cx) (d - 2ex^2) \right) - ibcx \sqrt{\frac{1}{c^2x^2} + 1} \sqrt{\frac{ex^2}{d}}}{9d^2x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^4\*Sqrt[d + e\*x^2]),x]

[Out] -1/9\*(Sqrt[d + e\*x^2]\*(3\*a\*(d - 2\*e\*x^2) + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-d + 2\*c^2\*d\*x^2 + 5\*e\*x^2) + 3\*b\*(d - 2\*e\*x^2)\*ArcCsch[c\*x]))/(d^2\*x^3) - ((I/9)\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*(2\*c^2\*d + 5\*e)\*EllipticE[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)] - 2\*(c^4\*d^2 + 2\*c^2\*d\*e - 3\*e^2)\*EllipticF[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)))/(Sqrt[c^2]\*d^2\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [F] time = 0.90, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{ex^6 + dx^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^4/(e\*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e\*x^6 + d\*x^4), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex^2 + d} x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^4/(e\*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(sqrt(e\*x^2 + d)\*x^4), x)

**maple** [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^4 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^4/(e\*x^2+d)^(1/2),x)

[Out] int((a+b\*arccsch(c\*x))/x^4/(e\*x^2+d)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a \left( \frac{2 \sqrt{ex^2 + d} e}{d^2 x} - \frac{\sqrt{ex^2 + d}}{dx^3} \right) + \frac{1}{3} b \left( \frac{(2e^2 x^4 + dex^2 - d^2) \log(\sqrt{c^2 x^2 + 1} + 1)}{\sqrt{ex^2 + d} d^2 x^3} \right) + 3 \int \frac{2}{3 \left( (c^2 d^2 x^4 + d^2 x^2) \sqrt{c^2 x^2 + 1} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^4/(e\*x^2+d)^(1/2),x, algorithm="maxima")

[Out] 1/3\*a\*(2\*sqrt(e\*x^2 + d)\*e/(d^2\*x) - sqrt(e\*x^2 + d)/(d\*x^3)) + 1/3\*b\*((2\*e^2\*x^4 + d\*e\*x^2 - d^2)\*log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x^2 + d)\*d^2\*x^3) + 3\*integrate(1/3\*(2\*c^2\*e^2\*x^4 + c^2\*d\*e\*x^2 - c^2\*d^2)/((c^2\*d^2\*x^4 + d^2\*x^2)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x^2 + d) + (c^2\*d^2\*x^4 + d^2\*x^2)\*sqrt(e\*x^2 + d)), x) - 3\*integrate(1/3\*(2\*c^2\*e^2\*x^6 + c^2\*d\*e\*x^4 + (3\*d^2\*log(c) - d^2)\*c^2\*x^2 + 3\*d^2\*log(c) + 3\*(c^2\*d^2\*x^2 + d^2)\*log(x))/((c^2\*d^2\*x^6 + d^2\*x^4)\*sqrt(e\*x^2 + d)), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^4 \sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^4\*(d + e\*x^2)^(1/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^4\*(d + e\*x^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^4 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*4/(e\*x\*\*2+d)\*\*(1/2),x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*\*4\*sqrt(d + e\*x\*\*2)), x)

$$3.147 \quad \int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=256

$$\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} - \frac{8bcd^{3/2} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2}}\right)}{3e^3 \sqrt{-c^2x^2}}$$

[Out] 1/3\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x))/e^3-1/6\*b\*(9\*c^2\*d+e)\*x\*arctan(e^(1/2)\*(-c^2\*x^2-1)^(1/2)/c/(e\*x^2+d)^(1/2))/c^2/e^(5/2)/(-c^2\*x^2)^(1/2)-8/3\*b\*c\*d^(3/2)\*x\*arctan((e\*x^2+d)^(1/2)/d^(1/2)/(-c^2\*x^2-1)^(1/2))/e^3/(-c^2\*x^2)^(1/2)-d^2\*(a+b\*arccsch(c\*x))/e^3/(e\*x^2+d)^(1/2)-2\*d\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/e^3+1/6\*b\*x\*(-c^2\*x^2-1)^(1/2)\*(e\*x^2+d)^(1/2)/c/e^2/(-c^2\*x^2)^(1/2)

**Rubi [A]** time = 1.13, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 6302, 12, 1615, 157, 63, 217, 203, 93, 204}

$$\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} - \frac{8bcd^{3/2} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2}}\right)}{3e^3 \sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] (b\*x\*Sqrt[-1 - c^2\*x^2]\*Sqrt[d + e\*x^2])/(6\*c\*e^2\*Sqrt[-(c^2\*x^2)]) - (d^2\*(a + b\*ArcCsch[c\*x]))/(e^3\*Sqrt[d + e\*x^2]) - (2\*d\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/e^3 + ((d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]))/(3\*e^3) - (b\*(9\*c^2\*d + e)\*x\*ArcTan[(Sqrt[e]\*Sqrt[-1 - c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(6\*c^2\*e^(5/2)\*Sqrt[-(c^2\*x^2)]) - (8\*b\*c\*d^(3/2)\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 - c^2\*x^2])])/(3\*e^3\*Sqrt[-(c^2\*x^2)])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 93

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)\*(c + d\*x)^n, x], x, (a + b\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

$- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}$   
 $], x]] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[m + n + 1, 0] \ \&\& \ \text{RationalQ}[n]$   
 $\ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x]$

### Rule 157

$\text{Int}[\frac{((c_.) + (d_.)*(x_))^{(n_)}*((e_.) + (f_.)*(x_))^{(p_)}*((g_.) + (h_.)*(x_))}{((a_.) + (b_.)*(x_))}, x\_Symbol] \rightarrow \text{Dist}[h/b, \text{Int}[(c + d*x)^n*(e + f*x)^p, x], x] + \text{Dist}[(b*g - a*h)/b, \text{Int}[(c + d*x)^n*(e + f*x)^p/(a + b*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, n, p\}, x]$

### Rule 203

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

### Rule 204

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_)^2], x\_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

### Rule 266

$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_))}^{(p_)}, x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

### Rule 1615

$\text{Int}[(\text{Px}_)*((a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}*((e_.) + (f_.)*(x_))^{(p_.)}, x\_Symbol] \rightarrow \text{With}\{q = \text{Expon}[\text{Px}, x], k = \text{Coeff}[\text{Px}, x, \text{Expon}[\text{Px}, x]]\}, \text{Simp}[(k*(a + b*x)^{(m + q - 1)}*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)})/(d*f*b^{(q - 1)}*(m + n + p + q + 1)), x] + \text{Dist}[1/(d*f*b^q*(m + n + p + q + 1)), \text{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*\text{ExpandToSum}[d*f*b^q*(m + n + p + q + 1)*\text{Px} - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^{(q - 2)}*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m + q + p)))*x], x], x] /; \text{NeQ}[m + n + p + q + 1, 0]] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x] \ \&\& \ \text{PolyQ}[\text{Px}, x] \ \&\& \ \text{IntegersQ}[2*m, 2*n, 2*p]$

### Rule 6302

$\text{Int}[(a_.) + \text{ArcSch}[(c_.)*(x_)]*(b_.)*((f_.)*(x_))^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x\_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcSch}[c*x], u, x] - \text{Dist}[(b*c*x)/\text{Sqrt}[-(c^2*x^2)], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[-1 - c^2*x^2]), x], x], x]] /; \text{FreeQ}\{a, b, c, d, e, f, m, p\}, x] \ \&\& \ ((\text{IGtQ}[p, 0] \ \&\& \ !(\text{ILtQ}[(m - 1)/2, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{IGtQ}[(m + 1)/2, 0] \ \&\& \ !(\text{ILtQ}[p, 0] \ \&\& \ \text{GtQ}[m + 2*p + 3, 0])) \ || \ (\text{ILtQ}[(m + 2*p + 1)/2, 0] \ \&\& \ !\text{ILtQ}[(m - 1)/2, 0]))$

### Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} \\
&= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} \\
&= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3e^3} \\
&= \frac{bx \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{6ce^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= \frac{bx \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{6ce^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= \frac{bx \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{6ce^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= \frac{bx \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{6ce^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= \frac{bx \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{6ce^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= \frac{bx \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{6ce^2 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3}
\end{aligned}$$

**Mathematica** [A] time = 0.64, size = 311, normalized size = 1.21

$$\frac{-2ac(8d^2 + 4dex^2 - e^2x^4) + bex\sqrt{\frac{1}{c^2x^2} + 1}(d + ex^2) - 2bccsch^{-1}(cx)(8d^2 + 4dex^2 - e^2x^4)}{6ce^3\sqrt{d + ex^2}} + \frac{bx\sqrt{\frac{1}{c^2x^2} + 1}\left(16c^5d^2 + 8c^4dex^2 - 8c^3e^2x^4\right)}{6ce^3\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] (b\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(d + e\*x^2) - 2\*a\*c\*(8\*d^2 + 4\*d\*e\*x^2 - e^2\*x^4) - 2\*b\*c\*(8\*d^2 + 4\*d\*e\*x^2 - e^2\*x^4)\*ArcCsch[c\*x])/(6\*c\*e^3\*Sqrt[d + e\*x^2]) + (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-(Sqrt[c^2]\*Sqrt[c^2\*d - e]\*Sqrt[e]\*(9\*c^2\*d + e)\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e]]\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])]) + 16\*c^5\*d^(3/2)\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]]))/(6\*c^4\*e^3\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [A] time = 1.03, size = 1719, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] [1/24\*((9\*b\*c^2\*d^2 + b\*d\*e + (9\*b\*c^2\*d\*e + b\*e^2)\*x^2)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 - 4\*(2\*c^4\*e\*x^3

$$\begin{aligned}
& + (c^4d + c^2e)x \sqrt{ex^2 + d} \sqrt{e} \sqrt{(c^2x^2 + 1)/(c^2x^2)} \\
& + e^2) + 8(bc^3e^2x^4 - 4bc^3de^2x^2 - 8bc^3d^2) \sqrt{ex^2 + d} \\
& \log((cx \sqrt{(c^2x^2 + 1)/(c^2x^2)} + 1)/(cx)) + 16(bc^3de^2x^2 + bc^3d^2) \sqrt{d} \\
& \log(((c^4d^2 + 6c^2de + e^2)x^4 + 8(c^2d^2 + d^2e)x^2 + 4((c^3d + ce)x^3 + 2cdx) \sqrt{ex^2 + d} \sqrt{d} \sqrt{(c^2x^2 + 1)/(c^2x^2)} \\
& + 8d^2)/x^4) + 4(2ac^3e^2x^4 - 8ac^3de^2x^2 - 16ac^3d^2 + (bc^2e^2x^3 + bc^2de^2x) \sqrt{(c^2x^2 + 1)/(c^2x^2)}) \sqrt{ex^2 + d} \\
& \sqrt{(c^3e^4x^2 + c^3de^3)}, 1/12((9bc^2d^2 + bde + (9bc^2de + be^2)x^2) \sqrt{-e} \arctan(1/2(2c^2e^2x^3 + (c^2d + e)x) \sqrt{ex^2 + d} \sqrt{-e} \sqrt{(c^2x^2 + 1)/(c^2x^2)})/(c^2e^2x^4 + (c^2de + e^2)x^2 + d^2)) \\
& + 4(bc^3e^2x^4 - 4bc^3de^2x^2 - 8bc^3d^2) \sqrt{ex^2 + d} \log((cx \sqrt{(c^2x^2 + 1)/(c^2x^2)} + 1)/(cx)) + 8(bc^3de^2x^2 + bc^3d^2) \sqrt{d} \\
& \log(((c^4d^2 + 6c^2de + e^2)x^4 + 8(c^2d^2 + d^2e)x^2 + 4((c^3d + ce)x^3 + 2cdx) \sqrt{ex^2 + d} \sqrt{d} \sqrt{(c^2x^2 + 1)/(c^2x^2)} \\
& + 8d^2)/x^4) + 2(2ac^3e^2x^4 - 8ac^3de^2x^2 - 16ac^3d^2 + (bc^2e^2x^3 + bc^2de^2x) \sqrt{(c^2x^2 + 1)/(c^2x^2)}) \sqrt{ex^2 + d} \\
& \sqrt{(c^3e^4x^2 + c^3de^3)}, -1/24(32(bc^3de^2x^2 + bc^3d^2) \sqrt{-d} \arctan(1/2((c^3d + ce)x^3 + 2cdx) \sqrt{ex^2 + d} \sqrt{-d} \sqrt{(c^2x^2 + 1)/(c^2x^2)})/(c^2de^2x^4 + (c^2d^2 + d^2e)x^2 + d^2)) \\
& - (9bc^2d^2 + bde + (9bc^2de + be^2)x^2) \sqrt{e} \log(8c^4e^2x^4 + c^4d^2 + 6c^2de + 8(c^4de + c^2e^2)x^2 - 4(2c^4e^2x^3 + (c^4d + c^2e)x) \sqrt{ex^2 + d} \sqrt{e} \sqrt{(c^2x^2 + 1)/(c^2x^2)} + e^2) \\
& - 8(bc^3e^2x^4 - 4bc^3de^2x^2 - 8bc^3d^2) \sqrt{ex^2 + d} \log((cx \sqrt{(c^2x^2 + 1)/(c^2x^2)} + 1)/(cx)) - 4(2ac^3e^2x^4 - 8ac^3de^2x^2 - 16ac^3d^2 + (bc^2e^2x^3 + bc^2de^2x) \sqrt{(c^2x^2 + 1)/(c^2x^2)}) \sqrt{ex^2 + d} \\
& \sqrt{(c^3e^4x^2 + c^3de^3)}, -1/12(16(bc^3de^2x^2 + bc^3d^2) \sqrt{-d} \arctan(1/2((c^3d + ce)x^3 + 2cdx) \sqrt{ex^2 + d} \sqrt{-d} \sqrt{(c^2x^2 + 1)/(c^2x^2)})/(c^2de^2x^4 + (c^2d^2 + d^2e)x^2 + d^2)) \\
& - (9bc^2d^2 + bde + (9bc^2de + be^2)x^2) \sqrt{-e} \arctan(1/2(2c^2e^2x^3 + (c^2d + e)x) \sqrt{ex^2 + d} \sqrt{-e} \sqrt{(c^2x^2 + 1)/(c^2x^2)})/(c^2e^2x^4 + (c^2de + e^2)x^2 + d^2)) \\
& - 4(bc^3e^2x^4 - 4bc^3de^2x^2 - 8bc^3d^2) \sqrt{ex^2 + d} \log((cx \sqrt{(c^2x^2 + 1)/(c^2x^2)} + 1)/(cx)) - 2(2ac^3e^2x^4 - 8ac^3de^2x^2 - 16ac^3d^2 + (bc^2e^2x^3 + bc^2de^2x) \sqrt{(c^2x^2 + 1)/(c^2x^2)}) \sqrt{ex^2 + d} \\
& \sqrt{(c^3e^4x^2 + c^3de^3)}]
\end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arcsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arcsch(c\*x) + a)\*x^5/(e\*x^2 + d)^(3/2), x)

**maple** [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arcsch}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arcsch(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int(x^5\*(a+b\*arcsch(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} \left( \frac{x^4}{\sqrt{ex^2 + d}e} - \frac{4dx^2}{\sqrt{ex^2 + d}e^2} - \frac{8d^2}{\sqrt{ex^2 + d}e^3} \right) a + \frac{1}{3} b \left( \frac{(e^2x^4 - 4dex^2 - 8d^2) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex^2 + d}e^3} + 3 \int \frac{1}{3((c^2e^3x^2 + e^3) \sqrt{ex^2 + d} + (c^2e^3x^2 + e^3) \sqrt{ex^2 + d})} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/3\*(x^4/(sqrt(e\*x^2 + d)\*e) - 4\*d\*x^2/(sqrt(e\*x^2 + d)\*e^2) - 8\*d^2/(sqrt(e\*x^2 + d)\*e^3))\*a + 1/3\*b\*((e^2\*x^4 - 4\*d\*e\*x^2 - 8\*d^2)\*log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x^2 + d)\*e^3) + 3\*integrate(1/3\*(c^2\*e^2\*x^5 - 4\*c^2\*d\*e\*x^3 - 8\*c^2\*d^2\*x)/((c^2\*e^3\*x^2 + e^3)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x^2 + d) + (c^2\*e^3\*x^2 + e^3)\*sqrt(e\*x^2 + d)), x) - 3\*integrate(1/3\*((3\*e^3\*log(c) + e^3)\*c^2\*x^7 - 12\*c^2\*d^2\*e\*x^3 - 8\*c^2\*d^3\*x - 3\*(c^2\*d\*e^2 - e^3\*log(c))\*x^5 + 3\*(c^2\*e^3\*x^7 + e^3\*x^5)\*log(x))/((c^2\*e^4\*x^4 + d\*e^3 + (c^2\*d\*e^3 + e^4)\*x^2)\*sqrt(e\*x^2 + d)), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2),x)

[Out] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out



$$3.148 \quad \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=160

$$\frac{\sqrt{d+ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d+ex^2}} + \frac{bx \tan^{-1}\left(\frac{\sqrt{e} \sqrt{-c^2 x^2 - 1}}{c \sqrt{d+ex^2}}\right)}{e^{3/2} \sqrt{-c^2 x^2}} + \frac{2bc \sqrt{d} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-c^2 x^2 - 1}}\right)}{e^2 \sqrt{-c^2 x^2}}$$

[Out] b\*x\*arctan(e^(1/2)\*(-c^2\*x^2-1)^(1/2)/c/(e\*x^2+d)^(1/2))/e^(3/2)/(-c^2\*x^2)^(1/2)+2\*b\*c\*x\*arctan((e\*x^2+d)^(1/2)/d^(1/2)/(-c^2\*x^2-1)^(1/2))\*d^(1/2)/e^2/(-c^2\*x^2)^(1/2)+d\*(a+b\*arccsch(c\*x))/e^2/(e\*x^2+d)^(1/2)+(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2)/e^2

**Rubi [A]** time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 6302, 12, 573, 157, 63, 217, 203, 93, 204}

$$\frac{\sqrt{d+ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{d (a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d+ex^2}} + \frac{bx \tan^{-1}\left(\frac{\sqrt{e} \sqrt{-c^2 x^2 - 1}}{c \sqrt{d+ex^2}}\right)}{e^{3/2} \sqrt{-c^2 x^2}} + \frac{2bc \sqrt{d} x \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-c^2 x^2 - 1}}\right)}{e^2 \sqrt{-c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] (d\*(a + b\*ArcCsch[c\*x]))/(e^2\*Sqrt[d + e\*x^2]) + (Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]))/e^2 + (b\*x\*ArcTan[(Sqrt[e]\*Sqrt[-1 - c^2\*x^2])/(c\*Sqrt[d + e\*x^2])])/(e^(3/2)\*Sqrt[-(c^2\*x^2)]) + (2\*b\*c\*Sqrt[d]\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 - c^2\*x^2])])/(e^2\*Sqrt[-(c^2\*x^2)])

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 93

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*(e + f*x)^p/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} - \frac{(bcx) \int \frac{2d+ex^2}{e^2 x \sqrt{-1-c^2 x^2} \sqrt{d+ex^2}} dx}{\sqrt{-c^2 x^2}} \\
&= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} - \frac{(bcx) \int \frac{2d+ex^2}{x \sqrt{-1-c^2 x^2} \sqrt{d+ex^2}} dx}{e^2 \sqrt{-c^2 x^2}} \\
&= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} - \frac{(bcx) \operatorname{Subst} \left( \int \frac{2d+ex}{x \sqrt{-1-c^2 x} \sqrt{d+ex}} dx \right)}{2e^2 \sqrt{-c^2 x^2}} \\
&= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} - \frac{(bcdx) \operatorname{Subst} \left( \int \frac{1}{x \sqrt{-1-c^2 x}} dx \right)}{e^2 \sqrt{-c^2 x^2}} \\
&= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} - \frac{(2bcdx) \operatorname{Subst} \left( \int \frac{1}{-d-x^2} dx \right)}{e^2 \sqrt{-c^2 x^2}} \\
&= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{2bc\sqrt{d} x \tan^{-1} \left( \frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{-1-c^2 x}} \right)}{e^2 \sqrt{-c^2 x^2}} \\
&= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^2} + \frac{bx \tan^{-1} \left( \frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+ex^2}} \right)}{e^{3/2} \sqrt{-c^2 x^2}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.37, size = 233, normalized size = 1.46

$$\frac{(2d + ex^2)(a + b \operatorname{csch}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} - \frac{bx \sqrt{\frac{1}{c^2 x^2} + 1} \left( 2c^3 \sqrt{d} \sqrt{-d - ex^2} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{c^2 x^2 + 1}}{\sqrt{-d - ex^2}} \right) - \sqrt{c^2} \sqrt{e} \sqrt{c^2 d - e} \sqrt{\frac{c^2(d + ex^2)}{c^2 d - e}} \right)}{c^2 e^2 \sqrt{c^2 x^2 + 1} \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] ((2\*d + e\*x^2)\*(a + b\*ArcCsch[c\*x]))/(e^2\*Sqrt[d + e\*x^2]) - (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-(Sqrt[c^2]\*Sqrt[c^2\*d - e]\*Sqrt[e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e])\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])]) + 2\*c^3\*Sqrt[d]\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]]))/(c^2\*e^2\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [B]** time = 0.84, size = 1274, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] [1/4\*((b\*e\*x^2 + b\*d)\*sqrt(e)\*log(8\*c^4\*e^2\*x^4 + c^4\*d^2 + 6\*c^2\*d\*e + 8\*(c^4\*d\*e + c^2\*e^2)\*x^2 + 4\*(2\*c^4\*e\*x^3 + (c^4\*d + c^2\*e)\*x)\*sqrt(e\*x^2 + d)\*sqrt(e)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + e^2) + 4\*(b\*c\*e\*x^2 + 2\*b\*c\*d)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*(b\*c\*e\*x^2 + b\*c\*d)\*sqrt(d)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 - 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2

```

*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d
))/(c*e^3*x^2 + c*d*e^2), -1/2*((b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*
e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2
)))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 2*(b*c*e*x^2 + 2*b*c*d)*sqr
t(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c*e*x^
2 + b*c*d)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)
*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d))/
(c*e^3*x^2 + c*d*e^2), 1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(-d)*arctan(1/2*((c^3
*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x
^2)))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + (b*e*x^2 + b*d)*sqrt(e)*l
og(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c
^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + e^2) + 4*(b*c*e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d)
)/(c*e^3*x^2 + c*d*e^2), 1/2*(2*(b*c*e*x^2 + b*c*d)*sqrt(-d)*arctan(1/2*((c
^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2
*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (b*e*x^2 + b*d)*sqrt(-e
)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 2*(b*c*
e*x^2 + 2*b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1
)/(c*x)) + 2*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2)]

```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

**maple [F]** time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$a \left( \frac{x^2}{\sqrt{ex^2 + de}} + \frac{2d}{\sqrt{ex^2 + de^2}} \right) + b \left( \frac{(ex^2 + 2d) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex^2 + de^2}} \right) + \int \frac{c^2ex^3 + 2c^2dx}{(c^2e^2x^2 + e^2)\sqrt{c^2x^2 + 1}\sqrt{ex^2 + d} + (c^2e^2x^2 + e^2)\sqrt{ex^2 + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] a*(x^2/(sqrt(e*x^2 + d)*e) + 2*d/(sqrt(e*x^2 + d)*e^2)) + b*((e*x^2 + 2*d)*
log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x^2 + d)*e^2) + integrate((c^2*e*x^3 + 2
*c^2*d*x)/((c^2*e^2*x^2 + e^2)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (c^2*e^2
*x^2 + e^2)*sqrt(e*x^2 + d)), x) - integrate((e^2*log(c) + e^2)*c^2*x^5 +
```

$2*c^2*d^2*x + (3*c^2*d*e + e^2*\log(c))*x^3 + (c^2*e^2*x^5 + e^2*x^3)*\log(x)$   
 $) / ((c^2*e^3*x^4 + d*e^2 + (c^2*d*e^2 + e^3)*x^2)*\sqrt{e*x^2 + d}), x)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2), x)

[Out] int((x^3\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(3/2), x)

[Out] Timed out

$$3.149 \quad \int \frac{x(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=82

$$\frac{a + b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}$$

[Out]  $-b*c*x*\arctan((e*x^2+d)^{(1/2)}/d^{(1/2)}/(-c^2*x^2-1)^{(1/2)})/e/d^{(1/2)}/(-c^2*x^2)^{(1/2)}+(-a-b*\operatorname{arccsch}(c*x))/e/(e*x^2+d)^{(1/2)}$

**Rubi [A]** time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {6300, 446, 93, 204}

$$\frac{a + b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In] `Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]`

[Out]  $-\left(\frac{a + b\operatorname{ArcCsch}[c*x]}{e\sqrt{d + e*x^2}}\right) - \frac{b*c*x*\operatorname{ArcTan}[\sqrt{d + e*x^2}]/(\sqrt{d}*\sqrt{-1 - c^2*x^2})}{\sqrt{d}*e*\sqrt{-c^2*x^2}}$

#### Rule 93

`Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]`

#### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

#### Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

#### Rule 6300

`Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCsch[c*x]))/(2*e*(p + 1)), x] - Dist[(b*c*x)/(2*e*(p + 1)*Sqrt[-(c^2*x^2)]), Int[(d + e*x^2)^(p + 1)/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]`

#### Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{(bcx) \int \frac{1}{x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}} dx}{e\sqrt{-c^2x^2}} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex^2}} dx, x, x^2\right)}{2e\sqrt{-c^2x^2}} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{(bcx) \operatorname{Subst}\left(\int \frac{1}{-d-x^2} dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 122, normalized size = 1.49

$$\frac{bcx\sqrt{\frac{1}{c^2x^2} + 1}\sqrt{-d - ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2+1}}{\sqrt{-d-ex^2}}\right)}{\sqrt{d}e\sqrt{c^2x^2+1}\sqrt{d+ex^2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] -((a + b\*ArcCsch[c\*x])/(e\*Sqrt[d + e\*x^2])) + (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]])/(Sqrt[d]\*e\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [B]** time = 0.69, size = 368, normalized size = 4.49

$$\frac{4\sqrt{ex^2+d}bd\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}+1}}{cx}\right) + 4\sqrt{ex^2+d}ad - (bex^2+bd)\sqrt{d}\log\left(\frac{(c^4d^2+6c^2de+e^2)x^4+8(c^2d^2+de)x^2+4((c^3d+ce^2)x^4)}{x^4}\right)}{4(de^2x^2+d^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] [-1/4\*(4\*sqrt(e\*x^2 + d)\*b\*d\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 4\*sqrt(e\*x^2 + d)\*a\*d - (b\*e\*x^2 + b\*d)\*sqrt(d)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 + 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4))/(d\*e^2\*x^2 + d^2\*e), -1/2\*(2\*sqrt(e\*x^2 + d)\*b\*d\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*sqrt(e\*x^2 + d)\*a\*d + (b\*e\*x^2 + b\*d)\*sqrt(-d)\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2))/(d\*e^2\*x^2 + d^2\*e)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x/(e\*x^2 + d)^(3/2), x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \left[ c^2 \int \frac{x}{(c^2 ex^2 + e) \sqrt{c^2 x^2 + 1} \sqrt{ex^2 + d} + (c^2 ex^2 + e) \sqrt{ex^2 + d}} dx + \frac{\log(\sqrt{c^2 x^2 + 1} + 1)}{\sqrt{ex^2 + d} e} + \int \frac{(e \log(c) - e) c^2 x^3}{(c^2 e^2 x^4} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -(c^2\*integrate(x/((c^2\*e\*x^2 + e)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x^2 + d) + (c^2\*e\*x^2 + e)\*sqrt(e\*x^2 + d)), x) + log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x^2 + d)\*e) + integrate(((e\*log(c) - e)\*c^2\*x^3 - (c^2\*d - e\*log(c))\*x + (c^2\*e\*x^3 + e\*x)\*log(x))/((c^2\*e^2\*x^4 + (c^2\*d\*e + e^2)\*x^2 + d\*e)\*sqrt(e\*x^2 + d)), x))\*b - a/(sqrt(e\*x^2 + d)\*e)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2),x)

[Out] int((x\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{acsch}(cx))}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Integral(x\*(a + b\*acsch(c\*x))/(d + e\*x\*\*2)\*\*(3/2), x)



$$3.150 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=26

$$\operatorname{Int} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(3/2), x)

**Rubi [A]** time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

[Out] Defer[Int][(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx = \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 32.96, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^(3/2)), x]

**fricas [A]** time = 0.68, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{e^2x^5 + 2dex^3 + d^2x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e^2\*x^5 + 2\*d\*e\*x^3 + d^2\*x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{3/2} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)^(3/2)\*x), x)

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x (e x^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(3/2),x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{\operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{3}{2}}} - \frac{1}{\sqrt{ex^2 + dd}} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -a\*(arcsinh(d/(sqrt(d\*e)\*abs(x)))/d^(3/2) - 1/(sqrt(e\*x^2 + d)\*d)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/((e\*x^2 + d)^(3/2)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x (e x^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)^(3/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)^(3/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out

$$3.151 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=26

$$\operatorname{Int} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(3/2), x)

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

[Out] Defer[Int][(a + b\*ArcCsch[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx = \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 38.59, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x^3\*(d + e\*x^2)^(3/2)), x]

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{e^2 x^7 + 2 d e x^5 + d^2 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e^2\*x^7 + 2\*d\*e\*x^5 + d^2\*x^3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)^(3/2)\*x^3), x)

**maple** [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^3 (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(3/2),x)

[Out] int((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a \left( \frac{3e \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{5}{2}}} - \frac{3e}{\sqrt{ex^2 + d} d^2} - \frac{1}{\sqrt{ex^2 + d} dx^2} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/2\*a\*(3\*e\*arsinh(d/(sqrt(d\*e)\*abs(x)))/d^(5/2) - 3\*e/(sqrt(e\*x^2 + d)\*d^2) - 1/(sqrt(e\*x^2 + d)\*d\*x^2)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/((e\*x^2 + d)^(3/2)\*x^3), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^3 (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^3\*(d + e\*x^2)^(3/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^3\*(d + e\*x^2)^(3/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*3/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out

$$3.152 \quad \int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=26

$$\operatorname{Int} \left( \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x)

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int][(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 8.80, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate[(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

**fricas [A]** time = 0.61, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(bx^4 \operatorname{arcsch}(cx) + ax^4) \sqrt{ex^2 + d}}{e^2 x^4 + 2 dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] integral((b\*x^4\*arccsch(c\*x) + a\*x^4)\*sqrt(e\*x^2 + d)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^4}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^4/(e\*x^2 + d)^(3/2), x)

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \left( \frac{x^3}{\sqrt{ex^2 + de}} + \frac{3dx}{\sqrt{ex^2 + de}e} - \frac{3d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{5}{2}}} \right) a + b \int \frac{x^4 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] 1/2\*(x^3/(sqrt(e\*x^2 + d)\*e) + 3\*d\*x/(sqrt(e\*x^2 + d)\*e^2) - 3\*d\*arcsinh(e\*x/sqrt(d\*e))/e^(5/2))\*a + b\*integrate(x^4\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(e\*x^2 + d)^(3/2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2),x)

[Out] int((x^4\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out

$$3.153 \quad \int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=26

$$\operatorname{Int} \left( \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x)

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int][(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 5.02, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

**fricas [A]** time = 0.50, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(bx^2 \operatorname{arcsch}(cx) + ax^2) \sqrt{ex^2 + d}}{e^2 x^4 + 2 dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsch(c\*x) + a\*x^2)\*sqrt(e\*x^2 + d)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2/(e\*x^2 + d)^(3/2), x)

**maple** [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{x}{\sqrt{ex^2 + de}} - \frac{\operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{3}{2}}} \right) + b \int \frac{x^2 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -a\*(x/(sqrt(e\*x^2 + d)\*e) - arcsinh(e\*x/sqrt(d\*e))/e^(3/2)) + b\*integrate(x^2\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e\*x^2 + d)^(3/2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2),x)

[Out] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acsch}(cx))}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Integral(x\*\*2\*(a + b\*acsch(c\*x))/(d + e\*x\*\*2)\*\*(3/2), x)



$$3.154 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=111

$$\frac{x(a + b \operatorname{csch}^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{bx\sqrt{d + ex^2} F\left(\tan^{-1}(cx) \mid 1 - \frac{e}{c^2 d}\right)}{d^2 \sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1} \sqrt{\frac{d + ex^2}{d(c^2 x^2 + 1)}}$$

[Out] x\*(a+b\*arccsch(c\*x))/d/(e\*x^2+d)^(1/2)-b\*x\*(1/(c^2\*x^2+1))^(1/2)\*(c^2\*x^2+1)^(1/2)\*EllipticF(c\*x/(c^2\*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))\*(e\*x^2+d)^(1/2)/d^2/(-c^2\*x^2)^(1/2)/(-c^2\*x^2-1)^(1/2)/((e\*x^2+d)/d/(c^2\*x^2+1))^(1/2)

**Rubi [A]** time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {191, 6292, 12, 418}

$$\frac{x(a + b \operatorname{csch}^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{bx\sqrt{d + ex^2} F\left(\tan^{-1}(cx) \mid 1 - \frac{e}{c^2 d}\right)}{d^2 \sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1} \sqrt{\frac{d + ex^2}{d(c^2 x^2 + 1)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*ArcCsch[c\*x])/(d + e\*x^2)^(3/2), x]

[Out] (x\*(a + b\*ArcCsch[c\*x]))/(d\*Sqrt[d + e\*x^2]) - (b\*x\*Sqrt[d + e\*x^2]\*EllipticF[ArcTan[c\*x], 1 - e/(c^2\*d)])/(d^2\*Sqrt[-(c^2\*x^2)]\*Sqrt[-1 - c^2\*x^2]\*Sqrt[(d + e\*x^2)/(d\*(1 + c^2\*x^2))])

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 418

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] := Simp[(Sqrt[a + b\*x^2]\*EllipticF[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)])/(a\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 6292

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex^2)^{3/2}} dx &= \frac{x(a + b \operatorname{csch}^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{1}{d\sqrt{-1-c^2x^2}\sqrt{d+ex^2}} dx}{\sqrt{-c^2x^2}} \\
&= \frac{x(a + b \operatorname{csch}^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}} dx}{d\sqrt{-c^2x^2}} \\
&= \frac{x(a + b \operatorname{csch}^{-1}(cx))}{d\sqrt{d + ex^2}} - \frac{bx\sqrt{d + ex^2} F(\tan^{-1}(cx) | 1 - \frac{e}{c^2d})}{d^2\sqrt{-c^2x^2}\sqrt{-1 - c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}$$

**Mathematica** [A] time = 0.20, size = 113, normalized size = 1.02

$$\frac{x(a + b \operatorname{csch}^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{bcx\sqrt{\frac{1}{c^2x^2} + 1}\sqrt{\frac{ex^2}{d} + 1} F\left(\sin^{-1}\left(\sqrt{-c^2}x\right) \middle| \frac{e}{c^2d}\right)}{\sqrt{-c^2}d\sqrt{c^2x^2 + 1}\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x^2)^(3/2), x]

[Out] (x\*(a + b\*ArcCsch[c\*x]))/(d\*Sqrt[d + e\*x^2]) + (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*EllipticF[ArcSin[Sqrt[-c^2]\*x], e/(c^2\*d)]/(Sqrt[-c^2]\*d\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas** [F] time = 0.99, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x^2 + d)^(3/2), x)

**maple** [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x)

[Out]  $\text{int}((a+b*\text{arccsch}(c*x))/(e*x^2+d)^{(3/2)}, x)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}}} dx + \frac{ax}{\sqrt{ex^2 + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\text{arccsch}(c*x))/(e*x^2+d)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out]  $b*\text{integrate}(\log(\text{sqrt}(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^{(3/2)}, x) + a*x/(\text{sqrt}(e*x^2 + d)*d)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a + b*\text{asinh}(1/(c*x)))/(d + e*x^2)^{(3/2)}, x)$

[Out]  $\text{int}((a + b*\text{asinh}(1/(c*x)))/(d + e*x^2)^{(3/2)}, x)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b*\text{acsch}(c*x))/(e*x**2+d)**(3/2), x)$

[Out]  $\text{Integral}((a + b*\text{acsch}(c*x))/(d + e*x**2)**(3/2), x)$

$$3.155 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=321

$$\frac{2ex(a + b \operatorname{csch}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{dx \sqrt{d + ex^2}} + \frac{2bex \sqrt{d + ex^2} F(\tan^{-1}(cx) | 1 - \frac{e}{c^2 d})}{d^3 \sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1} \sqrt{\frac{d + ex^2}{d(c^2 x^2 + 1)}}} + \frac{bc \sqrt{-c^2 x^2 - 1} \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2}} - \frac{bc^2 x \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2}}$$

[Out]  $(-a - b \operatorname{arccsch}(c x)) / d x / (e x^2 + d)^{(1/2)} - 2 e x x (a + b \operatorname{arccsch}(c x)) / d^2 / (e x^2 + d)^{(1/2)} + b c^3 x^2 (e x^2 + d)^{(1/2)} / d^2 / (-c^2 x^2)^{(1/2)} / (-c^2 x^2 - 1)^{(1/2)} + b c (-c^2 x^2 - 1)^{(1/2)} (e x^2 + d)^{(1/2)} / d^2 / (-c^2 x^2)^{(1/2)} - b c^2 x (1 / (c^2 x^2 + 1))^{(1/2)} (c^2 x^2 + 1)^{(1/2)} \operatorname{EllipticE}(c x / (c^2 x^2 + 1)^{(1/2)}, (1 - e / c^2 d)^{(1/2)}) (e x^2 + d)^{(1/2)} / d^2 / (-c^2 x^2)^{(1/2)} / (-c^2 x^2 - 1)^{(1/2)} / ((e x^2 + d) / d / (c^2 x^2 + 1))^{(1/2)} + 2 b e x x (1 / (c^2 x^2 + 1))^{(1/2)} (c^2 x^2 + 1)^{(1/2)} \operatorname{EllipticF}(c x / (c^2 x^2 + 1)^{(1/2)}, (1 - e / c^2 d)^{(1/2)}) (e x^2 + d)^{(1/2)} / d^3 / (-c^2 x^2)^{(1/2)} / (-c^2 x^2 - 1)^{(1/2)} / ((e x^2 + d) / d / (c^2 x^2 + 1))^{(1/2)}$

**Rubi [A]** time = 0.32, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {271, 191, 6302, 12, 583, 531, 418, 492, 411}

$$\frac{2ex(a + b \operatorname{csch}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{dx \sqrt{d + ex^2}} + \frac{bc^3 x^2 \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1}} + \frac{bc \sqrt{-c^2 x^2 - 1} \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2}} + \frac{2bex \sqrt{d + ex^2} F(\tan^{-1}(cx) | 1 - \frac{e}{c^2 d})}{d^3 \sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b \operatorname{ArcCsch}[c x]) / (x^2 (d + e x^2)^{(3/2)}) , x]$

[Out]  $(b c^3 x^2 \operatorname{Sqrt}[d + e x^2]) / (d^2 \operatorname{Sqrt}[-(c^2 x^2)] \operatorname{Sqrt}[-1 - c^2 x^2]) + (b c \operatorname{Sqrt}[-1 - c^2 x^2] \operatorname{Sqrt}[d + e x^2]) / (d^2 \operatorname{Sqrt}[-(c^2 x^2)]) - (a + b \operatorname{ArcCsch}[c x]) / (d x \operatorname{Sqrt}[d + e x^2]) - (2 e x x (a + b \operatorname{ArcCsch}[c x])) / (d^2 \operatorname{Sqrt}[d + e x^2]) - (b c^2 x \operatorname{Sqrt}[d + e x^2] \operatorname{EllipticE}[\operatorname{ArcTan}[c x], 1 - e / (c^2 d)]) / (d^2 \operatorname{Sqrt}[-(c^2 x^2)] \operatorname{Sqrt}[-1 - c^2 x^2] \operatorname{Sqrt}[(d + e x^2) / (d (1 + c^2 x^2))]) + (2 b e x x \operatorname{Sqrt}[d + e x^2] \operatorname{EllipticF}[\operatorname{ArcTan}[c x], 1 - e / (c^2 d)]) / (d^3 \operatorname{Sqrt}[-(c^2 x^2)] \operatorname{Sqrt}[-1 - c^2 x^2] \operatorname{Sqrt}[(d + e x^2) / (d (1 + c^2 x^2))])$

#### Rule 12

$\operatorname{Int}[(a\_)(u\_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match} Q[u, (b\_)(v\_)] /;$   $\operatorname{FreeQ}[b, x]$

#### Rule 191

$\operatorname{Int}[(a\_)(b\_)(x\_)^{(n\_)]^{(p\_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(x (a + b x^n)^{(p + 1)}) / a, x] /;$   $\operatorname{FreeQ}\{a, b, n, p\}, x \ \&\& \ \operatorname{EqQ}[1/n + p + 1, 0]$

#### Rule 271

$\operatorname{Int}[(x\_)^{(m\_)]((a\_)(b\_)(x\_)^{(n\_)]^{(p\_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(x^{(m + 1)} (a + b x^n)^{(p + 1)}) / (a^{(m + 1)}), x] - \operatorname{Dist}[(b^{(m + n)(p + 1)} + 1) / (a^{(m + 1)}), \operatorname{Int}[x^{(m + n)} (a + b x^n)^p, x], x] /;$   $\operatorname{FreeQ}\{a, b, m, n, p\}, x \ \&\& \ \operatorname{ILtQ}[\operatorname{Simplify}[(m + 1) / n + p + 1], 0] \ \&\& \ \operatorname{NeQ}[m, -1]$

#### Rule 411

$\operatorname{Int}[\operatorname{Sqrt}[(a\_)(b\_)(x\_)^2] / ((c\_)(d\_)(x\_)^2)^{(3/2)}, x\_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Sqrt}[a + b x^2] \operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Rt}[d/c, 2] x], 1 - (b c) / (a d)]) / (c \operatorname{Rt}[$

$d/c, 2] * \text{Sqrt}[c + d*x^2] * \text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))], x] /;$  FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

### Rule 418

$\text{Int}[1/(\text{Sqrt}[a_] + (b_)*(x_)^2) * \text{Sqrt}[c_] + (d_)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2] * \text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)]) / (a * \text{Rt}[d/c, 2] * \text{Sqrt}[c + d*x^2] * \text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /;$  FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

### Rule 492

$\text{Int}[(x_)^2/(\text{Sqrt}[a_] + (b_)*(x_)^2) * \text{Sqrt}[c_] + (d_)*(x_)^2], x\_Symbol] \rightarrow \text{Simp}[(x * \text{Sqrt}[a + b*x^2]) / (b * \text{Sqrt}[c + d*x^2]), x] - \text{Dist}[c/b, \text{Int}[\text{Sqrt}[a + b*x^2] / (c + d*x^2)^{3/2}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

### Rule 531

$\text{Int}[(a_) + (b_)*(x_)^{(n_)}]^{(p_)} * ((c_) + (d_)*(x_)^{(n_)})^{(q_)} * ((e_) + (f_)*(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Dist}[e, \text{Int}[(a + b*x^n)^p * (c + d*x^n)^q, x], x] + \text{Dist}[f, \text{Int}[x^n * (a + b*x^n)^p * (c + d*x^n)^q, x], x] /;$  FreeQ[{a, b, c, d, e, f, n, p, q}, x]

### Rule 583

$\text{Int}[(g_)*(x_)^{(m_)} * ((a_) + (b_)*(x_)^{(n_)})^{(p_)} * ((c_) + (d_)*(x_)^{(n_)})^{(q_)} * ((e_) + (f_)*(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(e*(g*x)^{m+1} * (a + b*x^n)^{p+1} * (c + d*x^n)^{q+1}) / (a*c*g^{m+1}), x] + \text{Dist}[1/(a*c*g^n * (m+1)), \text{Int}[(g*x)^{m+n} * (a + b*x^n)^p * (c + d*x^n)^q * \text{Simp}[a*f*c*(m+1) - e*(b*c + a*d)*(m+n+1) - e*n*(b*c*p + a*d*q) - b*e*d*(m+n*(p+q+2) + 1)*x^n, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

### Rule 6302

$\text{Int}[(a_) + \text{ArcCsch}[c_*(x_)] * (b_)] * ((f_)*(x_)^{(m_)} * ((d_) + (e_)*(x_)^2)^{(p_)}), x\_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(f*x)^m * (d + e*x^2)^p, x]\}, \text{Dist}[a + b * \text{ArcCsch}[c*x], u, x] - \text{Dist}[(b*c*x) / \text{Sqrt}[-(c^2*x^2)], \text{Int}[\text{SimplifyIntegrand}[u / (x * \text{Sqrt}[-1 - c^2*x^2]), x], x], x]] /;$  FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m-1)/2, 0] && GtQ[m+2\*p+3, 0])) || (IGtQ[(m+1)/2, 0] && !(ILtQ[p, 0] && GtQ[m+2\*p+3, 0])) || (ILtQ[(m+2\*p+1)/2, 0] && !ILtQ[(m-1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}} dx &= -\frac{a + b \operatorname{csch}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{csch}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-d-2ex^2}{d^2 x^2 \sqrt{-1-c^2 x^2} \sqrt{d+ex^2}} dx}{\sqrt{-c^2 x^2}} \\
&= -\frac{a + b \operatorname{csch}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{csch}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-d-2ex^2}{x^2 \sqrt{-1-c^2 x^2} \sqrt{d+ex^2}} dx}{d^2 \sqrt{-c^2 x^2}} \\
&= \frac{bc \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{csch}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-2de-c^2}{\sqrt{-1-c^2 x^2}} dx}{d^3 \sqrt{-c^2 x^2}} \\
&= \frac{bc \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{csch}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} + \frac{(2bcex) \int \frac{1}{\sqrt{-1-c^2 x^2}} dx}{d^2 \sqrt{-c^2 x^2}} \\
&= \frac{bc^3 x^2 \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2} \sqrt{-1 - c^2 x^2}} + \frac{bc \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{csch}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} \\
&= \frac{bc^3 x^2 \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2} \sqrt{-1 - c^2 x^2}} + \frac{bc \sqrt{-1 - c^2 x^2} \sqrt{d + ex^2}}{d^2 \sqrt{-c^2 x^2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{csch}^{-1}(cx))}{d^2 \sqrt{d + ex^2}}
\end{aligned}$$

**Mathematica [C]** time = 0.46, size = 201, normalized size = 0.63

$$\frac{-a(d + 2ex^2) + bcx \sqrt{\frac{1}{c^2 x^2} + 1} (d + ex^2) - b \operatorname{csch}^{-1}(cx) (d + 2ex^2)}{d^2 x \sqrt{d + ex^2}} + \frac{ibcx \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{\frac{ex^2}{d} + 1} \left( (2e - c^2 d) F\left(i \operatorname{sinh}^{-1}\left(\frac{\sqrt{d + ex^2}}{\sqrt{d}}\right)\right) \right)}{\sqrt{c^2} d^2 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^2\*(d + e\*x^2)^(3/2)),x]

[Out] (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(d + e\*x^2) - a\*(d + 2\*e\*x^2) - b\*(d + 2\*e\*x^2)\*ArcCsch[c\*x])/(d^2\*x\*Sqrt[d + e\*x^2]) + (I\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*(c^2\*d\*EllipticE[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)] + (-c^2\*d + 2\*e)\*EllipticF[I\*ArcSinh[Sqrt[c^2]\*x], e/(c^2\*d)))/(Sqrt[c^2]\*d^2\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [F]** time = 0.61, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{e^2 x^6 + 2 dex^4 + d^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e^2\*x^6 + 2\*d\*e\*x^4 + d^2\*x^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)^(3/2)\*x^2), x)

**maple** [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(3/2),x)

[Out] int((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(3/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-a \left( \frac{2ex}{\sqrt{ex^2 + d}d^2} + \frac{1}{\sqrt{ex^2 + d}dx} \right) - b \left( \frac{(2ex^2 + d) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{ex^2 + d}d^2x} \right) + \int \frac{2c^2ex^2 + c^2d}{(c^2d^2x^2 + d^2)\sqrt{c^2x^2 + 1}\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^2/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -a\*(2\*e\*x/(sqrt(e\*x^2 + d)\*d^2) + 1/(sqrt(e\*x^2 + d)\*d\*x)) - b\*((2\*e\*x^2 + d)\*log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(e\*x^2 + d)\*d^2\*x) + integrate((2\*c^2\*e\*x^2 + c^2\*d)/((c^2\*d^2\*x^2 + d^2)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x^2 + d) + (c^2\*d^2\*x^2 + d^2)\*sqrt(e\*x^2 + d)), x) + integrate(-(2\*c^2\*e^2\*x^6 + 3\*c^2\*d\*e\*x^4 - (d^2\*log(c) - d^2)\*c^2\*x^2 - d^2\*log(c) - (c^2\*d^2\*x^2 + d^2)\*log(x))/((c^2\*d^2\*e\*x^6 + d^3\*x^2 + (c^2\*d^3 + d^2\*e)\*x^4)\*sqrt(e\*x^2 + d)), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2 (ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x^2)^(3/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^2\*(d + e\*x^2)^(3/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*2/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out

$$3.156 \quad \int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=251

$$-\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} + \frac{bx \tan^{-1}\left(\frac{\sqrt{e} \sqrt{-c^2 x^2 - 1}}{c \sqrt{d + ex^2}}\right)}{e^{5/2} \sqrt{-c^2 x^2}} + \frac{8bc \sqrt{d} x \tan^{-1}\left(\frac{\sqrt{e} \sqrt{-c^2 x^2 - 1}}{c \sqrt{d + ex^2}}\right)}{3e^3}$$

[Out]  $-1/3*d^2*(a+b*\operatorname{arccsch}(c*x))/e^3/(e*x^2+d)^{(3/2)}+b*x*\operatorname{arctan}(e^{(1/2)}*(-c^2*x^2-1)^{(1/2)}/c/(e*x^2+d)^{(1/2)})/e^{(5/2)}/(-c^2*x^2)^{(1/2)}+8/3*b*c*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)}/(-c^2*x^2-1)^{(1/2)})*d^{(1/2)}/e^3/(-c^2*x^2)^{(1/2)}+2*d*(a+b*\operatorname{arccsch}(c*x))/e^3/(e*x^2+d)^{(1/2)}+1/3*b*c*d*x*(-c^2*x^2-1)^{(1/2)}/(c^2*d-e)/e^2/(-c^2*x^2)^{(1/2)}/(e*x^2+d)^{(1/2)}+(a+b*\operatorname{arccsch}(c*x))*(e*x^2+d)^{(1/2)}/e^3$

**Rubi [A]** time = 1.23, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$ , Rules used = {266, 43, 6302, 12, 1614, 157, 63, 217, 203, 93, 204}

$$-\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} + \frac{bcdx \sqrt{-c^2 x^2 - 1}}{3e^2 \sqrt{-c^2 x^2} (c^2 d - e) \sqrt{d + ex^2}} + \frac{8bc \sqrt{d} x \tan^{-1}\left(\frac{\sqrt{e} \sqrt{-c^2 x^2 - 1}}{c \sqrt{d + ex^2}}\right)}{3e^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^5*(a + b*\operatorname{ArcCsch}[c*x]))/(d + e*x^2)^{(5/2)}, x]$

[Out]  $(b*c*d*x*\operatorname{Sqrt}[-1 - c^2*x^2])/(3*(c^2*d - e)*e^2*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[d + e*x^2]) - (d^2*(a + b*\operatorname{ArcCsch}[c*x]))/(3*e^3*(d + e*x^2)^{(3/2)}) + (2*d*(a + b*\operatorname{ArcCsch}[c*x]))/(e^3*\operatorname{Sqrt}[d + e*x^2]) + (\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcCsch}[c*x]))/e^3 + (b*x*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[-1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(e^{(5/2)}*\operatorname{Sqrt}[-(c^2*x^2)]) + (8*b*c*\operatorname{Sqrt}[d]*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 - c^2*x^2])])/(3*e^3*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{Match}Q[u, (b_*)*(v_)] /;$   $\operatorname{FreeQ}[b, x]$

#### Rule 43

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$   $\operatorname{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ (\ !\operatorname{IntegerQ}[n] \ || \ (\operatorname{EqQ}[c, 0] \ \&\& \ \operatorname{LeQ}[7*m + 4*n + 4, 0]) \ || \ \operatorname{LtQ}[9*m + 5*(n + 1), 0] \ || \ \operatorname{GtQ}[m + n + 2, 0])$

#### Rule 63

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /;$   $\operatorname{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{LtQ}[-1, m, 0] \ \&\& \ \operatorname{LeQ}[-1, n, 0] \ \&\& \ \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \ \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 93

$\operatorname{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)})/(e_*) + (f_*)*(x_*)], x\_Symbol] \rightarrow \operatorname{With}[\{q = \operatorname{Denominator}[m]\}, \operatorname{Dist}[q, \operatorname{Subst}[\operatorname{Int}[x^{(q*(m + 1) - 1)}*(c - (a*d)/q + (d*x^q)/q)^n, x], x, (a + b*x)^{(1/q)}], x] /;$



$-1)/(b*e - a*f - (d*e - c*f)*x^q)$ , x], x,  $(a + b*x)^{1/q}/(c + d*x)^{1/q}$ ], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_)^(n\_))\*((e\_.) + (f\_.)\*(x\_)^(p\_))\*((g\_.) + (h\_.)\*(x\_))) / ((a\_.) + (b\_.)\*(x\_)), x\_Symbol] :> Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] + Dist[(b\*g - a\*h)/b, Int[((c + d\*x)^n\*(e + f\*x)^p)/(a + b\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 1614

Int[(Px\_)\*((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_))\*((e\_.) + (f\_.)\*(x\_)^(p\_.)), x\_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b\*x, x], R = PolynomialRemainder[Px, a + b\*x, x]}, Simp[(b\*R\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*ExpandToSum[(m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)\*Qx + a\*d\*f\*R\*(m + 1) - b\*R\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*R\*(m + n + p + 3)\*x, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 6302

Int[((a\_.) + ArcSch[(c\_.)\*(x\_)])\*(b\_.)\*((f\_.)\*(x\_)^(m\_.))\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{b c d x \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} \\
&= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{b c d x \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} \\
&= -\frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} - \frac{b c d x \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} \\
&= \frac{b c d x \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= \frac{b c d x \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= \frac{b c d x \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3} \\
&= \frac{b c d x \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{csch}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{csch}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx))}{e^3}
\end{aligned}$$

**Mathematica [A]** time = 0.56, size = 327, normalized size = 1.30

$$\frac{a(c^2 d - e)(8d^2 + 12dex^2 + 3e^2 x^4) + b(c^2 d - e) \operatorname{csch}^{-1}(cx)(8d^2 + 12dex^2 + 3e^2 x^4) + bcdex \sqrt{\frac{1}{c^2 x^2} + 1} (d + ex^2)}{3e^3 (c^2 d - e) (d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*d\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(d + e\*x^2) + a\*(c^2\*d - e)\*(8\*d^2 + 12\*d\*e\*x^2 + 3\*e^2\*x^4) + b\*(c^2\*d - e)\*(8\*d^2 + 12\*d\*e\*x^2 + 3\*e^2\*x^4)\*ArcCsch[c\*x])/(3\*(c^2\*d - e)\*e^3\*(d + e\*x^2)^(3/2)) - (b\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(-3\*Sqrt[c^2]\*Sqrt[c^2\*d - e]\*Sqrt[e]\*Sqrt[(c^2\*(d + e\*x^2))/(c^2\*d - e])\*ArcSinh[(c\*Sqrt[e]\*Sqrt[1 + c^2\*x^2])/(Sqrt[c^2]\*Sqrt[c^2\*d - e])]) + 8\*c^3\*Sqrt[d]\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]])/(3\*c^2\*e^3\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [B]** time = 1.11, size = 2421, normalized size = 9.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] [1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d)/(c^3*d^3*e^3 - c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2), -1/6*(3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 2*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d)/(c^3*d^3*e^3 - c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2), 1/12*(16*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + 3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d)/(c^3*d^3*e^3 - c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2), 1/6*(8*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - 3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 2*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e^2 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d)/(c^3*d^3*e^3 - c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2)]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^5/(e\*x^2 + d)^(5/2), x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} \left( \frac{3x^4}{(ex^2 + d)^{\frac{3}{2}}e} + \frac{12dx^2}{(ex^2 + d)^{\frac{3}{2}}e^2} + \frac{8d^2}{(ex^2 + d)^{\frac{3}{2}}e^3} \right) a + \frac{1}{3} b \left( \frac{(3e^2x^4 + 12dex^2 + 8d^2) \log(\sqrt{c^2x^2 + 1} + 1)}{(e^4x^2 + de^3)\sqrt{ex^2 + d}} + 3 \int \frac{1}{3} \left( \frac{3c^2x^4}{(e^4x^2 + de^3)\sqrt{ex^2 + d}} + \frac{12c^2dx^2}{(e^4x^2 + de^3)\sqrt{ex^2 + d}} + \frac{8c^2d^2}{(e^4x^2 + de^3)\sqrt{ex^2 + d}} \right) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/3\*(3\*x^4/((e\*x^2 + d)^(3/2)\*e) + 12\*d\*x^2/((e\*x^2 + d)^(3/2)\*e^2) + 8\*d^2/((e\*x^2 + d)^(3/2)\*e^3))\*a + 1/3\*b\*((3\*e^2\*x^4 + 12\*d\*e\*x^2 + 8\*d^2)\*log(sqrt(c^2\*x^2 + 1) + 1)/((e^4\*x^2 + d\*e^3)\*sqrt(e\*x^2 + d)) + 3\*integrate(1/3\*(3\*c^2\*e^2\*x^5 + 12\*c^2\*d\*e\*x^3 + 8\*c^2\*d^2\*x)/((c^2\*e^4\*x^4 + d\*e^3 + (c^2\*d\*e^3 + e^4)\*x^2)\*sqrt(c^2\*x^2 + 1)\*sqrt(e\*x^2 + d) + (c^2\*e^4\*x^4 + d\*e^3 + (c^2\*d\*e^3 + e^4)\*x^2)\*sqrt(e\*x^2 + d)), x) - 3\*integrate(1/3\*(3\*(e^3\*log(c) + e^3)\*c^2\*x^7 + 20\*c^2\*d^2\*e\*x^3 + 8\*c^2\*d^3\*x + 3\*(5\*c^2\*d\*e^2 + e^3\*log(c))\*x^5 + 3\*(c^2\*e^3\*x^7 + e^3\*x^5)\*log(x))/((c^2\*e^5\*x^6 + d^2\*e^3 + (2\*c^2\*d\*e^4 + e^5)\*x^4 + (c^2\*d^2\*e^3 + 2\*d\*e^4)\*x^2)\*sqrt(e\*x^2 + d)), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(5/2),x)

[Out] int((x^5\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.157 \quad \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=169

$$\frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{2bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d-c^2x^2-1}}\right)}{3\sqrt{d}e^2\sqrt{-c^2x^2}} - \frac{bcx\sqrt{-c^2x^2-1}}{3e\sqrt{-c^2x^2}(c^2d-e)\sqrt{d+ex^2}}$$

[Out]  $1/3*d*(a+b*\operatorname{arccsch}(c*x))/e^2/(e*x^2+d)^{(3/2)}-2/3*b*c*x*\operatorname{arctan}((e*x^2+d)^{(1/2)}/d^{(1/2)/(-c^2*x^2-1)^{(1/2)})/e^2/d^{(1/2)/(-c^2*x^2)^{(1/2)}+(-a-b*\operatorname{arccsch}(c*x))/e^2/(e*x^2+d)^{(1/2)}-1/3*b*c*x*(-c^2*x^2-1)^{(1/2)/(c^2*d-e)/e/(-c^2*x^2)^{(1/2)/(e*x^2+d)^{(1/2)}}$

**Rubi [A]** time = 0.27, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {266, 43, 6302, 12, 573, 152, 93, 204}

$$\frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{2bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d-c^2x^2-1}}\right)}{3\sqrt{d}e^2\sqrt{-c^2x^2}} - \frac{bcx\sqrt{-c^2x^2-1}}{3e\sqrt{-c^2x^2}(c^2d-e)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] `Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]`

[Out]  $-(b*c*x*\operatorname{Sqrt}[-1 - c^2*x^2])/(3*(c^2*d - e)*e*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[d + e*x^2]) + (d*(a + b*\operatorname{ArcCsch}[c*x]))/(3*e^2*(d + e*x^2)^{(3/2)}) - (a + b*\operatorname{ArcCsch}[c*x])/(e^2*\operatorname{Sqrt}[d + e*x^2]) - (2*b*c*x*\operatorname{ArcTan}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[-1 - c^2*x^2])])/(3*\operatorname{Sqrt}[d]*e^2*\operatorname{Sqrt}[-(c^2*x^2)])$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

#### Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

#### Rule 93

`Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]`

#### Rule 152

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g`

```
- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

### Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

### Rule 573

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
)*((e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

### Rule 6302

```
Int[((a_) + ArcCsch[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(
x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex^2)^{3/2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-2d - 3ex^2}{3e^2 x \sqrt{-1 - c^2 x^2} (d + ex^2)^{3/2}} dx}{\sqrt{-c^2 x^2}} \\
&= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex^2)^{3/2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(bcx) \int \frac{-2d - 3ex^2}{x \sqrt{-1 - c^2 x^2} (d + ex^2)^{3/2}} dx}{3e^2 \sqrt{-c^2 x^2}} \\
&= \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex^2)^{3/2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{(bcx) \operatorname{Subst} \left( \int \frac{-2d - 3ex}{x \sqrt{-1 - c^2 x} (d + ex)^{3/2}} dx, x, x \right)}{6e^2 \sqrt{-c^2 x^2}} \\
&= -\frac{bcx \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e \sqrt{-c^2 x^2} \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex^2)^{3/2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(bcx) \sqrt{-1 - c^2 x^2}}{3e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} \\
&= -\frac{bcx \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e \sqrt{-c^2 x^2} \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex^2)^{3/2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(bcx) \sqrt{-1 - c^2 x^2}}{3e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} \\
&= -\frac{bcx \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e \sqrt{-c^2 x^2} \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex^2)^{3/2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{(2bcx) \sqrt{-1 - c^2 x^2}}{3e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}} \\
&= -\frac{bcx \sqrt{-1 - c^2 x^2}}{3(c^2 d - e) e \sqrt{-c^2 x^2} \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{csch}^{-1}(cx))}{3e^2(d + ex^2)^{3/2}} - \frac{a + b \operatorname{csch}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{2bcx \sqrt{-1 - c^2 x^2}}{3e^2 \sqrt{-c^2 x^2} \sqrt{d + ex^2}}
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 201, normalized size = 1.19

$$\frac{a(c^2 d - e)(2d + 3ex^2) + bcex \sqrt{\frac{1}{c^2 x^2} + 1} (d + ex^2) + b(c^2 d - e) \operatorname{csch}^{-1}(cx)(2d + 3ex^2)}{3e^2(e - c^2 d)(d + ex^2)^{3/2}} + \frac{2bcx \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{-d - ex^2}}{3\sqrt{d} e^2 \sqrt{c^2 x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(d + e\*x^2) + a\*(c^2\*d - e)\*(2\*d + 3\*e\*x^2) + b\*(c^2\*d - e)\*(2\*d + 3\*e\*x^2)\*ArcCsch[c\*x])/(3\*e^2\*(-(c^2\*d) + e)\*(d + e\*x^2)^(3/2)) + (2\*b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]])/(3\*Sqrt[d]\*e^2\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [B]** time = 0.94, size = 786, normalized size = 4.65

$$\frac{2(2bc^2d^3 - 2bd^2e + 3(bc^2d^2e - bde^2)x^2)\sqrt{ex^2 + d} \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}+1}}{cx}\right) - (bc^2d^3 + (bc^2de^2 - be^3)x^4 - bd^2e + \dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2), x, algorithm="fricas")

```
[Out] [-1/6*(2*(2*b*c^2*d^3 - 2*b*d^2*e + 3*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 - 2*a*d^2*e + 3*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^2*d^4*e^2 - d^3*e^3 + (c^2*d^2*e^4 - d*e^5)*x^4 + 2*(c^2*d^3*e^3 - d^2*e^4)*x^2), -1/3*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + (2*b*c^2*d^3 - 2*b*d^2*e + 3*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*a*c^2*d^3 - 2*a*d^2*e + 3*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^2*d^4*e^2 - d^3*e^3 + (c^2*d^2*e^4 - d*e^5)*x^4 + 2*(c^2*d^3*e^3 - d^2*e^4)*x^2) ]
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

**maple** [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a \left( \frac{3x^2}{(ex^2 + d)^{\frac{3}{2}}e} + \frac{2d}{(ex^2 + d)^{\frac{3}{2}}e^2} \right) + b \int \frac{x^3 \log \left( \sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx} \right)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*integrate(x^3*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

$$3.158 \quad \int \frac{x(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=144

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{3e(d+ex^2)^{3/2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}} + \frac{bcx\sqrt{-c^2x^2-1}}{3d\sqrt{-c^2x^2}(c^2d-e)\sqrt{d+ex^2}}$$

[Out] 1/3\*(-a-b\*arccsch(c\*x))/e/(e\*x^2+d)^(3/2)-1/3\*b\*c\*x\*arctan((e\*x^2+d)^(1/2)/d^(1/2)/(-c^2\*x^2-1)^(1/2))/d^(3/2)/e/(-c^2\*x^2)^(1/2)+1/3\*b\*c\*x\*(-c^2\*x^2-1)^(1/2)/d/(c^2\*d-e)/(-c^2\*x^2)^(1/2)/(e\*x^2+d)^(1/2)

**Rubi [A]** time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {6300, 446, 96, 93, 204}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{3e(d+ex^2)^{3/2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}} + \frac{bcx\sqrt{-c^2x^2-1}}{3d\sqrt{-c^2x^2}(c^2d-e)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (b\*c\*x\*Sqrt[-1 - c^2\*x^2])/(3\*d\*(c^2\*d - e)\*Sqrt[-(c^2\*x^2)]\*Sqrt[d + e\*x^2]) - (a + b\*ArcCsch[c\*x])/(3\*e\*(d + e\*x^2)^(3/2)) - (b\*c\*x\*ArcTan[Sqrt[d + e\*x^2]/(Sqrt[d]\*Sqrt[-1 - c^2\*x^2])])/(3\*d^(3/2)\*e\*Sqrt[-(c^2\*x^2)])

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 96

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[(a\*d\*f\*(m + 1) + b\*c\*f\*(n + 1) + b\*d\*e\*(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])

### Rule 204

Int[(((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 446

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p\*(c + d\*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[

$b*c - a*d, 0]$  && IntegerQ[Simplify[(m + 1)/n]]

### Rule 6300

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))\*(x\_.)\*((d\_.) + (e\_.)\*(x\_.)^2)^(p\_.),  
x\_Symbol] :> Simp[((d + e\*x^2)^(p + 1)\*(a + b\*ArcCsch[c\*x]))/(2\*e\*(p + 1)),  
x] - Dist[(b\*c\*x)/(2\*e\*(p + 1)\*Sqrt[-(c^2\*x^2)]), Int[(d + e\*x^2)^(p + 1)/  
(x\*Sqrt[-1 - c^2\*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{x(a + bcsch^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= -\frac{a + bcsch^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{(bcx) \int \frac{1}{x\sqrt{-c^2x^2}(d+ex^2)^{3/2}} dx}{3e\sqrt{-c^2x^2}} \\ &= -\frac{a + bcsch^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-c^2x}(d+ex)^{3/2}} dx, x, x^2\right)}{6e\sqrt{-c^2x^2}} \\ &= \frac{bcx\sqrt{-1 - c^2x^2}}{3d(c^2d - e)\sqrt{-c^2x^2}\sqrt{d + ex^2}} - \frac{a + bcsch^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{x\sqrt{-c^2x}\sqrt{d+ex}} dx, x, \frac{1}{\sqrt{d+ex^2}}\right)}{6de\sqrt{-c^2x^2}} \\ &= \frac{bcx\sqrt{-1 - c^2x^2}}{3d(c^2d - e)\sqrt{-c^2x^2}\sqrt{d + ex^2}} - \frac{a + bcsch^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{(bcx) \text{Subst}\left(\int \frac{1}{-d-x^2} dx, x, \frac{1}{\sqrt{d+ex^2}}\right)}{3de\sqrt{-c^2x^2}} \\ &= \frac{bcx\sqrt{-1 - c^2x^2}}{3d(c^2d - e)\sqrt{-c^2x^2}\sqrt{d + ex^2}} - \frac{a + bcsch^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{bcx \tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}} \end{aligned}$$

**Mathematica [A]** time = 0.23, size = 185, normalized size = 1.28

$$\frac{ad(e - c^2d) + bcex\sqrt{\frac{1}{c^2x^2} + 1}(d + ex^2) + bd(e - c^2d)csch^{-1}(cx)}{3de(c^2d - e)(d + ex^2)^{3/2}} + \frac{bcx\sqrt{\frac{1}{c^2x^2} + 1}\sqrt{-d - ex^2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{c^2x^2+1}}{\sqrt{-d-ex^2}}\right)}{3d^{3/2}e\sqrt{c^2x^2 + 1}\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (a\*d\*(-(c^2\*d) + e) + b\*c\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(d + e\*x^2) + b\*d\*(-(c^2\*d) + e)\*ArcCsch[c\*x])/(3\*d\*(c^2\*d - e)\*e\*(d + e\*x^2)^(3/2)) + (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[-d - e\*x^2]\*ArcTan[(Sqrt[d]\*Sqrt[1 + c^2\*x^2])/Sqrt[-d - e\*x^2]])/(3\*d^(3/2)\*e\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [B]** time = 0.66, size = 698, normalized size = 4.85

$$\frac{4(bc^2d^3 - bd^2e)\sqrt{ex^2 + d} \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2} + 1}}{cx}\right) - (bc^2d^3 + (bc^2de^2 - be^3)x^4 - bd^2e + 2(bc^2d^2e - bde^2)x^2)\sqrt{d}}{12(c^2d^5e - d^4e^2 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] [-1/12\*(4\*(b\*c^2\*d^3 - b\*d^2\*e)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) - (b\*c^2\*d^3 + (b\*c^2\*d\*e^2 - b\*e^3)\*x^4 - b\*d^2\*e + 2\*(b\*c^2\*d^2\*e - b\*d\*e^2)\*x^2)\*sqrt(d)\*log(((c^4\*d^2 + 6\*c^2\*d\*e + e^2)\*x^4 + 8\*(c^2\*d^2 + d\*e)\*x^2 + 4\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d))\*sqrt(d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 8\*d^2)/x^4) + 4\*(a\*c^2\*d^3 - a\*d^2\*e - (b\*c\*d\*e^2\*x^3 + b\*c\*d^2\*e\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^2\*d^5\*e - d^4\*e^2 + (c^2\*d^3\*e^3 - d^2\*e^4)\*x^4 + 2\*(c^2\*d^4\*e^2 - d^3\*e^3)\*x^2), -1/6\*((b\*c^2\*d^3 + (b\*c^2\*d\*e^2 - b\*e^3)\*x^4 - b\*d^2\*e + 2\*(b\*c^2\*d^2\*e - b\*d\*e^2)\*x^2)\*sqrt(-d)\*arctan(1/2\*((c^3\*d + c\*e)\*x^3 + 2\*c\*d\*x)\*sqrt(e\*x^2 + d)\*sqrt(-d)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))/(c^2\*d\*e\*x^4 + (c^2\*d^2 + d\*e)\*x^2 + d^2)) + 2\*(b\*c^2\*d^3 - b\*d^2\*e)\*sqrt(e\*x^2 + d)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + 2\*(a\*c^2\*d^3 - a\*d^2\*e - (b\*c\*d\*e^2\*x^3 + b\*c\*d^2\*e\*x)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)))\*sqrt(e\*x^2 + d))/(c^2\*d^5\*e - d^4\*e^2 + (c^2\*d^3\*e^3 - d^2\*e^4)\*x^4 + 2\*(c^2\*d^4\*e^2 - d^3\*e^3)\*x^2)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x/(e\*x^2 + d)^(5/2), x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{arcsch}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{x \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{5}{2}}} dx - \frac{a}{3(ex^2 + d)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] b\*integrate(x\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e\*x^2 + d)^(5/2), x) - 1/3\*a/((e\*x^2 + d)^(3/2)\*e)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(5/2), x)
```

```
[Out] Timed out
```

$$3.159 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=26

$$\operatorname{Int} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(5/2), x)

**Rubi** [A] time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

[Out] Defer[Int][(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx = \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx$$

**Mathematica** [A] time = 45.84, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*(d + e\*x^2)^(5/2)), x]

**fricas** [A] time = 0.69, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{e^3 x^7 + 3 d e^2 x^5 + 3 d^2 e x^3 + d^3 x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e^3\*x^7 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^3 + d^3\*x), x)

**giac** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{5/2} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)^(5/2)\*x), x)

**maple** [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x(e x^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(5/2),x)

[Out] int((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} a \left( \frac{3 \operatorname{arsinh}\left(\frac{d}{\sqrt{de}|x|}\right)}{d^{\frac{5}{2}}} - \frac{3}{\sqrt{ex^2 + d} d^2} - \frac{1}{(ex^2 + d)^{\frac{3}{2}} d} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] -1/3\*a\*(3\*arcsinh(d/(sqrt(d\*e)\*abs(x)))/d^(5/2) - 3/(sqrt(e\*x^2 + d)\*d^2) - 1/((e\*x^2 + d)^(3/2)\*d)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/((e\*x^2 + d)^(5/2)\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x(e x^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)^(5/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(d + e\*x^2)^(5/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.160 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=26

$$\operatorname{Int} \left( \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(5/2), x)

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

[Out] Defer[Int][(a + b\*ArcCsch[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx = \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 59.84, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x^3\*(d + e\*x^2)^(5/2)), x]

fricas [A] time = 0.76, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a)}{e^3 x^9 + 3 d e^2 x^7 + 3 d^2 e x^5 + d^3 x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(5/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e^3\*x^9 + 3\*d\*e^2\*x^7 + 3\*d^2\*e\*x^5 + d^3\*x^3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{5/2} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/((e\*x^2 + d)^(5/2)\*x^3), x)

**maple** [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^3 (e x^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(5/2),x)

[Out] int((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} a \left( \frac{15 e \operatorname{arsinh}\left(\frac{d}{\sqrt{d e} |x|}\right)}{d^{\frac{7}{2}}} - \frac{15 e}{\sqrt{e x^2 + d} d^3} - \frac{5 e}{(e x^2 + d)^{\frac{3}{2}} d^2} - \frac{3}{(e x^2 + d)^{\frac{3}{2}} d x^2} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{e^2 x^2} + 1} + \frac{1}{c x}\right)}{(e x^2 + d)^{\frac{5}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^3/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/6\*a\*(15\*e\*arcsinh(d/(sqrt(d\*e)\*abs(x)))/d^(7/2) - 15\*e/(sqrt(e\*x^2 + d)\*d^3) - 5\*e/((e\*x^2 + d)^(3/2)\*d^2) - 3/((e\*x^2 + d)^(3/2)\*d\*x^2)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/((e\*x^2 + d)^(5/2)\*x^3), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{c x}\right)}{x^3 (e x^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^3\*(d + e\*x^2)^(5/2)),x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^3\*(d + e\*x^2)^(5/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*3/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.161 \quad \int \frac{x^6 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=26

$$\operatorname{Int} \left( \frac{x^6 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable(x^6\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2), x)

**Rubi [A]** time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^6 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^6\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Defer[Int] [(x^6\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^6 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx = \int \frac{x^6 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Mathematica [A]** time = 13.40, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^6\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Integrate[(x^6\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

**fricas [A]** time = 0.79, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(bx^6 \operatorname{arcsch}(cx) + ax^6) \sqrt{ex^2 + d}}{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2), x, algorithm="fricas")

[Out] integral((b\*x^6\*arccsch(c\*x) + a\*x^6)\*sqrt(e\*x^2 + d)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^6}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^6/(e\*x^2 + d)^(5/2), x)

**maple** [A] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x^6 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x^6\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} \left( \frac{3x^5}{(ex^2 + d)^{\frac{3}{2}}e} + \frac{5dx \left( \frac{3x^2}{(ex^2+d)^{\frac{3}{2}}e} + \frac{2d}{(ex^2+d)^{\frac{3}{2}}e^2} \right)}{e} + \frac{5dx}{\sqrt{ex^2 + d}e^3} - \frac{15d \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{7}{2}}} \right) a + b \int \frac{x^6 \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/6\*(3\*x^5/((e\*x^2 + d)^(3/2)\*e) + 5\*d\*x\*(3\*x^2/((e\*x^2 + d)^(3/2)\*e) + 2\*d/((e\*x^2 + d)^(3/2)\*e^2))/e + 5\*d\*x/(sqrt(e\*x^2 + d)\*e^3) - 15\*d\*arcsinh(e\*x/sqrt(d\*e))/e^(7/2))\*a + b\*integrate(x^6\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e\*x^2 + d)^(5/2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^6 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(5/2),x)

[Out] int((x^6\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.162 \quad \int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=26

$$\operatorname{Int} \left( \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2), x)

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Defer[Int] [(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx = \int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Mathematica [A]** time = 12.60, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] Integrate[(x^4\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

**fricas [A]** time = 0.62, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(bx^4 \operatorname{arcsch}(cx) + ax^4) \sqrt{ex^2 + d}}{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2), x, algorithm="fricas")

[Out] integral((b\*x^4\*arccsch(c\*x) + a\*x^4)\*sqrt(e\*x^2 + d)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^4}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^4/(e\*x^2 + d)^(5/2), x)

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} \left( x \left( \frac{3x^2}{(ex^2 + d)^{\frac{3}{2}} e} + \frac{2d}{(ex^2 + d)^{\frac{3}{2}} e^2} \right) + \frac{x}{\sqrt{ex^2 + d} e^2} - \frac{3 \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{e^{\frac{5}{2}}} \right) a + b \int \frac{x^4 \log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] -1/3\*(x\*(3\*x^2/((e\*x^2 + d)^(3/2)\*e) + 2\*d/((e\*x^2 + d)^(3/2)\*e^2)) + x/(sqrt(e\*x^2 + d)\*e^2) - 3\*arcsinh(e\*x/sqrt(d\*e))/e^(5/2))\*a + b\*integrate(x^4\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e\*x^2 + d)^(5/2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(5/2),x)

[Out] int((x^4\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.163 \quad \int \frac{x^2(a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=359

$$\frac{x^3(a + b \operatorname{csch}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{bx\sqrt{d + ex^2} F(\tan^{-1}(cx) | 1 - \frac{e}{c^2d})}{3d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2 - 1}(c^2d - e)\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} + \frac{bcx^2\sqrt{-c^2x^2 - 1}}{3d\sqrt{-c^2x^2}(c^2d - e)\sqrt{d + ex^2}} - \frac{bc^2x\sqrt{-c^2x^2 - 1}}{3de\sqrt{-c^2x^2}}$$

[Out]  $1/3*x^3*(a+b*\operatorname{arccsch}(c*x))/d/(e*x^2+d)^{(3/2)}+1/3*b*c*x^2*(-c^2*x^2-1)^{(1/2)}/d/(c^2*d-e)/(-c^2*x^2)^{(1/2)}/(e*x^2+d)^{(1/2)}+1/3*b*c^3*x^2*(e*x^2+d)^{(1/2)}/d/(c^2*d-e)/e/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}-1/3*b*c^2*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*\operatorname{EllipticE}(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d/(c^2*d-e)/e/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}+1/3*b*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*\operatorname{EllipticF}(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d^2/(c^2*d-e)/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}$

**Rubi [A]** time = 0.31, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {264, 6302, 12, 471, 422, 418, 492, 411}

$$\frac{x^3(a + b \operatorname{csch}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{bx\sqrt{d + ex^2} F(\tan^{-1}(cx) | 1 - \frac{e}{c^2d})}{3d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2 - 1}(c^2d - e)\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} + \frac{bc^3x^2\sqrt{d + ex^2}}{3de\sqrt{-c^2x^2}\sqrt{-c^2x^2 - 1}(c^2d - e)} + \frac{bc^2x\sqrt{-c^2x^2 - 1}}{3d\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^2*(a + b*\operatorname{ArcCsch}[c*x]))/(d + e*x^2)^{(5/2)}, x]$

[Out]  $(b*c*x^2*\operatorname{Sqrt}[-1 - c^2*x^2])/(3*d*(c^2*d - e)*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[d + e*x^2]) + (b*c^3*x^2*\operatorname{Sqrt}[d + e*x^2])/(3*d*(c^2*d - e)*e*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[-1 - c^2*x^2]) + (x^3*(a + b*\operatorname{ArcCsch}[c*x]))/(3*d*(d + e*x^2)^{(3/2)}) - (b*c^2*x*\operatorname{Sqrt}[d + e*x^2]*\operatorname{EllipticE}[\operatorname{ArcTan}[c*x], 1 - e/(c^2*d)])/(3*d*(c^2*d - e)*e*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (b*x*\operatorname{Sqrt}[d + e*x^2]*\operatorname{EllipticF}[\operatorname{ArcTan}[c*x], 1 - e/(c^2*d)])/(3*d^2*(c^2*d - e)*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))])$

### Rule 12

$\operatorname{Int}[(a_*)(u_), x\_Symbol] := \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 264

$\operatorname{Int}[(c_*)(x_)^{(m_*)}((a_*) + (b_*)(x_)^{(n_*)})^{(p_)}, x\_Symbol] := \operatorname{Simp}[(c*x)^{(m+1)}*(a + b*x^n)^{(p+1)}/(a*c*(m+1)), x] /;$  FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n + p + 1, 0] && NeQ[m, -1]

### Rule 411

$\operatorname{Int}[\operatorname{Sqrt}[(a_*) + (b_*)(x_)^2]/((c_*) + (d_*)(x_)^2)^{(3/2)}, x\_Symbol] := \operatorname{Simp}[(\operatorname{Sqrt}[a + b*x^2]*\operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\operatorname{Rt}[d/c, 2]*\operatorname{Sqrt}[c + d*x^2]*\operatorname{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /;$  FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

### Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

#### Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

#### Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^q_], x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e,
q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

#### Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

#### Rule 6302

```
Int[((a_.) + ArcCsch[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

#### Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{3d (d + ex^2)^{3/2}} - \frac{(bcx) \int \frac{x^2}{3d \sqrt{-1-c^2x^2} (d+ex^2)^{3/2}} dx}{\sqrt{-c^2x^2}} \\
&= \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{3d (d + ex^2)^{3/2}} - \frac{(bcx) \int \frac{x^2}{\sqrt{-1-c^2x^2} (d+ex^2)^{3/2}} dx}{3d \sqrt{-c^2x^2}} \\
&= \frac{bcx^2 \sqrt{-1-c^2x^2}}{3d (c^2d - e) \sqrt{-c^2x^2} \sqrt{d + ex^2}} + \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{(bcx) \int \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}} dx}{3d (-c^2d + e) \sqrt{-c^2x^2}} \\
&= \frac{bcx^2 \sqrt{-1-c^2x^2}}{3d (c^2d - e) \sqrt{-c^2x^2} \sqrt{d + ex^2}} + \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{3d (d + ex^2)^{3/2}} - \frac{(bcx) \int \frac{1}{\sqrt{-1-c^2x^2} \sqrt{d+ex^2}} dx}{3d (-c^2d + e) \sqrt{-c^2x^2}} \\
&= \frac{bcx^2 \sqrt{-1-c^2x^2}}{3d (c^2d - e) \sqrt{-c^2x^2} \sqrt{d + ex^2}} + \frac{bc^3 x^2 \sqrt{d + ex^2}}{3d (c^2d - e) e \sqrt{-c^2x^2} \sqrt{-1 - c^2x^2}} + \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{3d (d + ex^2)^{3/2}} \\
&= \frac{bcx^2 \sqrt{-1-c^2x^2}}{3d (c^2d - e) \sqrt{-c^2x^2} \sqrt{d + ex^2}} + \frac{bc^3 x^2 \sqrt{d + ex^2}}{3d (c^2d - e) e \sqrt{-c^2x^2} \sqrt{-1 - c^2x^2}} + \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{3d (d + ex^2)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 189, normalized size = 0.53

$$\frac{x^2 \left( ax (c^2d - e) + bc \sqrt{\frac{1}{c^2x^2} + 1} (d + ex^2) + bx (c^2d - e) \operatorname{csch}^{-1}(cx) \right)}{3d (c^2d - e) (d + ex^2)^{3/2}} - \frac{bcx \sqrt{\frac{1}{c^2x^2} + 1} \sqrt{\frac{ex^2}{d} + 1} E \left( \sin^{-1} \left( \sqrt{\frac{-e}{d}} x \right) \middle| \frac{c^2}{e} \right)}{3d \sqrt{c^2x^2 + 1} \sqrt{\frac{-e}{d}} (c^2d - e) \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(5/2), x]

[Out] (x^2\*(a\*(c^2\*d - e)\*x + b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*(d + e\*x^2) + b\*(c^2\*d - e)\*x\*ArcCsch[c\*x]))/(3\*d\*(c^2\*d - e)\*(d + e\*x^2)^(3/2)) - (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 + (e\*x^2)/d]\*EllipticE[ArcSin[Sqrt[-(e/d)]\*x], (c^2\*d)/e])/ (3\*d\*(c^2\*d - e)\*Sqrt[-(e/d)]\*Sqrt[1 + c^2\*x^2]\*Sqrt[d + e\*x^2])

**fricas [F]** time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(bx^2 \operatorname{arcsch}(cx) + ax^2) \sqrt{ex^2 + d}}{e^3 x^6 + 3de^2 x^4 + 3d^2 ex^2 + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2), x, algorithm="fricas")

[Out] integral((b\*x^2\*arccsch(c\*x) + a\*x^2)\*sqrt(e\*x^2 + d)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex^2 + d)^{5/2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*x^2/(e\*x^2 + d)^(5/2), x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3} a \left( \frac{x}{(ex^2 + d)^{\frac{3}{2}} e} - \frac{x}{\sqrt{ex^2 + d} de} \right) + b \int \frac{x^2 \log \left( \sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx} \right)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] -1/3\*a\*(x/((e\*x^2 + d)^(3/2)\*e) - x/(sqrt(e\*x^2 + d)\*d\*e)) + b\*integrate(x^2\*log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e\*x^2 + d)^(5/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(5/2),x)

[Out] int((x^2\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(5/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(5/2),x)

[Out] Timed out

$$3.164 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex^2)^{5/2}} dx$$

**Optimal.** Leaf size=278

$$\frac{2x(a + b \operatorname{csch}^{-1}(cx))}{3d^2 \sqrt{d + ex^2}} + \frac{x(a + b \operatorname{csch}^{-1}(cx))}{3d(d + ex^2)^{3/2}} - \frac{bc\sqrt{e}x\sqrt{-c^2x^2 - 1} E\left(\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) \middle| 1 - \frac{c^2d}{e}\right)}{3d^{3/2}\sqrt{-c^2x^2} (c^2d - e) \sqrt{d + ex^2} \sqrt{\frac{d(c^2x^2 + 1)}{d + ex^2}}} - \frac{bx(3c^2d - 2e) \sqrt{d + ex^2}}{3d^3\sqrt{-c^2x^2} \sqrt{-c^2x^2 - 1}}$$

[Out]  $\frac{1}{3}x*(a+b*\operatorname{arccsch}(c*x))/d/(e*x^2+d)^{(3/2)}+2/3*x*(a+b*\operatorname{arccsch}(c*x))/d^2/(e*x^2+d)^{(1/2)}-1/3*b*c*x*(1/(1+e*x^2/d))^{(1/2)}*(1+e*x^2/d)^{(1/2)}*\operatorname{EllipticE}(x*e^{(1/2)}/d^{(1/2)}/(1+e*x^2/d)^{(1/2)},(1-c^2*d/e)^{(1/2)})*e^{(1/2)}*(-c^2*x^2-1)^{(1/2)}/d^{(3/2)}/(c^2*d-e)/(-c^2*x^2)^{(1/2)}/(d*(c^2*x^2+1)/(e*x^2+d))^{(1/2)}/(e*x^2+d)^{(1/2)}-1/3*b*(3*c^2*d-2*e)*x*(1/(c^2*x^2+1))^{(1/2)}*(c^2*x^2+1)^{(1/2)}*\operatorname{EllipticF}(c*x/(c^2*x^2+1)^{(1/2)},(1-e/c^2/d)^{(1/2)})*(e*x^2+d)^{(1/2)}/d^3/(c^2*d-e)/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}/((e*x^2+d)/d/(c^2*x^2+1))^{(1/2)}$

**Rubi [A]** time = 0.18, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$ , Rules used = {192, 191, 6292, 12, 525, 418, 411}

$$\frac{2x(a + b \operatorname{csch}^{-1}(cx))}{3d^2 \sqrt{d + ex^2}} + \frac{x(a + b \operatorname{csch}^{-1}(cx))}{3d(d + ex^2)^{3/2}} - \frac{bx(3c^2d - 2e) \sqrt{d + ex^2} F\left(\tan^{-1}(cx) \middle| 1 - \frac{e}{c^2d}\right)}{3d^3\sqrt{-c^2x^2} \sqrt{-c^2x^2 - 1} (c^2d - e) \sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}} - \frac{bc\sqrt{e}x\sqrt{-c^2x^2 - 1} E\left(\tan^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d}}\right) \middle| 1 - \frac{c^2d}{e}\right)}{3d^3\sqrt{-c^2x^2} (c^2d - e)}$$

Antiderivative was successfully verified.

[In] `Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2), x]`

[Out]  $(x*(a + b*\operatorname{ArcCsch}[c*x]))/(3*d*(d + e*x^2)^{(3/2)}) + (2*x*(a + b*\operatorname{ArcCsch}[c*x]))/(3*d^2*\operatorname{Sqrt}[d + e*x^2]) - (b*c*\operatorname{Sqrt}[e]*x*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*x)/\operatorname{Sqrt}[d]], 1 - (c^2*d)/e])/(3*d^{(3/2)}*(c^2*d - e)*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[(d*(1 + c^2*x^2))/(d + e*x^2)]*\operatorname{Sqrt}[d + e*x^2]) - (b*(3*c^2*d - 2*e)*x*\operatorname{Sqrt}[d + e*x^2]*\operatorname{EllipticF}[\operatorname{ArcTan}[c*x], 1 - e/(c^2*d)])/(3*d^3*(c^2*d - e)*\operatorname{Sqrt}[-(c^2*x^2)]*\operatorname{Sqrt}[-1 - c^2*x^2]*\operatorname{Sqrt}[(d + e*x^2)/(d*(1 + c^2*x^2))])$

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

#### Rule 191

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]`

#### Rule 192

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]`

#### Rule 411

`Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[`

$d/c, 2] * \text{Sqrt}[c + d*x^2] * \text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

### Rule 418

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*(x_)^2] * \text{Sqrt}[(c_) + (d_)*(x_)^2]), x\_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2] * \text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*\text{Rt}[d/c, 2] * \text{Sqrt}[c + d*x^2] * \text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& !\text{SimplerSqrtQ}[b/a, d/c]$

### Rule 525

$\text{Int}[(e_ + (f_)*(x_)^2)/(\text{Sqrt}[(a_) + (b_)*(x_)^2] * ((c_) + (d_)*(x_)^2)^{(3/2)}), x\_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(\text{Sqrt}[a + b*x^2] * \text{Sqrt}[c + d*x^2]), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

### Rule 6292

$\text{Int}[(a_ + \text{ArcCsch}[(c_)*(x_)])*(b_)*((d_ + (e_)*(x_)^2)^{(p_)}), x\_Symbol] \rightarrow \text{With}[\{u = \text{IntHide}[(d + e*x^2)^p, x]\}, \text{Dist}[a + b*\text{ArcCsch}[c*x], u, x] - \text{Dist}[(b*c*x)/\text{Sqrt}[-(c^2*x^2)], \text{Int}[\text{SimplifyIntegrand}[u/(x*\text{Sqrt}[-1 - c^2*x^2]), x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& (\text{IGtQ}[p, 0] || \text{ILtQ}[p + 1/2, 0])$

### Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{csch}^{-1}(cx)}{(d + ex^2)^{5/2}} dx &= \frac{x(a + b \operatorname{csch}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{csch}^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{3d+2ex^2}{3d^2\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}} dx}{\sqrt{-c^2x^2}} \\ &= \frac{x(a + b \operatorname{csch}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{csch}^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(bcx) \int \frac{3d+2ex^2}{\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}} dx}{3d^2\sqrt{-c^2x^2}} \\ &= \frac{x(a + b \operatorname{csch}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{csch}^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{(bc(3c^2d - 2e)x) \int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}} dx}{3d^2(c^2d - e)\sqrt{-c^2x^2}} \\ &= \frac{x(a + b \operatorname{csch}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{csch}^{-1}(cx))}{3d^2\sqrt{d + ex^2}} - \frac{bc\sqrt{e}x\sqrt{-1 - c^2x^2} E\left(\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\right)}{3d^{3/2}(c^2d - e)\sqrt{-c^2x^2}\sqrt{\frac{d(1+c^2x^2)}{d+ex^2}}\sqrt{d}} \end{aligned}$$

**Mathematica [C]** time = 0.56, size = 248, normalized size = 0.89

$$\frac{x(a(c^2d - e)(3d + 2ex^2) - bcex\sqrt{\frac{1}{c^2x^2} + 1}(d + ex^2) + b(c^2d - e)\operatorname{csch}^{-1}(cx)(3d + 2ex^2)) - ibcx\sqrt{\frac{1}{c^2x^2} + 1}\sqrt{\frac{e}{d}}}{3d^2(c^2d - e)(d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(d + e\*x^2)^(5/2), x]

[Out] (x\*(-(b\*c\*e\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*(d + e\*x^2)) + a\*(c^2\*d - e)\*(3\*d + 2\*e\*x^2) + b\*(c^2\*d - e)\*(3\*d + 2\*e\*x^2)\*ArcCsch[c\*x]))/(3\*d^2\*(c^2\*d - e)\*(d

$+ e*x^2)^{(3/2)} - ((I/3)*b*c*sqrt[1 + 1/(c^2*x^2)]*x*sqrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + 2*(c^2*d - e)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)])) / (sqrt[c^2]*d^2*(c^2*d - e)*sqrt[1 + c^2*x^2]*sqrt[d + e*x^2])$

**fricas** [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arcsch}(cx) + a)}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)/(e^3\*x^6 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^2 + d^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(e\*x^2 + d)^(5/2), x)

**maple** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

[Out] int((a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3} a \left( \frac{2x}{\sqrt{ex^2 + d} d^2} + \frac{x}{(ex^2 + d)^{\frac{3}{2}} d} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{c^2x^2} + 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/(e\*x^2+d)^(5/2),x, algorithm="maxima")

[Out] 1/3\*a\*(2\*x/(sqrt(e\*x^2 + d)\*d^2) + x/((e\*x^2 + d)^(3/2)\*d)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x))/(e\*x^2 + d)^(5/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{(ex^2 + d)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

### 3.165 $\int (fx)^m (d + ex^2)^3 (a + bcsch^{-1}(cx)) dx$

**Optimal.** Leaf size=596

$$\frac{d^3(fx)^{m+1} (a + bcsch^{-1}(cx))}{f(m+1)} + \frac{3d^2e(fx)^{m+3} (a + bcsch^{-1}(cx))}{f^3(m+3)} + \frac{3de^2(fx)^{m+5} (a + bcsch^{-1}(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7} (a + bcsch^{-1}(cx))}{f^7(m+7)}$$

[Out]  $d^3(fx)^{m+1} (a + bcsch^{-1}(cx)) / f(m+1) + 3d^2e(fx)^{m+3} (a + bcsch^{-1}(cx)) / f^3(m+3) + 3de^2(fx)^{m+5} (a + bcsch^{-1}(cx)) / f^5(m+5) + e^3(fx)^{m+7} (a + bcsch^{-1}(cx)) / f^7(m+7)$

**Rubi [A]** time = 2.57, antiderivative size = 577, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {270, 6302, 1809, 1267, 459, 365, 364}

$$\frac{3d^2e(fx)^{m+3} (a + bcsch^{-1}(cx))}{f^3(m+3)} + \frac{d^3(fx)^{m+1} (a + bcsch^{-1}(cx))}{f(m+1)} + \frac{3de^2(fx)^{m+5} (a + bcsch^{-1}(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7} (a + bcsch^{-1}(cx))}{f^7(m+7)}$$

Antiderivative was successfully verified.

[In] Int[(fx)^m\*(d + e\*x^2)^3\*(a + b\*ArcCsch[c\*x]), x]

[Out]  $(b * e * (e^2 * (15 + 8 * m + m^2)^2 - 3 * c^2 * d * e * (3 + m)^2 * (42 + 13 * m + m^2) + 3 * c^4 * d^2 * (840 + 638 * m + 179 * m^2 + 22 * m^3 + m^4))) * x * (f * x)^{(1 + m)} * \text{Sqrt}[-1 - c^2 * x^2] / (c^5 * f * (2 + m) * (3 + m) * (4 + m) * (5 + m) * (6 + m) * (7 + m) * \text{Sqrt}[-(c^2 * x^2)]) - (b * e^2 * (e * (5 + m)^2 - 3 * c^2 * d * (42 + 13 * m + m^2))) * x * (f * x)^{(3 + m)} * \text{Sqrt}[-1 - c^2 * x^2] / (c^3 * f^3 * (4 + m) * (5 + m) * (6 + m) * (7 + m) * \text{Sqrt}[-(c^2 * x^2)]) + (b * e^3 * x * (f * x)^{(5 + m)} * \text{Sqrt}[-1 - c^2 * x^2]) / (c * f^5 * (6 + m) * (7 + m) * \text{Sqrt}[-(c^2 * x^2)]) + (d^3 * (f * x)^{(1 + m)} * (a + b * \text{ArcCsch}[c * x])) / (f * (1 + m)) + (3 * d^2 * e * (f * x)^{(3 + m)} * (a + b * \text{ArcCsch}[c * x])) / (f^3 * (3 + m)) + (3 * d * e^2 * (f * x)^{(5 + m)} * (a + b * \text{ArcCsch}[c * x])) / (f^5 * (5 + m)) + (e^3 * (f * x)^{(7 + m)} * (a + b * \text{ArcCsch}[c * x])) / (f^7 * (7 + m)) - (b * c * (d^3 / (1 + m)^2 - (e * (e^2 * (15 + 8 * m + m^2)^2 - 3 * c^2 * d * e * (3 + m)^2 * (42 + 13 * m + m^2) + 3 * c^4 * d^2 * (840 + 638 * m + 179 * m^2 + 22 * m^3 + m^4))) / (c^6 * (2 + m) * (3 + m) * (4 + m) * (5 + m) * (6 + m) * (7 + m))) * x * (f * x)^{(1 + m)} * \text{Sqrt}[1 + c^2 * x^2] * \text{Hypergeometric2F1}[1/2, (1 + m)/2, (3 + m)/2, -(c^2 * x^2)] / (f * \text{Sqrt}[-(c^2 * x^2)] * \text{Sqrt}[-1 - c^2 * x^2])$

**Rule 270**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[Exp andIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

**Rule 364**

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a

)]/(c\*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

### Rule 365

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

### Rule 459

Int[((e\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 1267

Int[((f\_.)\*(x\_))^(m\_.)\*((d\_) + (e\_.)\*(x\_)^2)^(q\_.)\*((a\_) + (b\_.)\*(x\_)^2 + (c\_.)\*(x\_)^4)^(p\_.), x\_Symbol] := Simp[(c^p\*(f\*x)^(m + 4\*p - 1)\*(d + e\*x^2)^(q + 1))/(e\*f^(4\*p - 1)\*(m + 4\*p + 2\*q + 1)), x] + Dist[1/(e\*(m + 4\*p + 2\*q + 1)), Int[(f\*x)^m\*(d + e\*x^2)^q\*ExpandToSum[e\*(m + 4\*p + 2\*q + 1)\*((a + b\*x^2 + c\*x^4)^p - c^p\*x^(4\*p)) - d\*c^p\*(m + 4\*p - 1)\*x^(4\*p - 2), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[p, 0] && !IntegerQ[q] && NeQ[m + 4\*p + 2\*q + 1, 0]

### Rule 1809

Int[(Pq\_)\*((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^2)^(p\_), x\_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f\*(c\*x)^(m + q - 1)\*(a + b\*x^2)^(p + 1))/(b\*c^(q - 1)\*(m + q + 2\*p + 1)), x] + Dist[1/(b\*(m + q + 2\*p + 1)), Int[(c\*x)^m\*(a + b\*x^2)^p\*ExpandToSum[b\*(m + q + 2\*p + 1)\*Pq - b\*f\*(m + q + 2\*p + 1)\*x^q - a\*f\*(m + q - 1)\*x^(q - 2), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2\*p + 1, 0] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && (!IGtQ[m, 0] || IGtQ[p + 1/2, -1])

### Rule 6302

Int[((a\_.) + ArcSch[c\_.\*(x\_)]\*(b\_.))\*((f\_.)\*(x\_))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcSch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^2)^3 (a + b \operatorname{arcsch}(cx)) dx &= \frac{d^3 (fx)^{1+m} (a + b \operatorname{arcsch}(cx))}{f(1+m)} + \frac{3d^2 e (fx)^{3+m} (a + b \operatorname{arcsch}(cx))}{f^3(3+m)} + \frac{3de}{f^5} \\
&= \frac{be^3 x (fx)^{5+m} \sqrt{-1 - c^2 x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2 x^2}} + \frac{d^3 (fx)^{1+m} (a + b \operatorname{arcsch}(cx))}{f(1+m)} + \frac{3d^2 e (fx)^{3+m}}{f^3(3+m)} \\
&= -\frac{be^2 (e(5+m)^2 - 3c^2 d (42 + 13m + m^2)) x (fx)^{3+m} \sqrt{-1 - c^2 x^2}}{c^3 f^3 (4+m)(5+m)(6+m)(7+m)\sqrt{-c^2 x^2}} + \frac{be^2 (e(5+m)^2 - 3c^2 d (42 + 13m + m^2))}{c^3 f^3 (4+m)(5+m)(6+m)(7+m)} \\
&= \frac{be (e^2 (15 + 8m + m^2)^2 - 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (8m + 5))}{c^5 f (2+m)(3+m)(4+m)(5+m)} \\
&= \frac{be (e^2 (15 + 8m + m^2)^2 - 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (8m + 5))}{c^5 f (2+m)(3+m)(4+m)(5+m)} \\
&= \frac{be (e^2 (15 + 8m + m^2)^2 - 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (8m + 5))}{c^5 f (2+m)(3+m)(4+m)(5+m)}
\end{aligned}$$

**Mathematica** [F] time = 0.23, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2)^3 (a + b \operatorname{arcsch}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^3\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)^3\*(a + b\*ArcCsch[c\*x]), x]

**fricas** [F] time = 0.78, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(ae^3x^6 + 3ade^2x^4 + 3ad^2ex^2 + ad^3 + (be^3x^6 + 3bde^2x^4 + 3bd^2ex^2 + bd^3) \operatorname{arcsch}(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^3\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral((a\*e^3\*x^6 + 3\*a\*d\*e^2\*x^4 + 3\*a\*d^2\*e\*x^2 + a\*d^3 + (b\*e^3\*x^6 + 3\*b\*d\*e^2\*x^4 + 3\*b\*d^2\*e\*x^2 + b\*d^3)\*arccsch(c\*x))\*(f\*x)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^3 (b \operatorname{arcsch}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^3\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^3\*(b\*arccsch(c\*x) + a)\*(f\*x)^m, x)



**maple** [F] time = 0.42, size = 0, normalized size = 0.00

$$\int (fx)^m (ex^2 + d)^3 (a + b \operatorname{arccsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)^3\*(a+b\*arccsch(c\*x)),x)

[Out] int((f\*x)^m\*(e\*x^2+d)^3\*(a+b\*arccsch(c\*x)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^3\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] a\*e^3\*f^m\*x^7\*x^m/(m + 7) + 3\*a\*d\*e^2\*f^m\*x^5\*x^m/(m + 5) + 3\*a\*d^2\*e\*f^m\*x^3\*x^m/(m + 3) + (f\*x)^(m + 1)\*a\*d^3/(f\*(m + 1)) - (((m^3 + 9\*m^2 + 23\*m + 15)\*b\*e^3\*f^m\*x^7 + 3\*(m^3 + 11\*m^2 + 31\*m + 21)\*b\*d\*e^2\*f^m\*x^5 + 3\*(m^3 + 13\*m^2 + 47\*m + 35)\*b\*d^2\*e\*f^m\*x^3 + (m^3 + 15\*m^2 + 71\*m + 105)\*b\*d^3\*f^m\*x)\*x^m\*log(x) - ((m^3 + 9\*m^2 + 23\*m + 15)\*b\*e^3\*f^m\*x^7 + 3\*(m^3 + 11\*m^2 + 31\*m + 21)\*b\*d\*e^2\*f^m\*x^5 + 3\*(m^3 + 13\*m^2 + 47\*m + 35)\*b\*d^2\*e\*f^m\*x^3 + (m^3 + 15\*m^2 + 71\*m + 105)\*b\*d^3\*f^m\*x)\*x^m\*log(sqrt(c^2\*x^2 + 1) + 1))/((m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105) + integrate(((m^3 + 9\*m^2 + 23\*m + 15)\*b\*c^2\*e^3\*f^m\*x^8 + 3\*(m^3 + 11\*m^2 + 31\*m + 21)\*b\*c^2\*d\*e^2\*f^m\*x^6 + 3\*(m^3 + 13\*m^2 + 47\*m + 35)\*b\*c^2\*d^2\*e\*f^m\*x^4 + (m^3 + 15\*m^2 + 71\*m + 105)\*b\*c^2\*d^3\*f^m\*x^2)\*x^m/((m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*c^2\*x^2 + m^4 + 16\*m^3 + 86\*m^2 + ((m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*c^2\*x^2 + m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*sqrt(c^2\*x^2 + 1) + 176\*m + 105), x) - integrate(((m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*b\*c^2\*e^3\*f^m\*x^8\*log(c) + (3\*(m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*c^2\*d\*e^2\*f^m\*log(c) + (m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*e^3\*f^m\*log(c) - (m^3 + 9\*m^2 + 23\*m + 15)\*e^3\*f^m)\*b\*x^6 + 3\*((m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*c^2\*d^2\*e\*f^m\*log(c) + (m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*d\*e^2\*f^m\*log(c) - (m^3 + 11\*m^2 + 31\*m + 21)\*d\*e^2\*f^m)\*b\*x^4 + ((m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*c^2\*d^3\*f^m\*log(c) + 3\*(m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*d^2\*e\*f^m\*log(c) - 3\*(m^3 + 13\*m^2 + 47\*m + 35)\*d^2\*e\*f^m)\*b\*x^2 + ((m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*d^3\*f^m\*log(c) - (m^3 + 15\*m^2 + 71\*m + 105)\*d^3\*f^m)\*b)\*x^m/((m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105)\*c^2\*x^2 + m^4 + 16\*m^3 + 86\*m^2 + 176\*m + 105), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (fx)^m (ex^2 + d)^3 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(d + e\*x^2)^3\*(a + b\*asinh(1/(c\*x))),x)

[Out] int((f\*x)^m\*(d + e\*x^2)^3\*(a + b\*asinh(1/(c\*x))),x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(e\*x\*\*2+d)\*\*3\*(a+b\*acsch(c\*x)),x)

[Out] Timed out

### 3.166 $\int (fx)^m (d + ex^2)^2 (a + bcsch^{-1}(cx)) dx$

**Optimal.** Leaf size=379

$$\frac{d^2(fx)^{m+1} (a + bcsch^{-1}(cx))}{f(m+1)} + \frac{2de(fx)^{m+3} (a + bcsch^{-1}(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5} (a + bcsch^{-1}(cx))}{f^5(m+5)} + \frac{be^2x\sqrt{-c^2x^2-1}(fx)^m}{cf^3(m+4)(m+5)}$$

[Out]  $d^2*(f*x)^{(1+m)}*(a+b*arccsch(c*x))/f/(1+m)+2*d*e*(f*x)^{(3+m)}*(a+b*arccsch(c*x))/f^3/(3+m)+e^2*(f*x)^{(5+m)}*(a+b*arccsch(c*x))/f^5/(5+m)-b*e*(e*(3+m)^2-2*c^2*d*(m^2+9*m+20))*x*(f*x)^{(1+m)}*(-c^2*x^2-1)^{(1/2)}/c^3/f/(4+m)/(5+m)/(m^2+5*m+6)/(-c^2*x^2)^{(1/2)}+b*e^2*x*(f*x)^{(3+m)}*(-c^2*x^2-1)^{(1/2)}/c/f^3/(4+m)/(5+m)/(-c^2*x^2)^{(1/2)}-b*(c^4*d^2*(2+m)*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(3+m)^2-2*c^2*d*(m^2+9*m+20)))*x*(f*x)^{(1+m)}*hypergeom([1/2, 1/2+1/2*m], [3/2+1/2*m], -c^2*x^2)*(c^2*x^2+1)^{(1/2)}/c^3/f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(-c^2*x^2)^{(1/2)}/(-c^2*x^2-1)^{(1/2)}$

**Rubi [A]** time = 0.49, antiderivative size = 360, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {270, 6302, 12, 1267, 459, 365, 364}

$$\frac{d^2(fx)^{m+1} (a + bcsch^{-1}(cx))}{f(m+1)} + \frac{2de(fx)^{m+3} (a + bcsch^{-1}(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5} (a + bcsch^{-1}(cx))}{f^5(m+5)} - \frac{bcx\sqrt{c^2x^2+1}(fx)^m}{cf^3(m+4)(m+5)}$$

Antiderivative was successfully verified.

[In] Int[(f\*x)^m\*(d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]), x]

[Out]  $-((b*e*(e*(3+m)^2-2*c^2*d*(20+9*m+m^2))*x*(f*x)^{(1+m)}*Sqrt[-1-c^2*x^2])/(c^3*f*(2+m)*(3+m)*(4+m)*(5+m)*Sqrt[-(c^2*x^2)])+(b*e^2*x*(f*x)^{(3+m)}*Sqrt[-1-c^2*x^2])/(c*f^3*(4+m)*(5+m)*Sqrt[-(c^2*x^2)])+(d^2*(f*x)^{(1+m)}*(a+b*ArcCsch[c*x]))/(f*(1+m))+(2*d*e*(f*x)^{(3+m)}*(a+b*ArcCsch[c*x]))/(f^3*(3+m))+(e^2*(f*x)^{(5+m)}*(a+b*ArcCsch[c*x]))/(f^5*(5+m))-(b*c*(d^2/(1+m)^2+(e*(e*(3+m)^2-2*c^2*d*(20+9*m+m^2)))/(c^4*(2+m)*(3+m)*(4+m)*(5+m)))*x*(f*x)^{(1+m)}*Sqrt[1+c^2*x^2]*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, -(c^2*x^2)]/(f*Sqrt[-(c^2*x^2)]*Sqrt[-1-c^2*x^2])$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 270

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

#### Rule 364

Int[((c\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -(b\*x^n)/a])/(c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILTQ[p, 0] || GtQ[a, 0])

#### Rule 365

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

### Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

### Rule 1267

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
+ 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x]
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]
&& !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

### Rule 6302

```
Int[((a_) + ArcSch[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(
x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSch[c*x], u, x] - Dist[(b*c*x)/Sqrt[-(c^2*x^2)], Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
|| (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

### Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^2)^2 (a + bcsch^{-1}(cx)) dx &= \frac{d^2(fx)^{1+m} (a + bcsch^{-1}(cx))}{f(1+m)} + \frac{2de(fx)^{3+m} (a + bcsch^{-1}(cx))}{f^3(3+m)} + \frac{e^2(fx)^{5+m} (a + bcsch^{-1}(cx))}{f^5(5+m)} \\
&= \frac{d^2(fx)^{1+m} (a + bcsch^{-1}(cx))}{f(1+m)} + \frac{2de(fx)^{3+m} (a + bcsch^{-1}(cx))}{f^3(3+m)} + \frac{e^2(fx)^{5+m} (a + bcsch^{-1}(cx))}{f^5(5+m)} \\
&= \frac{be^2x(fx)^{3+m} \sqrt{-1 - c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{d^2(fx)^{1+m} (a + bcsch^{-1}(cx))}{f(1+m)} + \frac{2de(fx)^{3+m} (a + bcsch^{-1}(cx))}{f^3(3+m)} \\
&= -\frac{be(e(3+m)^2 - 2c^2d(20 + 9m + m^2))x(fx)^{1+m} \sqrt{-1 - c^2x^2}}{c^3f(2+m)(4+m)(15 + 8m + m^2)\sqrt{-c^2x^2}} + \frac{e^2(fx)^{5+m} (a + bcsch^{-1}(cx))}{f^5(5+m)} \\
&= -\frac{be(e(3+m)^2 - 2c^2d(20 + 9m + m^2))x(fx)^{1+m} \sqrt{-1 - c^2x^2}}{c^3f(2+m)(4+m)(15 + 8m + m^2)\sqrt{-c^2x^2}} + \frac{e^2(fx)^{5+m} (a + bcsch^{-1}(cx))}{f^5(5+m)} \\
&= -\frac{be(e(3+m)^2 - 2c^2d(20 + 9m + m^2))x(fx)^{1+m} \sqrt{-1 - c^2x^2}}{c^3f(2+m)(4+m)(15 + 8m + m^2)\sqrt{-c^2x^2}} + \frac{e^2(fx)^{5+m} (a + bcsch^{-1}(cx))}{f^5(5+m)}
\end{aligned}$$

**Mathematica** [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2)^2 (a + bcsch^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)^2\*(a + b\*ArcCsch[c\*x]), x]

**fricas** [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2) \operatorname{arcsch}(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral((a\*e^2\*x^4 + 2\*a\*d\*e\*x^2 + a\*d^2 + (b\*e^2\*x^4 + 2\*b\*d\*e\*x^2 + b\*d^2)\*arccsch(c\*x))\*(f\*x)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^2 (b \operatorname{arcsch}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^2\*(b\*arccsch(c\*x) + a)\*(f\*x)^m, x)

**maple** [F] time = 0.41, size = 0, normalized size = 0.00

$$\int (fx)^m (ex^2 + d)^2 (a + b \operatorname{arccsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)), x)

[Out] int((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^2f^m x^5 x^m}{m+5} + \frac{2adef^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} ad^2}{f(m+1)} - \frac{((m^2 + 4m + 3)be^2f^m x^5 + 2(m^2 + 6m + 5)bdef^m x^3 + (m^2 + 8m + 1))}{(m^3 + 9m^2 + 23m + 15) \sqrt{c^2 x^2 + 1} + 1} + \text{integrate}(((m^2 + 4m + 3)*b*c^2*e^2*f^m*x^6 + 2*(m^2 + 6m + 5)*b*c^2*d*e*f^m*x^4 + (m^2 + 8m + 15)*b*c^2*d^2*f^m*x^2)*x^m/((m^3 + 9m^2 + 23m + 15)*c^2*x^2 + m^3 + 9m^2 + 23m + 15)*\sqrt{c^2*x^2 + 1} + 23m + 15), x) - \text{integrate}(((m^3 + 9m^2 + 23m + 15)*b*c^2*e^2*f^m*x^6*\log(c) + (2*(m^3 + 9m^2 + 23m + 15)*c^2*d*e*f^m*\log(c) + ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^2\*(a+b\*arccsch(c\*x)), x, algorithm="maxima")

[Out] a\*e^2\*f^m\*x^5\*x^m/(m + 5) + 2\*a\*d\*e\*f^m\*x^3\*x^m/(m + 3) + (f\*x)^(m + 1)\*a\*d^2/(f\*(m + 1)) - (((m^2 + 4\*m + 3)\*b\*e^2\*f^m\*x^5 + 2\*(m^2 + 6\*m + 5)\*b\*d\*e\*f^m\*x^3 + (m^2 + 8\*m + 15)\*b\*d^2\*f^m\*x)\*x^m\*log(x) - ((m^2 + 4\*m + 3)\*b\*e^2\*f^m\*x^5 + 2\*(m^2 + 6\*m + 5)\*b\*d\*e\*f^m\*x^3 + (m^2 + 8\*m + 15)\*b\*d^2\*f^m\*x)\*x^m\*log(sqrt(c^2\*x^2 + 1) + 1))/(m^3 + 9\*m^2 + 23\*m + 15) + integrate(((m^2 + 4\*m + 3)\*b\*c^2\*e^2\*f^m\*x^6 + 2\*(m^2 + 6\*m + 5)\*b\*c^2\*d\*e\*f^m\*x^4 + (m^2 + 8\*m + 15)\*b\*c^2\*d^2\*f^m\*x^2)\*x^m/((m^3 + 9\*m^2 + 23\*m + 15)\*c^2\*x^2 + m^3 + 9\*m^2 + 23\*m + 15)\*sqrt(c^2\*x^2 + 1) + 23\*m + 15), x) - integrate(((m^3 + 9\*m^2 + 23\*m + 15)\*b\*c^2\*e^2\*f^m\*x^6\*log(c) + (2\*(m^3 + 9\*m^2 + 23\*m + 15)\*c^2\*d\*e\*f^m\*log(c) + (

$$m^3 + 9m^2 + 23m + 15)e^{2f^m} \log(c) - (m^2 + 4m + 3)e^{2f^m} b x^4 + ((m^3 + 9m^2 + 23m + 15)c^2 d^2 f^m \log(c) + 2(m^3 + 9m^2 + 23m + 15) d e f^m \log(c) - 2(m^2 + 6m + 5) d e f^m b x^2 + ((m^3 + 9m^2 + 23m + 15) d^2 f^m \log(c) - (m^2 + 8m + 15) d^2 f^m b) x^m / ((m^3 + 9m^2 + 23m + 15) c^2 x^2 + m^3 + 9m^2 + 23m + 15), x)$$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (fx)^m (ex^2 + d)^2 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))), x)

[Out] int((f\*x)^m\*(d + e\*x^2)^2\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{acsch}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(e\*x\*\*2+d)\*\*2\*(a+b\*acsch(c\*x)), x)

[Out] Integral((f\*x)\*\*m\*(a + b\*acsch(c\*x))\*(d + e\*x\*\*2)\*\*2, x)

### 3.167 $\int (fx)^m (d + ex^2) (a + bcsch^{-1}(cx)) dx$

**Optimal.** Leaf size=220

$$\frac{d(fx)^{m+1} (a + bcsch^{-1}(cx))}{f(m+1)} + \frac{e(fx)^{m+3} (a + bcsch^{-1}(cx))}{f^3(m+3)} + \frac{bx\sqrt{c^2x^2+1} (fx)^{m+1} (e(m+1)^2 - c^2d(m+2)(m+3))}{cf(m+1)^2(m+2)(m+3)\sqrt{-c^2x^2-1}}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccsch(c*x))/f
^3/(3+m)+b*e*x*(f*x)^(1+m)*(-c^2*x^2-1)^(1/2)/c/f/(m^2+5*m+6)/(-c^2*x^2)^(1
/2)+b*(e*(1+m)^2-c^2*d*(2+m)*(3+m))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m
], [3/2+1/2*m], -c^2*x^2*(c^2*x^2+1)^(1/2)/c/f/(1+m)^2/(2+m)/(3+m)/(-c^2*x^2
)^(1/2)/(-c^2*x^2-1)^(1/2)
```

**Rubi [A]** time = 0.22, antiderivative size = 208, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {14, 6302, 12, 459, 365, 364}

$$\frac{d(fx)^{m+1} (a + bcsch^{-1}(cx))}{f(m+1)} + \frac{e(fx)^{m+3} (a + bcsch^{-1}(cx))}{f^3(m+3)} - \frac{bcx\sqrt{c^2x^2+1} (fx)^{m+1} \left( \frac{d}{(m+1)^2} - \frac{e}{c^2(m+2)(m+3)} \right) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} + \frac{m}{2}, \frac{1}{2} + \frac{m}{2}, -c^2x^2\right)}{f\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]), x]
```

```
[Out] (b*e*x*(f*x)^(1 + m)*Sqrt[-1 - c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[-(c^2*x^
2)]) + (d*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m
)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) - (b*c*(d/(1 + m)^2 - e/(c^2*(2 + m)*
(3 + m)))*x*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)]/(f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

#### Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

#### Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

+ 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 6302

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_.)]\*(b\_.))\*((f\_.)\*(x\_.))^(m\_.)\*((d\_.) + (e\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> With[{u = IntHide[(f\*x)^m\*(d + e\*x^2)^p, x]}, Dist[a + b\*ArcCsch[c\*x], u, x] - Dist[(b\*c\*x)/Sqrt[-(c^2\*x^2)], Int[SimplifyIntegrand[u/(x\*Sqrt[-1 - c^2\*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2\*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2\*p + 3, 0])) || (ILtQ[(m + 2\*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

### Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^2) (a + bcsch^{-1}(cx)) dx &= \frac{d(fx)^{1+m} (a + bcsch^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + bcsch^{-1}(cx))}{f^3(3+m)} - \frac{(bcx)}{f} \\ &= \frac{d(fx)^{1+m} (a + bcsch^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + bcsch^{-1}(cx))}{f^3(3+m)} - \frac{(bcx)}{f} \\ &= \frac{bex(fx)^{1+m} \sqrt{-1 - c^2x^2}}{cf(6 + 5m + m^2) \sqrt{-c^2x^2}} + \frac{d(fx)^{1+m} (a + bcsch^{-1}(cx))}{f(1+m)} + \frac{e(fx)^3}{f} \\ &= \frac{bex(fx)^{1+m} \sqrt{-1 - c^2x^2}}{cf(6 + 5m + m^2) \sqrt{-c^2x^2}} + \frac{d(fx)^{1+m} (a + bcsch^{-1}(cx))}{f(1+m)} + \frac{e(fx)^3}{f} \\ &= \frac{bex(fx)^{1+m} \sqrt{-1 - c^2x^2}}{cf(6 + 5m + m^2) \sqrt{-c^2x^2}} + \frac{d(fx)^{1+m} (a + bcsch^{-1}(cx))}{f(1+m)} + \frac{e(fx)^3}{f} \end{aligned}$$

**Mathematica** [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2) (a + bcsch^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)\*(a + b\*ArcCsch[c\*x]), x]

**fricas** [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \operatorname{arcsch}(cx)\right) (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*(f\*x)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)(b \operatorname{arcsch}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="giac")

[Out] integrate((e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*(f\*x)^m, x)

**maple** [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^m (ex^2 + d)(a + b \operatorname{arccsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x)

[Out] int((f\*x)^m\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae f^m x^3 x^m}{m+3} + \frac{(fx)^{m+1} ad}{f(m+1)} - \frac{(be f^m (m+1)x^3 + bdf^m (m+3)x)x^m \log(x) - (be f^m (m+1)x^3 + bdf^m (m+3)x)x^m \log}{m^2 + 4m + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] a\*e\*f^m\*x^3\*x^m/(m + 3) + (f\*x)^(m + 1)\*a\*d/(f\*(m + 1)) - ((b\*e\*f^m\*(m + 1)\*x^3 + b\*d\*f^m\*(m + 3)\*x)\*x^m\*log(x) - (b\*e\*f^m\*(m + 1)\*x^3 + b\*d\*f^m\*(m + 3)\*x)\*x^m\*log(sqrt(c^2\*x^2 + 1) + 1))/(m^2 + 4\*m + 3) + integrate((b\*c^2\*e\*f^m\*(m + 1)\*x^4 + b\*c^2\*d\*f^m\*(m + 3)\*x^2)\*x^m/((m^2 + 4\*m + 3)\*c^2\*x^2 + m^2 + (m^2 + 4\*m + 3)\*c^2\*x^2 + m^2 + 4\*m + 3)\*sqrt(c^2\*x^2 + 1) + 4\*m + 3), x) - integrate(((m^2 + 4\*m + 3)\*b\*c^2\*e\*f^m\*x^4\*log(c) + ((m^2 + 4\*m + 3)\*c^2\*d\*f^m\*log(c) + (m^2 + 4\*m + 3)\*e\*f^m\*log(c) - e\*f^m\*(m + 1))\*b\*x^2 + (m^2 + 4\*m + 3)\*d\*f^m\*log(c) - d\*f^m\*(m + 3))\*b)\*x^m/((m^2 + 4\*m + 3)\*c^2\*x^2 + m^2 + 4\*m + 3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (fx)^m (ex^2 + d) \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int((f\*x)^m\*(d + e\*x^2)\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{acsch}(cx))(d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(e\*x\*\*2+d)\*(a+b\*acsch(c\*x)),x)

[Out] Integral((f\*x)\*\*m\*(a + b\*acsch(c\*x))\*(d + e\*x\*\*2), x)



$$3.168 \quad \int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{d + ex^2} dx$$

Optimal. Leaf size=26

$$\operatorname{Int} \left( \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{d + ex^2}, x \right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d), x)

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{d + ex^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2), x]

[Out] Defer[Int][((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2), x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{d + ex^2} dx = \int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{d + ex^2} dx$$

Mathematica [A] time = 1.87, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{d + ex^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2), x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2), x]

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{ex^2 + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d), x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arccsch}(cx))}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d), x)

[Out] int((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d), x, algorithm="maxima")

[Out] integrate((b\*arccsch(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^m \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f\*x)^m\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2), x)

[Out] int(((f\*x)^m\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{acsch}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d), x)

[Out] Integral((f\*x)\*\*m\*(a + b\*acsch(c\*x))/(d + e\*x\*\*2), x)

$$3.169 \quad \int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx$$

**Optimal.** Leaf size=26

$$\operatorname{Int} \left( \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2}, x \right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

**Rubi [A]** time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2,x]

[Out] Defer[Int][((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2, x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx$$

**Mathematica [A]** time = 6.96, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2,x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^2, x]

**fricas [A]** time = 0.88, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{e^2 x^4 + 2 d e x^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)\*(f\*x)^m/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d)^2, x)

**maple** [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

[Out] int((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^2,x, algorithm="maxima")

[Out] integrate((b\*arccsch(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d)^2, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^m \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f\*x)^m\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2,x)

[Out] int(((f\*x)^m\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*2,x)

[Out] Timed out

$$3.170 \quad \int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

**Optimal.** Leaf size=28

$$\operatorname{Int}\left((d + ex^2)^{3/2} (fx)^m (a + b \operatorname{csch}^{-1}(cx)), x\right)$$

[Out] Unintegrable((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x)

**Rubi [A]** time = 0.12, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] Defer[Int] [(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

Rubi steps

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx = \int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

**Mathematica [A]** time = 1.15, size = 0, normalized size = 0.00

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[(f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*ArcCsch[c\*x]), x]

**fricas [A]** time = 0.56, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \operatorname{arcsch}(cx)\right) \sqrt{ex^2 + d} (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x, algorithm="fricas")

[Out] integral((a\*e\*x^2 + a\*d + (b\*e\*x^2 + b\*d)\*arccsch(c\*x))\*sqrt(e\*x^2 + d)\*(f\*x)^m, x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)), x, algorithm="giac")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)\*(f\*x)^m, x)

**maple** [A] time = 0.45, size = 0, normalized size = 0.00

$$\int (fx)^m (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arccsch}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x)

[Out] int((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arsch}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(e\*x^2+d)^(3/2)\*(a+b\*arccsch(c\*x)),x, algorithm="maxima")

[Out] integrate((e\*x^2 + d)^(3/2)\*(b\*arccsch(c\*x) + a)\*(f\*x)^m, x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (fx)^m (ex^2 + d)^{3/2} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))),x)

[Out] int((f\*x)^m\*(d + e\*x^2)^(3/2)\*(a + b\*asinh(1/(c\*x))), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(e\*x\*\*2+d)\*\*(3/2)\*(a+b\*acsch(c\*x)),x)

[Out] Timed out

$$3.171 \quad \int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\sqrt{d + ex^2} (fx)^m (a + b \operatorname{csch}^{-1}(cx)), x\right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x)

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

[Out] Defer[Int][(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

Rubi steps

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx = \int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Mathematica [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{csch}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

[Out] Integrate[(f\*x)^m\*Sqrt[d + e\*x^2]\*(a + b\*ArcCsch[c\*x]), x]

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a) (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*(f\*x)^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))\*(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*(f\*x)^m, x)

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{arccsch}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)`

[Out] `int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)`

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)`

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (fx)^m \sqrt{ex^2 + d} \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)`

[Out] `int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)`

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{acsch}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)`

[Out] `Integral((f*x)**m*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)`



$$3.172 \quad \int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

**Optimal.** Leaf size=28

$$\operatorname{Int} \left( \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}}, x \right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x)

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Defer[Int] [((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx = \int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

**Mathematica [A]** time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/Sqrt[d + e\*x^2], x]

**fricas [A]** time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{\sqrt{ex^2 + d}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="fricas")

[Out] integral((b\*arccsch(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

**maple** [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arccsch}(cx))}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x)

[Out] int((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arsch}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(1/2), x, algorithm="maxima")

[Out] integrate((b\*arccsch(c\*x) + a)\*(f\*x)^m/sqrt(e\*x^2 + d), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^m \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f\*x)^m\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(1/2), x)

[Out] int(((f\*x)^m\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(1/2), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{acsch}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(1/2), x)

[Out] Integral((f\*x)\*\*m\*(a + b\*acsch(c\*x))/sqrt(d + e\*x\*\*2), x)

$$3.173 \quad \int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Optimal.** Leaf size=28

$$\operatorname{Int} \left( \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x)

**Rubi [A]** time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Defer[Int][((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

**Mathematica [A]** time = 1.87, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

[Out] Integrate[((f\*x)^m\*(a + b\*ArcCsch[c\*x]))/(d + e\*x^2)^(3/2), x]

**fricas [A]** time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( \frac{\sqrt{ex^2 + d} (b \operatorname{arcsch}(cx) + a) (fx)^m}{e^2 x^4 + 2 dex^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2), x, algorithm="fricas")

[Out] integral(sqrt(e\*x^2 + d)\*(b\*arccsch(c\*x) + a)\*(f\*x)^m/(e^2\*x^4 + 2\*d\*e\*x^2 + d^2), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a) (fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d)^(3/2), x)

**maple** [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arccsch}(cx))}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x)

[Out] int((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)^m\*(a+b\*arccsch(c\*x))/(e\*x^2+d)^(3/2),x, algorithm="maxima")

[Out] integrate((b\*arccsch(c\*x) + a)\*(f\*x)^m/(e\*x^2 + d)^(3/2), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(fx)^m \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{(ex^2 + d)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((f\*x)^m\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2),x)

[Out] int(((f\*x)^m\*(a + b\*asinh(1/(c\*x))))/(d + e\*x^2)^(3/2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x)\*\*m\*(a+b\*acsch(c\*x))/(e\*x\*\*2+d)\*\*(3/2),x)

[Out] Timed out

$$3.174 \quad \int \frac{x^{11}(a+b\operatorname{csch}^{-1}(cx))}{\sqrt{1-c^4x^4}} dx$$

**Optimal.** Leaf size=395

$$\frac{(1-c^4x^4)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b\operatorname{csch}^{-1}(cx))}{2c^{12}} - \frac{b\sqrt{c^2x^2+1}}{90c^{13}x}$$

[Out] 1/3\*(-c^4\*x^4+1)^(3/2)\*(a+b\*arccsch(c\*x))/c^12-1/10\*(-c^4\*x^4+1)^(5/2)\*(a+b\*arccsch(c\*x))/c^12+7/90\*b\*(-c^2\*x^2+1)^(3/2)\*(c^2\*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)^(1/2)-13/150\*b\*(-c^2\*x^2+1)^(5/2)\*(c^2\*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)^(1/2)+3/70\*b\*(-c^2\*x^2+1)^(7/2)\*(c^2\*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)^(1/2)-1/90\*b\*(-c^2\*x^2+1)^(9/2)\*(c^2\*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)^(1/2)+4/15\*b\*arctanh((-c^2\*x^2+1)^(1/2))\*(c^2\*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)^(1/2)-4/15\*b\*(-c^2\*x^2+1)^(1/2)\*(c^2\*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)^(1/2)-1/2\*(a+b\*arccsch(c\*x))\*(-c^4\*x^4+1)^(1/2)/c^12

**Rubi [A]** time = 2.34, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {266, 43, 6310, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$\frac{(1-c^4x^4)^{5/2}(a+b\operatorname{csch}^{-1}(cx))}{10c^{12}} + \frac{(1-c^4x^4)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1-c^4x^4}(a+b\operatorname{csch}^{-1}(cx))}{2c^{12}} - \frac{b\sqrt{c^2x^2+1}}{90c^{13}x}$$

Antiderivative was successfully verified.

[In] Int[(x^11\*(a + b\*ArcCsch[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out] (-4\*b\*Sqrt[1 - c^2\*x^2]\*Sqrt[1 + c^2\*x^2])/(15\*c^13\*Sqrt[1 + 1/(c^2\*x^2)]\*x) + (7\*b\*(1 - c^2\*x^2)^(3/2)\*Sqrt[1 + c^2\*x^2])/(90\*c^13\*Sqrt[1 + 1/(c^2\*x^2)]\*x) - (13\*b\*(1 - c^2\*x^2)^(5/2)\*Sqrt[1 + c^2\*x^2])/(150\*c^13\*Sqrt[1 + 1/(c^2\*x^2)]\*x) + (3\*b\*(1 - c^2\*x^2)^(7/2)\*Sqrt[1 + c^2\*x^2])/(70\*c^13\*Sqrt[1 + 1/(c^2\*x^2)]\*x) - (b\*(1 - c^2\*x^2)^(9/2)\*Sqrt[1 + c^2\*x^2])/(90\*c^13\*Sqrt[1 + 1/(c^2\*x^2)]\*x) - (Sqrt[1 - c^4\*x^4]\*(a + b\*ArcCsch[c\*x]))/(2\*c^12) + ((1 - c^4\*x^4)^(3/2)\*(a + b\*ArcCsch[c\*x]))/(3\*c^12) - ((1 - c^4\*x^4)^(5/2)\*(a + b\*ArcCsch[c\*x]))/(10\*c^12) + (4\*b\*Sqrt[1 + c^2\*x^2]\*ArcTanh[Sqrt[1 - c^2\*x^2]])/(15\*c^13\*Sqrt[1 + 1/(c^2\*x^2)]\*x)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*(c + d\*x)^n/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n

+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 266

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)\*(a + b\*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

### Rule 783

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[(d + e\*x)^(m + p)\*(f + g\*x)\*(a/d + (c\*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c\*d^2 + a\*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

### Rule 848

Int[((d\_) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))^(n\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[(d + e\*x)^(m + p)\*(f + g\*x)^n\*(a/d + (c\*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e\*f - d\*g, 0] && EqQ[c\*d^2 + a\*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

### Rule 6310

Int[((a\_.) + ArcCsch[(c\_.)\*(x\_)])\*(b\_.)\*(u\_), x\_Symbol] := With[{v = IntHide[u, x]}, Dist[a + b\*ArcCsch[c\*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2\*Sqrt[1 + 1/(c^2\*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]

### Rule 6721

Int[(u\_.)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(b^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(x^(n\*FracPart[p])\*(1 + a/(x^n\*b))^FracPart[p]), Int[u\*x^(n\*p)\*(1 + a/(x^n\*b))^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rubi steps

$$\begin{aligned}
\int \frac{x^{11} (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{4c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{4c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{4c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{4c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{4c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{4c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{csch}^{-1}(cx))}{4c^{12}} \\
&= -\frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{1 + \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{3c^{12}} \\
&= -\frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{1 + \frac{1}{c^2 x^2}} x} + \frac{7b(1 - c^2 x^2)^{3/2} \sqrt{1 + c^2 x^2}}{90c^{13} \sqrt{1 + \frac{1}{c^2 x^2}} x} - \frac{13b(1 - c^2 x^2)^{5/2} \sqrt{1 + c^2 x^2}}{150c^{13} \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{1 + \frac{1}{c^2 x^2}} x} + \frac{7b(1 - c^2 x^2)^{3/2} \sqrt{1 + c^2 x^2}}{90c^{13} \sqrt{1 + \frac{1}{c^2 x^2}} x} - \frac{13b(1 - c^2 x^2)^{5/2} \sqrt{1 + c^2 x^2}}{150c^{13} \sqrt{1 + \frac{1}{c^2 x^2}} x}
\end{aligned}$$

**Mathematica [A]** time = 0.31, size = 214, normalized size = 0.54

$$\frac{105a\sqrt{1 - c^4 x^4} (3c^8 x^8 + 4c^4 x^4 + 8) + 840b \log(c^2 x^3 + x) + 105b\sqrt{1 - c^4 x^4} (3c^8 x^8 + 4c^4 x^4 + 8) \operatorname{csch}^{-1}(cx)}{3150c^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11\*(a + b\*ArcCsch[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out] -1/3150\*(105\*a\*Sqrt[1 - c^4\*x^4]\*(8 + 4\*c^4\*x^4 + 3\*c^8\*x^8) + (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 - c^4\*x^4]\*(768 - 36\*c^2\*x^2 + 78\*c^4\*x^4 - 5\*c^6\*x^6 + 35\*c^8\*x^8))/(1 + c^2\*x^2) + 105\*b\*Sqrt[1 - c^4\*x^4]\*(8 + 4\*c^4\*x^4 +

$3c^8x^8) \cdot \text{ArcCsch}[cx] + 840b \cdot \text{Log}[x + c^2x^3] - 840b \cdot \text{Log}[1 + c^2x^2 + c\sqrt{1 + 1/(c^2x^2)}] \cdot x \cdot \sqrt{1 - c^4x^4}] / c^{12}$

**fricas** [A] time = 0.61, size = 382, normalized size = 0.97

$$105 \left( 3bc^{10}x^{10} + 3bc^8x^8 + 4bc^6x^6 + 4bc^4x^4 + 8bc^2x^2 + 8b \right) \sqrt{-c^4x^4 + 1} \log \left( \frac{cx \sqrt{\frac{c^2x^2+1}{c^2x^2}} + 1}{cx} \right) + (35bc^9x^9 - 5bc^7x^7)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
[Out] -1/3150*(105*(3*b*c^10*x^10 + 3*b*c^8*x^8 + 4*b*c^6*x^6 + 4*b*c^4*x^4 + 8*b*c^2*x^2 + 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (35*b*c^9*x^9 - 5*b*c^7*x^7 + 78*b*c^5*x^5 - 36*b*c^3*x^3 + 76*8*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 420*(b*c^2*x^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 420*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 105*(3*a*c^10*x^10 + 3*a*c^8*x^8 + 4*a*c^6*x^6 + 4*a*c^4*x^4 + 8*a*c^2*x^2 + 8*a)*sqrt(-c^4*x^4 + 1))/(c^14*x^2 + c^12)
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value
```

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x^{11} (a + b \operatorname{arccsch}(cx))}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
[Out] int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{30} a \left( \frac{3(-c^4x^4 + 1)^{\frac{5}{2}}}{c^{12}} - \frac{10(-c^4x^4 + 1)^{\frac{3}{2}}}{c^{12}} + \frac{15\sqrt{-c^4x^4 + 1}}{c^{12}} \right) + \frac{1}{30} b \left( \frac{(3c^{12}x^{12} + c^8x^8 + 4c^4x^4 - 8) \log(\sqrt{c^2x^2 + 1})}{\sqrt{c^2x^2 + 1} \sqrt{cx + 1} \sqrt{-cx + 1} c^{12}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sqrt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(cx + 1)*sqrt(-cx + 1)*c^12) - 30*integrate((x^11*log(c) + x^11*log(x))*e^(-1/2*log(c^2*x^2 + 1)) - 1
```



$/2*\log(c*x + 1) - 1/2*\log(-c*x + 1)), x) - 30*\integrate(1/30*(3*c^{10}*x^{11} - 3*c^8*x^9 + 4*c^6*x^7 - 4*c^4*x^5 + 8*c^2*x^3 - 8*x)/(\sqrt{c^2*x^2 + 1}*\sqrt{c*x + 1}*\sqrt{-c*x + 1})*c^{10} + \sqrt{c*x + 1}*\sqrt{-c*x + 1}*c^{10}), x)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11} \left( a + b \operatorname{asinh} \left( \frac{1}{cx} \right) \right)}{\sqrt{1 - c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^11\*(a + b\*asinh(1/(c\*x))))/(1 - c^4\*x^4)^(1/2), x)

[Out] int((x^11\*(a + b\*asinh(1/(c\*x))))/(1 - c^4\*x^4)^(1/2), x)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*11\*(a+b\*acsch(c\*x))/(-c\*\*4\*x\*\*4+1)\*\*(1/2), x)

[Out] Timed out

$$3.175 \quad \int \frac{x^7 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$$

**Optimal.** Leaf size=264

$$\frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} - \frac{b \sqrt{c^2 x^2 + 1} (1 - c^2 x^2)^{5/2}}{30c^9 x \sqrt{\frac{1}{c^2 x^2} + 1}} + \frac{b \sqrt{c^2 x^2 + 1} (1 - c^2 x^2)^{5/2}}{18c^9 x \sqrt{\frac{1}{c^2 x^2} + 1}}$$

[Out]  $\frac{1}{6}(-c^4 x^4 + 1)^{3/2} (a + b \operatorname{arccsch}(c x)) / c^8 + \frac{1}{18} b (-c^2 x^2 + 1)^{3/2} (c^2 x^2 + 1)^{1/2} / c^9 x / (1 + 1/c^2 x^2)^{1/2} - \frac{1}{30} b (-c^2 x^2 + 1)^{5/2} (c^2 x^2 + 1)^{1/2} / c^9 x / (1 + 1/c^2 x^2)^{1/2} + \frac{1}{3} b \operatorname{arctanh}((-c^2 x^2 + 1)^{1/2}) (c^2 x^2 + 1)^{1/2} / c^9 x / (1 + 1/c^2 x^2)^{1/2} - \frac{1}{3} b (-c^2 x^2 + 1)^{1/2} (c^2 x^2 + 1)^{1/2} / c^9 x / (1 + 1/c^2 x^2)^{1/2} - \frac{1}{2} (a + b \operatorname{arccsch}(c x)) (-c^4 x^4 + 1)^{1/2} / c^8$

**Rubi [A]** time = 1.92, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {266, 43, 6310, 12, 6721, 6742, 848, 50, 63, 208, 783}

$$\frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} - \frac{b \sqrt{c^2 x^2 + 1} (1 - c^2 x^2)^{5/2}}{30c^9 x \sqrt{\frac{1}{c^2 x^2} + 1}} + \frac{b \sqrt{c^2 x^2 + 1} (1 - c^2 x^2)^{5/2}}{18c^9 x \sqrt{\frac{1}{c^2 x^2} + 1}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^7 (a + b \operatorname{ArcCsch}[c x])) / \operatorname{Sqrt}[1 - c^4 x^4], x]$

[Out]  $-(b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{Sqrt}[1 + c^2 x^2]) / (3 c^9 \operatorname{Sqrt}[1 + 1 / (c^2 x^2)] x) + (b (1 - c^2 x^2)^{3/2} \operatorname{Sqrt}[1 + c^2 x^2]) / (18 c^9 \operatorname{Sqrt}[1 + 1 / (c^2 x^2)] x) - (b (1 - c^2 x^2)^{5/2} \operatorname{Sqrt}[1 + c^2 x^2]) / (30 c^9 \operatorname{Sqrt}[1 + 1 / (c^2 x^2)] x) - (\operatorname{Sqrt}[1 - c^4 x^4] (a + b \operatorname{ArcCsch}[c x])) / (2 c^8) + ((1 - c^4 x^4)^{3/2} (a + b \operatorname{ArcCsch}[c x])) / (6 c^8) + (b \operatorname{Sqrt}[1 + c^2 x^2] \operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2 x^2]]) / (3 c^9 \operatorname{Sqrt}[1 + 1 / (c^2 x^2)] x)$

### Rule 12

$\operatorname{Int}[(a_*) (u_*), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_*) (v_*)] /; \operatorname{FreeQ}[b, x]$

### Rule 43

$\operatorname{Int}[(a_*) + (b_*) (x_*)^{(m_*)} ((c_*) + (d_*) (x_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b x)^m (c + d x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n, x\} \&\& \operatorname{NeQ}[b c - a d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (\operatorname{!IntegerQ}[n] \parallel (\operatorname{EqQ}[c, 0] \&\& \operatorname{LeQ}[7 m + 4 n + 4, 0]) \parallel \operatorname{LtQ}[9 m + 5 (n + 1), 0] \parallel \operatorname{GtQ}[m + n + 2, 0])$

### Rule 50

$\operatorname{Int}[(a_*) + (b_*) (x_*)^{(m_*)} ((c_*) + (d_*) (x_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b x)^{m+1} (c + d x)^n / (b (m + n + 1)), x] + \operatorname{Dist}[(n (b c - a d)) / (b (m + n + 1)), \operatorname{Int}[(a + b x)^m (c + d x)^{n-1}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \&\& \operatorname{NeQ}[b c - a d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[m + n + 1, 0] \&\& (\operatorname{IGtQ}[m, 0] \&\& (\operatorname{!IntegerQ}[n] \parallel (\operatorname{GtQ}[m, 0] \&\& \operatorname{LtQ}[m - n, 0]))) \&\& \operatorname{!ILtQ}[m + n + 2, 0] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 63

$\operatorname{Int}[(a_*) + (b_*) (x_*)^{(m_*)} ((c_*) + (d_*) (x_*))^{(n_*)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{p(m+1)-1} (c - (a d)/b +$

$(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}, x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]]/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

#### Rule 266

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p, x], x, x^n], x] /; \text{FreeQ}\{a, b, m, n, p\}, x\} \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

#### Rule 783

$\text{Int}[(d_ + (e_)*(x_))^{(m_)}*((f_ + (g_)*(x_))^{(n_)}*((a_ + (c_)*(x_)^2)^{(p_)}), x\_Symbol] \rightarrow \text{Int}[(d + e*x)^{(m + p)}*(f + g*x)*(a/d + (c*x)/e)^p, x] /; \text{FreeQ}\{a, c, d, e, f, g, m\}, x\} \&\& \text{EqQ}[c*d^2 + a*e^2, 0] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[a, 0] \&\& \text{GtQ}[d, 0] \&\& \text{EqQ}[m + p, 0]))$

#### Rule 848

$\text{Int}[(d_ + (e_)*(x_))^{(m_)}*((f_ + (g_)*(x_))^{(n_)}*((a_ + (c_)*(x_)^2)^{(p_)}), x\_Symbol] \rightarrow \text{Int}[(d + e*x)^{(m + p)}*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; \text{FreeQ}\{a, c, d, e, f, g, m, n\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{EqQ}[c*d^2 + a*e^2, 0] \&\& (\text{IntegerQ}[p] \parallel (\text{GtQ}[a, 0] \&\& \text{GtQ}[d, 0] \&\& \text{EqQ}[m + p, 0]))$

#### Rule 6310

$\text{Int}[(a_ + \text{ArcCsch}[(c_)*(x_)]*(b_))*(u_), x\_Symbol] \rightarrow \text{With}\{v = \text{IntHide}[u, x]\}, \text{Dist}[a + b*\text{ArcCsch}[c*x], v, x] + \text{Dist}[b/c, \text{Int}[\text{SimplifyIntegrand}[v/(x^2*\text{Sqrt}[1 + 1/(c^2*x^2)]), x], x], x] /; \text{InverseFunctionFreeQ}[v, x] /; \text{FreeQ}\{a, b, c\}, x]$

#### Rule 6721

$\text{Int}[(u_)*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x\_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(x^{(n*\text{FracPart}[p])}*(1 + a/(x^n*b))^{\text{FracPart}[p]}), \text{Int}[u*x^{(n*p)}*(1 + a/(x^n*b))^p, x], x] /; \text{FreeQ}\{a, b, p\}, x\} \&\& \text{!IntegerQ}[p] \&\& \text{ILtQ}[n, 0] \&\& \text{!RationalFunctionQ}[u, x] \&\& \text{IntegerQ}[p + 1/2]$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

#### Rubi steps

$$\begin{aligned}
\int \frac{x^7 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} + \frac{b \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{6c^8 \sqrt{1 + \frac{1}{c^2 x^2}}} dx}{c} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} + \frac{b \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c^9} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} + \frac{(b \sqrt{1 + c^2 x^2}) \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} - \frac{(b \sqrt{1 + c^2 x^2}) \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} - \frac{(b \sqrt{1 + c^2 x^2}) \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} - \frac{(b \sqrt{1 + c^2 x^2}) \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} - \frac{(b \sqrt{1 + c^2 x^2}) \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} - \frac{(b \sqrt{1 + c^2 x^2}) \int \frac{(-2 - c^4 x^4) \sqrt{1 - c^4 x^4}}{\sqrt{1 + \frac{1}{c^2 x^2}}} dx}{6c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{b \sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{csch}^{-1}(cx))}{6c^8} \\
&= -\frac{b \sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} + \frac{b (1 - c^2 x^2)^{3/2} \sqrt{1 + c^2 x^2}}{18c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} - \frac{b (1 - c^2 x^2)^{5/2} \sqrt{1 + c^2 x^2}}{30c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} \\
&= -\frac{b \sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} + \frac{b (1 - c^2 x^2)^{3/2} \sqrt{1 + c^2 x^2}}{18c^9 \sqrt{1 + \frac{1}{c^2 x^2}}} - \frac{b (1 - c^2 x^2)^{5/2} \sqrt{1 + c^2 x^2}}{30c^9 \sqrt{1 + \frac{1}{c^2 x^2}}}
\end{aligned}$$

**Mathematica [A]** time = 0.42, size = 180, normalized size = 0.68

$$\frac{15a\sqrt{1 - c^4 x^4} (c^4 x^4 + 2) + 15b\sqrt{1 - c^4 x^4} (c^4 x^4 + 2) \operatorname{csch}^{-1}(cx) + 30b \log(c^2 x^3 + x) + \frac{bcx \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{1 - c^4 x^4} (3c^4 x^4 - c^2 x^2 + 1)}{c^2 x^2 + 1}}{90c^8}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(a + b\*ArcCsch[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out] -1/90\*(15\*a\*Sqrt[1 - c^4\*x^4]\*(2 + c^4\*x^4) + (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 - c^4\*x^4]\*(28 - c^2\*x^2 + 3\*c^4\*x^4))/(1 + c^2\*x^2) + 15\*b\*Sqrt[1 - c^4\*x^4]\*(2 + c^4\*x^4)\*ArcCsch[c\*x] + 30\*b\*Log[x + c^2\*x^3] - 30\*b\*Log[1 + c^2\*x^2 + c\*Sqrt[1 + 1/(c^2\*x^2)]]\*x\*Sqrt[1 - c^4\*x^4])/c^8

**fricas** [A] time = 0.59, size = 324, normalized size = 1.23

$$15 \left( bc^6 x^6 + bc^4 x^4 + 2bc^2 x^2 + 2b \right) \sqrt{-c^4 x^4 + 1} \log \left( \frac{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1}{cx} \right) + \left( 3bc^5 x^5 - bc^3 x^3 + 28bcx \right) \sqrt{-c^4 x^4 + 1} \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x, algorithm="fricas")

[Out] -1/90\*(15\*(b\*c^6\*x^6 + b\*c^4\*x^4 + 2\*b\*c^2\*x^2 + 2\*b)\*sqrt(-c^4\*x^4 + 1)\*log((c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c\*x)) + (3\*b\*c^5\*x^5 - b\*c^3\*x^3 + 28\*b\*c\*x)\*sqrt(-c^4\*x^4 + 1)\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) - 15\*(b\*c^2\*x^2 + b)\*log((c^2\*x^2 + sqrt(-c^4\*x^4 + 1)\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c^2\*x^2 + 1)) + 15\*(b\*c^2\*x^2 + b)\*log(-(c^2\*x^2 - sqrt(-c^4\*x^4 + 1)\*c\*x\*sqrt((c^2\*x^2 + 1)/(c^2\*x^2)) + 1)/(c^2\*x^2 + 1)) + 15\*(a\*c^6\*x^6 + a\*c^4\*x^4 + 2\*a\*c^2\*x^2 + 2\*a)\*sqrt(-c^4\*x^4 + 1))/(c^10\*x^2 + c^8)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index\_m & i,const vecteur & l) Error: Bad Argument Value

**maple** [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x^7 (a + b \operatorname{arccsch}(cx))}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x)

[Out] int(x^7\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} a \left( \frac{(-c^4 x^4 + 1)^{\frac{3}{2}}}{c^8} - \frac{3 \sqrt{-c^4 x^4 + 1}}{c^8} \right) + \frac{1}{6} b \left( \frac{(c^8 x^8 + c^4 x^4 - 2) \log(\sqrt{c^2 x^2 + 1} + 1)}{\sqrt{c^2 x^2 + 1} \sqrt{cx + 1} \sqrt{-cx + 1} c^8} - 6 \int (x^7 \log(c) + x^7 \log(x)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x, algorithm="maxima")

[Out] 1/6\*a\*((-c^4\*x^4 + 1)^(3/2)/c^8 - 3\*sqrt(-c^4\*x^4 + 1)/c^8) + 1/6\*b\*((c^8\*x^8 + c^4\*x^4 - 2)\*log(sqrt(c^2\*x^2 + 1) + 1)/(sqrt(c^2\*x^2 + 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*c^8) - 6\*integrate((x^7\*log(c) + x^7\*log(x))\*e^(-1/2\*log(c^2\*x^2 + 1) - 1/2\*log(c\*x + 1) - 1/2\*log(-c\*x + 1)), x) - 6\*integrate(1/6\*(c^6\*x^7 - c^4\*x^5 + 2\*c^2\*x^3 - 2\*x)/(sqrt(c^2\*x^2 + 1)\*sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*c^6 + sqrt(c\*x + 1)\*sqrt(-c\*x + 1)\*c^6), x))

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{\sqrt{1 - c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

```
[Out] int((x^7*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2), x)
```

```
[Out] Timed out
```

$$3.176 \quad \int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$$

**Optimal.** Leaf size=130

$$-\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} + \frac{bx\sqrt{1 - c^4 x^4}}{2c^3\sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1}} - \frac{bx \tan^{-1}\left(\frac{\sqrt{1 - c^4 x^4}}{\sqrt{-c^2 x^2 - 1}}\right)}{2c^3\sqrt{-c^2 x^2}}$$

[Out]  $-1/2*b*x*\arctan((-c^4*x^4+1)^{(1/2)/(-c^2*x^2-1)^{(1/2)})/c^3/(-c^2*x^2)^{(1/2)}$   
 $-1/2*(a+b*\operatorname{arccsch}(c*x))*(-c^4*x^4+1)^{(1/2)/c^4+1/2*b*x*(-c^4*x^4+1)^{(1/2)/c}$   
 $^3/(-c^2*x^2)^{(1/2)/(-c^2*x^2-1)^{(1/2)}$

**Rubi [A]** time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {261, 6310, 12, 1572, 1252, 848, 50, 63, 208}

$$-\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} - \frac{b\sqrt{1 - c^2 x^2} \sqrt{c^2 x^2 + 1}}{2c^5 x \sqrt{\frac{1}{c^2 x^2} + 1}} + \frac{b\sqrt{c^2 x^2 + 1} \tanh^{-1}\left(\sqrt{1 - c^2 x^2}\right)}{2c^5 x \sqrt{\frac{1}{c^2 x^2} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*ArcCsch[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out]  $-(b*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[1 + c^2*x^2])/(2*c^5*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x) -$   
 $(\operatorname{Sqrt}[1 - c^4*x^4]*(a + b*\operatorname{ArcCsch}[c*x]))/(2*c^4) + (b*\operatorname{Sqrt}[1 + c^2*x^2]*\operatorname{Arc}$   
 $\operatorname{Tanh}[\operatorname{Sqrt}[1 - c^2*x^2]])/(2*c^5*\operatorname{Sqrt}[1 + 1/(c^2*x^2)]*x)$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[  
 $((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + \operatorname{Dist}[(n*(b*c - a*d))/$   
 $(b*(m + n + 1)), \operatorname{Int}[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; \operatorname{FreeQ}\{a, b,$   
 $c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[m + n + 1, 0] \&\& !($   
 $\operatorname{IGtQ}[m, 0] \&\& (!\operatorname{IntegerQ}[n] || (\operatorname{GtQ}[m, 0] \&\& \operatorname{LtQ}[m - n, 0]))) \&\& !\operatorname{ILtQ}[m + n$   
 $+ 2, 0] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[  
 $\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)*(c - (a*d)/b +$   
 $(d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}$   
 $[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Den}$   
 $\operatorname{ominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 261

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := Simp[(a + b\*x^n)^(p + 1)/(b\*n\*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&

NeQ[p, -1]

### Rule 848

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))^(n\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[(d + e\*x)^(m + p)\*(f + g\*x)^n\*(a/d + (c\*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e\*f - d\*g, 0] && EqQ[c\*d^2 + a\*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

### Rule 1252

Int[(x\_)^(m\_)\*((d\_) + (e\_)\*(x\_)^2)^(q\_)\*((a\_) + (c\_)\*(x\_)^4)^(p\_), x\_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)\*(d + e\*x)^q\*(a + c\*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

### Rule 1572

Int[(x\_)^(m\_)\*((d\_) + (e\_)\*(x\_)^(mn\_))^(q\_)\*((a\_) + (c\_)\*(x\_)^(n2\_))^(p\_), x\_Symbol] :> Dist[(e^IntPart[q]\*(d + e\*x^mn)^FracPart[q])/(x^(mn\*FracPart[q]))\*(1 + d/(x^mn\*e))^FracPart[q], Int[x^(m + mn\*q)\*(1 + d/(x^mn\*e))^q\*(a + c\*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && EqQ[n2, -2\*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n2]

### Rule 6310

Int[((a\_) + ArcCsch[(c\_)\*(x\_)])\*(b\_)\*(u\_), x\_Symbol] :> With[{v = IntHide[u, x]}, Dist[a + b\*ArcCsch[c\*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[v/(x^2\*Sqrt[1 + 1/(c^2\*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]

### Rubi steps



$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{csch}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} + \frac{b \int -\frac{\sqrt{1 - c^4 x^4}}{2c^4 \sqrt{1 + \frac{1}{c^2 x^2}} x^2} dx}{c} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} - \frac{b \int \frac{\sqrt{1 - c^4 x^4}}{\sqrt{1 + \frac{1}{c^2 x^2}} x^2} dx}{2c^5} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 + c^2 x^2}) \int \frac{\sqrt{1 - c^4 x^4}}{x\sqrt{1 + c^2 x^2}} dx}{2c^5 \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 + c^2 x^2}) \operatorname{Subst}\left(\int \frac{\sqrt{1 - c^4 x^2}}{x\sqrt{1 + c^2 x}} dx, x, x^2\right)}{4c^5 \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 + c^2 x^2}) \operatorname{Subst}\left(\int \frac{\sqrt{1 - c^2 x}}{x} dx, x, x^2\right)}{4c^5 \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 + \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 + c^2 x^2}) \operatorname{Subst}\left(\int \frac{\sqrt{1 - c^2 x}}{x} dx, x, x^2\right)}{4c^5 \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 + \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} + \frac{(b\sqrt{1 + c^2 x^2}) \operatorname{Subst}\left(\int \frac{\sqrt{1 - c^2 x}}{x} dx, x, x^2\right)}{2c^7 \sqrt{1 + \frac{1}{c^2 x^2}} x} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{2c^5 \sqrt{1 + \frac{1}{c^2 x^2}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{csch}^{-1}(cx))}{2c^4} + \frac{b\sqrt{1 + c^2 x^2} \tanh^{-1}\left(\frac{\sqrt{1 - c^2 x}}{x}\right)}{2c^5 \sqrt{1 + \frac{1}{c^2 x^2}} x}
\end{aligned}$$

**Mathematica [A]** time = 0.34, size = 141, normalized size = 1.08

$$\frac{a\sqrt{1 - c^4 x^4} + b\sqrt{1 - c^4 x^4} \operatorname{csch}^{-1}(cx) + b \log(c^2 x^3 + x) + \frac{bcx \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{1 - c^4 x^4}}{c^2 x^2 + 1} - b \log\left(c^2 x^2 + cx \sqrt{\frac{1}{c^2 x^2} + 1} \sqrt{1 - c^4 x^4}\right)}{2c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*ArcCsch[c\*x]))/Sqrt[1 - c^4\*x^4], x]

[Out] -1/2\*(a\*Sqrt[1 - c^4\*x^4] + (b\*c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 - c^4\*x^4])/(1 + c^2\*x^2) + b\*Sqrt[1 - c^4\*x^4]\*ArcCsch[c\*x] + b\*Log[x + c^2\*x^3] - b\*Log[1 + c^2\*x^2 + c\*Sqrt[1 + 1/(c^2\*x^2)]\*x\*Sqrt[1 - c^4\*x^4]])/c^4

**fricas [B]** time = 0.68, size = 265, normalized size = 2.04

$$\frac{2\sqrt{-c^4 x^4 + 1} bcx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 2\sqrt{-c^4 x^4 + 1} (bc^2 x^2 + b) \log\left(\frac{cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} + 1}{cx}\right) - (bc^2 x^2 + b) \log\left(\frac{c^2 x^2 + \sqrt{-c^4 x^4 + 1} cx \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}}{c^2 x^2 + 1}\right)}{4(c^6 x^2 + c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x, algorithm="fricas")  
 [Out] 
$$-1/4*(2*\sqrt{-c^4*x^4 + 1}*b*c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 2*\sqrt{-c^4*x^4 + 1}*(b*c^2*x^2 + b)*\log((c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c*x)) - (b*c^2*x^2 + b)*\log((c^2*x^2 + \sqrt{-c^4*x^4 + 1}*c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c^2*x^2 + 1)) + (b*c^2*x^2 + b)*\log(-(c^2*x^2 - \sqrt{-c^4*x^4 + 1}*c*x*\sqrt{(c^2*x^2 + 1)/(c^2*x^2)} + 1)/(c^2*x^2 + 1)) + 2*\sqrt{-c^4*x^4 + 1}*(a*c^2*x^2 + a))/(c^6*x^2 + c^4)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x, algorithm="giac")  
 [Out] integrate((b\*arccsch(c\*x) + a)\*x^3/sqrt(-c^4\*x^4 + 1), x)

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{arcsch}(cx))}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x)  
 [Out] int(x^3\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} b \left( \frac{(c^4x^4 - 1) \log(\sqrt{c^2x^2 + 1} + 1)}{\sqrt{c^2x^2 + 1} \sqrt{cx + 1} \sqrt{-cx + 1} c^4} - 2 \int (x^3 \log(c) + x^3 \log(x)) e^{\left(-\frac{1}{2} \log(c^2x^2+1) - \frac{1}{2} \log(cx+1) - \frac{1}{2} \log(-cx+1)\right)} dx - 2 \int \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+b\*arccsch(c\*x))/(-c^4\*x^4+1)^(1/2),x, algorithm="maxima")  
 [Out] 
$$1/2*b*((c^4*x^4 - 1)*\log(\sqrt{c^2*x^2 + 1} + 1)/(\sqrt{c^2*x^2 + 1}*\sqrt{c*x + 1}*\sqrt{-c*x + 1}*c^4) - 2*\integrate((x^3*\log(c) + x^3*\log(x))*e^{(-1/2*\log(c^2*x^2 + 1) - 1/2*\log(c*x + 1) - 1/2*\log(-c*x + 1))}, x) - 2*\integrate(1/2*(c^2*x^3 - x)/(\sqrt{c^2*x^2 + 1}*\sqrt{c*x + 1}*\sqrt{-c*x + 1}*c^2 + \sqrt{c*x + 1}*\sqrt{-c*x + 1}*c^2), x) - 1/2*\sqrt{-c^4*x^4 + 1}*a/c^4$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \left( a + b \operatorname{asinh}\left(\frac{1}{cx}\right) \right)}{\sqrt{1 - c^4x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*asinh(1/(c\*x))))/(1 - c^4\*x^4)^(1/2),x)  
 [Out] int((x^3\*(a + b\*asinh(1/(c\*x))))/(1 - c^4\*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acsch}(cx))}{\sqrt{-(cx - 1)(cx + 1)(c^2x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))  
, x)
```

$$3.177 \quad \int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx$$

**Optimal.** Leaf size=29

$$\operatorname{Int} \left( \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}}, x \right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x/(-c^4\*x^4+1)^(1/2), x)

**Rubi [A]** time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

[Out] Defer[Int] [(a + b\*ArcCsch[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

Rubi steps

$$\int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx = \int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx$$

**Mathematica [A]** time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arcsch}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x\*Sqrt[1 - c^4\*x^4]), x]

**fricas [A]** time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral} \left( -\frac{\sqrt{-c^4 x^4 + 1} (b \operatorname{arcsch}(cx) + a)}{c^4 x^5 - x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(-c^4\*x^4+1)^(1/2), x, algorithm="fricas")

[Out] integral(-sqrt(-c^4\*x^4 + 1)\*(b\*arccsch(c\*x) + a)/(c^4\*x^5 - x), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{-c^4 x^4 + 1} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(-c^4\*x^4+1)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(sqrt(-c^4\*x^4 + 1)\*x), x)

**maple** [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x/(-c^4\*x^4+1)^(1/2), x)

[Out] int((a+b\*arccsch(c\*x))/x/(-c^4\*x^4+1)^(1/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}a\left(\log\left(\sqrt{-c^4x^4+1}+1\right)-\log\left(\sqrt{-c^4x^4+1}-1\right)\right)+b\int\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+\frac{1}{cx}\right)}{\sqrt{-(c^2x^2+1)(cx+1)(cx-1)}x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x/(-c^4\*x^4+1)^(1/2), x, algorithm="maxima")

[Out] -1/4\*a\*(log(sqrt(-c^4\*x^4 + 1) + 1) - log(sqrt(-c^4\*x^4 + 1) - 1)) + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(sqrt(-(c^2\*x^2 + 1)\*(c\*x + 1)\*(c\*x - 1))\*x), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x\sqrt{1 - c^4x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x\*(1 - c^4\*x^4)^(1/2)), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x\*(1 - c^4\*x^4)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x/(-c\*\*4\*x\*\*4+1)\*\*(1/2), x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*sqrt(-(c\*x - 1)\*(c\*x + 1)\*(c\*\*2\*x\*\*2 + 1))), x)

$$3.178 \quad \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

**Optimal.** Leaf size=29

$$\operatorname{Int}\left(\frac{a + b \operatorname{csch}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}}, x\right)$$

[Out] Unintegrable((a+b\*arccsch(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x)

**Rubi [A]** time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$ , Rules used = {}

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b\*ArcCsch[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

[Out] Defer[Int][(a + b\*ArcCsch[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

Rubi steps

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx = \int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

**Mathematica [A]** time = 3.49, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{csch}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b\*ArcCsch[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

[Out] Integrate[(a + b\*ArcCsch[c\*x])/(x^5\*Sqrt[1 - c^4\*x^4]), x]

**fricas [A]** time = 0.75, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-c^4 x^4 + 1}(b \operatorname{arcsch}(cx) + a)}{c^4 x^9 - x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x, algorithm="fricas")

[Out] integral(-sqrt(-c^4\*x^4 + 1)\*(b\*arccsch(c\*x) + a)/(c^4\*x^9 - x^5), x)

**giac [A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{-c^4 x^4 + 1} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x, algorithm="giac")

[Out] integrate((b\*arccsch(c\*x) + a)/(sqrt(-c^4\*x^4 + 1)\*x^5), x)

**maple** [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^5 \sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*arccsch(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x)

[Out] int((a+b\*arccsch(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x)

**maxima** [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8} \left( c^4 \log(\sqrt{-c^4 x^4 + 1} + 1) - c^4 \log(\sqrt{-c^4 x^4 + 1} - 1) + \frac{2\sqrt{-c^4 x^4 + 1}}{x^4} \right) a + b \int \frac{\log\left(\sqrt{\frac{1}{c^2 x^2} + 1} + \frac{1}{cx}\right)}{\sqrt{-(c^2 x^2 + 1)(cx + 1)(cx - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*arccsch(c\*x))/x^5/(-c^4\*x^4+1)^(1/2), x, algorithm="maxima")

[Out] -1/8\*(c^4\*log(sqrt(-c^4\*x^4 + 1) + 1) - c^4\*log(sqrt(-c^4\*x^4 + 1) - 1) + 2\*sqrt(-c^4\*x^4 + 1)/x^4)\*a + b\*integrate(log(sqrt(1/(c^2\*x^2) + 1) + 1/(c\*x)))/(sqrt(-(c^2\*x^2 + 1)\*(c\*x + 1)\*(c\*x - 1))\*x^5), x)

**mupad** [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*asinh(1/(c\*x)))/(x^5\*(1 - c^4\*x^4)^(1/2)), x)

[Out] int((a + b\*asinh(1/(c\*x)))/(x^5\*(1 - c^4\*x^4)^(1/2)), x)

**sympy** [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^5 \sqrt{-(cx - 1)(cx + 1)(c^2 x^2 + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*acsch(c\*x))/x\*\*5/(-c\*\*4\*x\*\*4+1)\*\*(1/2), x)

[Out] Integral((a + b\*acsch(c\*x))/(x\*\*5\*sqrt(-(c\*x - 1)\*(c\*x + 1)\*(c\*\*2\*x\*\*2 + 1))), x)





# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp,Log,
    Sin,Cos,Tan,Cot,Sec,Csc,
    ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
    Sinh,Cosh,Tanh,Coth,Sech,Csch,
    ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
  },func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  },func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
  ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`' or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```



```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print(">>>>Enter expnType, expn=", expn)
        print(">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands(
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```

```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```